8 Arithmetic Progressions

Exercise 8.1

Question 1. In which of the following situations, does the list of numbers involved make an arithmet ic progression, and why?

(i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs ₹150 for the first metre and rises by ₹ 50 for each subsequent metre.

(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.

Solution (i) According to question, the fare for journey of first km *i.e.,* 1 km is ₹ 15 and next 2 km, 3 km, 4 km ... are respectively ₹ (15 + 8), ₹ (15 + 2 × 8), ₹ (15 + 3 × 8), ... so on.

i.e., 15, 23, 31, 39, ...

Here, each term is obtained by adding 8 to the preceding term except first term. So, it is an AP.

(ii) Let the amount of air present in the cylinder be y units.

So, according to question, the terms giving the air present in the cylinder is given by

$$y,\ y-\frac{y}{4}=\frac{3y}{4},\ \frac{3y}{4}-\frac{1}{4}\times\frac{3y}{4}=\frac{12y-3y}{16}=\frac{9y}{16},\ \ldots$$

or $$y,\ \frac{3y}{4},\ \frac{9y}{16},\ \ldots$$

Here, $$\frac{3y}{4}-y=-\frac{y}{4} \qquad (\because T_2-T_1)$$

and $$\frac{9y}{16}-\frac{3y}{4}=\frac{9y-12y}{16}=-\frac{3y}{16} \qquad (\because T_3-T_2)$$

$\Rightarrow$ $$\frac{3y}{4}-y\neq\frac{9y}{16}-\frac{3y}{4}$$

$\Rightarrow$ It does not form an AP, because common difference is not same.

(iii) According to question, the cost of digging for the first metre, second metre, third metre and so on are respectively ₹ 150, ₹ (150 + 50), ₹ (200 + 50), ... and so on.

i.e., 150, 200, 250, ...

Here, each term is obtained by adding 50 to the preceding term except first term. So, it is an AP.

(iv) According to question, the amount of money in the account in the first year, second year, third year and so on are respectively

$$10000, 10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^2, ...$$

i.e., $$10000, 10000\times\frac{108}{100}, 10000\times\frac{108}{100}\times\frac{108}{100}, \ldots$$

i.e., 10000, 10800, 11664, ...

Now, $d = 10800 - 10000 = 800$

and $d = 11664 - 10800 = 864$

$\therefore$ $800 \neq 864$

Since, the common difference are not same.

So, it does not form an AP.

Question 2. Write first four terms of the AP, when the first term a and the common difference d are given as follows

(i) $a = 10, d = 10$ (ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$ (iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Solution (i) Given, $a = 10, d = 10$

$t_1 = a, t_2 = a + d, t_3 = a + 2d, t_4 = a + 3d$, ... represents an AP for different values of a and d.

Now $t_1 = 10, t_2 = 10 + 10 = 20, t_3 = 10 + 2\times 10 = 30,$

$t_4 = 10 + 3 \times 10 = 40$

Thus, the first four terms of AP are 10, 20, 30, 40.

(ii) Given $a = -2, d = 0$

Now, $t_1 = a, t_2 = a + d, t_3 = a + 2d, t_4 = a + 3d$ represent first four terms of an AP.

i.e., $t_1 = -2, t_2 = -2 + 0 = -2, t_3 = -2 + 2(0) = -2, t_4 = -2 + 3(0) = -2$

(iii) Given, $a = 4, d = -3$

Now, $t_1 = a, t_2 = a + d, t_3 = a + 2d, t_4 = a + 3d$ represent first four terms of an AP.

i.e., $t_1 = 4, t_2 = 4 - 3 = 1, t_3 = 4 + 2(-3) = 4 - 6 = -2$

$t_4 = 4 + 3(-3) = 4 - 9 = -5$

Thus, the first four terms of an AP are 4, 1, –2, –5.

(iv) Given, $a = -1, d = \frac{1}{2}$

Now $t_1 = a, t_2 = a + d, t_3 = a + 2d, t_4 = a + 3d$ represent first four terms of an AP.

i.e., $t_1 = -1, t_2 = -1 + \frac{1}{2} = -\frac{1}{2}, t_3 = -1 + 2\left(\frac{1}{2}\right) = 0, t_4 = -1 + 3\left(\frac{1}{2}\right) = \frac{1}{2}$

Thus, the first four terms of an AP are $-1, -\frac{1}{2}, 0, \frac{1}{2}$.

(v) Given, $a = -1.25, d = -0.25$

Now, $t_1 = a, t_2 = a + d, t_3 = a + 2d, t_4 = a + 3d$ represent first four terms of an AP.

i.e., $t_1 = -1.25, t_2 = -1.25 - 0.25 = -1.50, t_3 = -1.25 + 2(-0.25)$

$= -1.25 - 0.50$

$= -1.75$

$t_4 = -1.25 + 3(-0.25)$

$= -1.25 - 0.75$

$= -2.00$

Thus, the first four terms of an AP are $-1.25, -1.50, -1.75, -2.00$.

Question 3. For the following APs, write the first term and the common difference

(i) $3, 1, -1, -3, \ldots$ (ii) $-5, -1, 3, 7, \ldots$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$ (iv) $0.6, 1.7, 2.8, 3.9, \ldots$

Solution (i) First term $a = t_1 = 3$,

common difference d = 2nd term – 1st term

$= 1 - 3 = -2$

(ii) First term $a = t_1 = -5$

Common difference d = 2nd term – 1st term $= -1 - (-5) = -1 + 5 = 4$

(iii) First term $a = t_1 = \frac{1}{3}$

Common difference d = 2nd term – 1st term $= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) First term $a = t_1 = 0.6$

Common difference d = 2nd term – 1st term $= 1.7 - 0.6 = 1.1$

Question 4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) $2, 4, 8, 16, \ldots$ (ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

(iii) $-1.2, -3.2, -5.2, -7.2, \ldots$ (iv) $-10, -6, -2, 2, \ldots$

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$

(vi) 0.2, 0.22, 0.222, 0.2222, ... (vii) 0, – 4, – 8, – 12, ...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$ (ix) 1, 3, 9, 27, ...

(x) $a, 2a, 3a, 4a, \ldots$ (xi) $a, a^2, a^3, a^4, \ldots$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ (xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$

(xiv) $1^2, 3^2, 5^2, 7^2, \ldots$ (xv) $1^2, 5^2, 7^2, 73, \ldots$

Solution (i) Here, $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$

$$a_2 - a_1 = 4 - 2 = 2$$
$$a_3 - a_2 = 8 - 4 = 4$$
$$a_4 - a_3 = 16 - 8 = 8$$

Clearly $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an AP, because the common difference is not same.

(ii) Here, $a_1 = 2, a_2 = \frac{5}{2}, a_3 = 3, a_4 = \frac{7}{2}, \ldots$

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$
$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$
$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$$

Clearly, the difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = \frac{1}{2}$

Next three terms of AP are

$$a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$$
$$a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2} = 4.5$$
$$a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = \frac{9+1}{2} = \frac{10}{2} = 5$$

(iii) Here, $a_1 = -1.2, a_2 = -3.2, a_3 = -5.2, a_4 = -7.2$

$$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$$
$$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$$
$$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$$

Clearly, the difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = -2$.

Next three terms of AP are

$$a_5 = a_4 + d = -7.2 + (-2) = -9.2$$
$$a_6 = a_5 + d = -9.2 + (-2) = -11.2$$
$$a_7 = a_6 + d = -11.2 + (-2) = -13.2$$

(iv) $-10, -6, -2, 2, \ldots$

Here, $a_1 = -10, a_2 = -6, a_3 = -2, a_4 = 2$

$$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$$
$$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$$
$$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$$

Clearly, the difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = 4$.

Next three terms of AP are

$$a_5 = a_4 + d = 2 + 4 = 6$$
$$a_6 = a_5 + d = 6 + 4 = 10$$
$$a_7 = a_6 + d = 10 + 4 = 14$$

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$

Here, $a_1 = 3, a_2 = 3 + \sqrt{2}, a_3 = 3 + 2\sqrt{2}, a_4 = 3 + 3\sqrt{2}$

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$
$$a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) = \sqrt{2}$$
$$a_4 - a_3 = 3 + 3\sqrt{2} - (3 + 2\sqrt{2}) = \sqrt{2}$$

Clearly, the difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = \sqrt{2}$.

Next three terms of AP are

$$a_5 = a_4 + d = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$$
$$a_6 = a_5 + d = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$
$$a_7 = a_6 + d = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$$

(vi) $0.2, 0.22, 0.222, 0.2222, \ldots$

Here, $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$

$$a_2 - a_1 = 0.22 - 0.2 = 0.02$$
$$a_3 - a_2 = 0.222 - 0.22 = 0.002$$
$$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$$

Clearly $\quad a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an AP, because the common difference is not same.

(vii) $0, -4, -8, -12, \ldots$

Here, $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

$$a_2 - a_1 = -4 - 0 = -4$$
$$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$$
$$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$$

Clearly, the difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = -4$.

Next three terms of AP are

$$a_5 = a_4 + d = -12 + (-4) = -16$$
$$a_6 = a_5 + d = -16 + (-4) = -20$$
$$a_7 = a_6 + d = -20 + (-4) = -24$$

(viii) Here, $a_1 = -\frac{1}{2}, a_2 = -\frac{1}{2}, a_3 = -\frac{1}{2}, a_4 = -\frac{1}{2}$

$$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$$

Clearly, difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = 0$

Next three terms of AP are

$$a_5 = a_4 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

$$a_6 = a_5 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

$$a_7 = a_6 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$$

(ix) 1, 3, 9, 27, ...

Here, $a_1 = 1, a_2 = 3, a_3 = 9, a_4 = 27$

$$a_2 - a_1 = 3 - 1 = 2$$

$$a_3 - a_2 = 9 - 3 = 6$$

$$a_4 - a_3 = 27 - 9 = 18$$

Clearly, $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an AP, because the common difference is not same.

(x) $a, 2a, 3a, 4a, ...$

Here, $a_1 = a, a_2 = 2a, a_3 = 3a, a_4 = 4a$

$$a_2 - a_1 = 2a - a = a$$

$$a_3 - a_2 = 3a - 2a = a$$

$$a_4 - a_3 = 4a - 3a = a$$

Clearly, difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = a$

Next three terms of AP are

$$a_5 = a_4 + d = 4a + a = 5a$$

$$a_6 = a_5 + d = 5a + a = 6a$$

$$a_7 = a_6 + d = 6a + a = 7a$$

(xi) $a, a^2, a^3, a^4, ...$

Here, $a_1 = a, a_2 = a^2, a_3 = a^3, a_4 = a^4$

$$a_2 - a_1 = a^2 - a = a(a-1)$$

$$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$$

$$a_4 - a_3 = a^4 - a^3 = a^3(a-1)$$

Clearly, $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an AP, because the common difference is not same.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

Here, $a_1 = \sqrt{2}, a_2 = \sqrt{8} = 2\sqrt{2}, a_3 = \sqrt{18}$

$= 3\sqrt{2}, a_4 = \sqrt{32} = 4\sqrt{2}$

$$a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$
$$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$
$$a_4 - a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$$

Clearly, difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = \sqrt{2}$

Next three terms of AP are

$$a_5 = a_4 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$$
$$a_6 = a_5 + d = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$$
$$a_7 = a_6 + d = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

Here, $a_1 = \sqrt{3}, a_2 = \sqrt{6}, a_3 = \sqrt{9}, a_4 = \sqrt{12}$

$$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3 \times 2} - \sqrt{3} = \sqrt{3}\,(\sqrt{2} - 1)$$

and $\quad a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{3 \times 2} = \sqrt{3}\,(\sqrt{3} - \sqrt{2})$

$$a_4 - a_3 = \sqrt{12} - \sqrt{9}$$
$$= \sqrt{4 \times 3} - \sqrt{9}$$
$$= 2\sqrt{3} - 3$$
$$= \sqrt{3}\,(2 - \sqrt{3})$$

Clearly, $\quad a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an AP, because the common difference is not same.

(xiv) $1^2, 3^2, 5^2, 7^2, ...$

Here, d$a_1 = 1^2 = 1, a_2 = 3^2 = 9,$

$$a_3 = 5^2 = 25, a_4 = 7^2 = 49$$
$$a_2 - a_1 = 9 - 1 = 8$$
$$a_3 - a_2 = 25 - 9 = 16$$
$$a_4 - a_3 = 49 - 25 = 24$$

Clearly, $\quad a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers does not form an AP, because the common difference is not same.

(xv) $1^2, 5^2, 7^2, 73$

Here, $a_1 = 1^2 = 1, a_2 = 5^2 = 25, a_3 = 7^2 = 49, a_4 = 73$

$$a_2 - a_1 = 25 - 1 = 24$$
$$a_3 - a_2 = 49 - 25 = 24$$
$$a_4 - a_3 = 73 - 49 = 24$$

Clearly, difference of successive terms is constant, therefore list of numbers form an AP. So, common difference, $d = 24$

Next three terms of AP are

$$a_5 = a_4 + d = 73 + 24 = 97$$
$$a_6 = a_5 + d = 97 + 24 = 121$$
$$a_7 = a_6 + d = 121 + 24 = 145$$

8 Arithmetic Progressions

Exercise 18.2

Question 1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the nth term of the AP

	a	d	n	a_n
(i)	7	3	8	...
(ii)	– 18	...	10	0
(iii)	...	–3	18	–5
(iv)	– 18.9	2.5	...	3.6
(v)	3.5	0	105	...

Solution (i) The n th term of an AP is $a_n = a + (n-1)d$

$$= 7 + (8-1)\,3 = 7 + 7 \times 3 = 7 + 21 = 28$$

(ii) The nth term of an AP is $a_n = a + (n-1)d$

$\Rightarrow$ $0 = -18 + (10-1)\,d$

$\Rightarrow$ $18 = 9d \Rightarrow d = \frac{18}{9} = 2$

(iii) The n th term of an AP is $a_n = a + (n-1)d$

$\Rightarrow$ $-5 = a + (18-1)(-3)$

$\Rightarrow$ $-5 = a + 17(-3)$

$\Rightarrow$ $-5 = a - 51$

$\Rightarrow$ $a = -5 + 51 = 46$

(iv) The n th term of an AP is $a_n = a + (n-1)d$

$\Rightarrow$ $3.6 = -18.9 + (n-1)\,2.5$

$3.6 + 18.9 = (n-1)\,2.5$

$\Rightarrow$ $22.5 = (n-1)\,2.5$

$\Rightarrow$ $n - 1 = \frac{22.5}{2.5}$

$\Rightarrow$ $n - 1 = 9$

$\Rightarrow$ $n = 9 + 1 = 10$

(v) The n th term of an AP is $a_n = a + (n-1)d = 3.5 + (105-1) \times 0$

$= 3.5 + 0 = 3.5$

Question 2. Choose the correct choice in the following and justify.

(i) 30th term of the AP 10, 7, 4, ... , is

(a) 97 (b) 77

(c) – 77 (d) – 87

(ii) 11th term of the AP $-3, -\frac{1}{2}, 2, \ldots$ is

(a) 28 (b) 22

(c) -38 (d) $-48\frac{1}{2}$

Solution (i) (c) Here, first term $a = 10$, common difference, $d = 7 - 10 = -3$, number of terms $n = 30$

Since, the nth term of an AP is $a_n = a + (n-1)d$

$$\Rightarrow \quad a_{30} = 10 + (30-1)(-3)$$
$$= 10 + 29(-3) = 10 - 87 = -77$$

(ii) (b) Here, $a = -3$, $d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$

Since, the nth term of an AP is

$$a_n = a + (n-1)d$$
$$\therefore \quad a_{11} = -3 + (11-1)\frac{5}{2}$$
$$= -3 + 10 \times \frac{5}{2}$$
$$= -3 + 25 = 22$$

Question 3. In the following APs, find the missing terms in the boxes.

(i) 2, ☐, 26 (ii) ☐, 13, ☐, 3

(iii) 5, ☐, ☐, $9\frac{1}{2}$ (iv) -4, ☐, ☐, ☐, ☐, 6

(v) ☐, 38, ☐, ☐, ☐, -22

Solution (i) Let 2, ☐, 26 be a, $a + d$ and $a + 2d$ respectively are in AP.

$$\therefore \quad a = 2 \text{ and } a + 2d = 26$$
$$\Rightarrow \quad 2 + 2d = 26 \qquad (\because a = 2)$$
$$\Rightarrow \quad 2d = 26 - 2 = 24$$
$$\Rightarrow \quad d = \frac{24}{2} = 12$$

Hence, the missing term $= a + d = 2 + 12 = 14$

(ii) Let ☐, 13, ☐, 3 be a, $a + d$, $a + 2d$ and $a + 3d$ respectively are in AP.

$$\therefore \quad a + d = 13 \qquad \ldots(i)$$
$$\text{and} \quad a + 3d = 3 \qquad \ldots(ii)$$

On subtracting Eq. (i) from Eq. (ii), we get

$$2d = -10 \Rightarrow d = -5$$

Put, $d = -5$ in Eq. (ii), we get

$$a + 3(-5) = 3$$
$$a - 15 = 3$$
$$\Rightarrow \quad a = 3 + 15 = 18$$

Hence, the missing term are $a = 18$ and $a + 2d = 18 + 2(-5)$

$$= 18 - 10 = 8$$

(iii) Let 5, $\square$, $\square$, $9\frac{1}{2}$ be $a, a + d, a + 2d$ and $a + 3d$ respectively are in AP.

$\therefore \quad a = 5 \quad$ ...(i)

and $\quad a + 3d = 9\frac{1}{2} = \frac{19}{2} \quad$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$3d = \frac{19}{2} - 5 = \frac{19 - 10}{2} = \frac{9}{2}$$

$$\Rightarrow \quad 3d = \frac{9}{2}$$

$$\Rightarrow \quad d = \frac{1}{3} \times \frac{9}{2} = \frac{3}{2}$$

Hence, the missing terms are $a + d = 5 + \frac{3}{2} = \frac{10 + 3}{2} = \frac{13}{2} = 6\frac{1}{2}$

and $\quad a + 2d = 5 + 2 \times \frac{3}{2} = 5 + 3 = 8$

(iv) Let -4, $\square$, $\square$, $\square$, $\square$, 6 be $a, a + d, a + 2d, a + 3d$, $a + 4d$ and $a + 5d$ respectively are in AP.

$\therefore \quad a = -4 \quad$ ...(i)

and $\quad a + 5d = 6 \quad$ ...(ii)

On putting $a = -4$ in Eq. (ii), we get

$$-4 + 5d = 6$$

$$\Rightarrow \quad 5d = 6 + 4 = 10$$

$$\Rightarrow \quad d = \frac{10}{5} = 2$$

Hence, the missing terms are

$$a + d = -4 + 2 = -2$$

$$a + 2d = -4 + 2 \times 2 = -4 + 4 = 0$$

$$a + 3d = -4 + 3 \times 2 = -4 + 6 = 2$$

and $\quad a + 4d = -4 + 4 \times 2 = -4 + 8 = 4$

(v) Let $\square$, 38, $\square$, $\square$, $\square$, -22 be $a, a + d, a + 2d, a + 3d, a + 4d$ and $a + 5d$ respectively are in AP.

$\therefore \quad a + d = 38 \quad$ ...(i)

and $\quad a + 5d = -22 \quad$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$4d = -60 \Rightarrow d = -\frac{60}{4} = -15$$

On putting $d = -15$ in Eq. (i), we get

$$a + (-15) = 38 \Rightarrow a - 15 = 38$$

$$\Rightarrow \quad a = 38 + 15 = 53$$

Hence, the missing terms are $a = 53$

$$a + 2d = 53 + 2 \times (-15) = 53 - 30 = 23$$

$$a + 3d = 53 + 3 \times (-15) = 53 - 45 = 8$$

and $\quad a + 4d = 53 + 4 \times (-15) = 53 - 60 = -7$

Question 4. Which term of the AP 3, 8, 13, 18, ... is 78?

Solution Let n th term be 78.

Given, 3, 8, 13, 18 .... are in AP.

First term, $a = 3$, common difference, $d = 8 - 3 = 5$

$$\because \quad n\text{th term } a_n = 78$$
$$\therefore \quad a + (n-1)d = 78$$
$$\Rightarrow \quad 3 + (n-1)5 = 78$$
$$\Rightarrow \quad (n-1)5 = 78 - 3$$
$$\Rightarrow \quad 5(n-1) = 75$$
$$\Rightarrow \quad n - 1 = 15$$
$$\Rightarrow \quad n = 15 + 1$$
$$\Rightarrow \quad n = 16$$

Hence, 16th term be 78.

Question 5. Find the number of terms in each of the following APs

(i) 7, 13, 19, ..., 205 (ii) 18, $15\frac{1}{2}$, 13, ..., – 47

Solution (i) Suppose, there are n terms in the given AP. Then, n th term $a_n = 205$, first term $a = 7$, common difference $d = 13 - 7 = 6$

$$\because \quad a + (n-1)d = a_n$$
$$\therefore \quad a + (n-1)d = 205$$
$$\Rightarrow \quad 7 + (n-1)6 = 205$$
$$\Rightarrow \quad 6(n-1) = 205 - 7$$
$$\Rightarrow \quad 6(n-1) = 198$$
$$\Rightarrow \quad n - 1 = \frac{198}{6} = 33$$
$$\Rightarrow \quad n = 33 + 1 = 34$$

Hence, the given AP contains 34 terms.

(ii) Suppose, there are n terms in the given AP.

Given, first term $a = 18$, common difference $d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18$

$$= \frac{31 - 36}{2} = \frac{-5}{2}$$

Then, nth term $a_n = -47$

$$\Rightarrow \quad a + (n-1)\left(\frac{-5}{2}\right) = -47 \qquad [\because a + (n-1)d = a_n]$$
$$\Rightarrow \quad 18 + (n-1)\left(\frac{-5}{2}\right) = -47$$
$$\Rightarrow \quad \left(\frac{-5}{2}\right)(n-1) = -47 - 18 = -65$$
$$\Rightarrow \quad (n-1) = -65 \times \frac{-2}{5} = -13 \times -2 = 26$$
$$\Rightarrow \quad n = 26 + 1 = 27$$

Hence, the given AP contains 27 terms.

Question 6. Check whether – 150 is a terms of the AP 11, 8, 5, 2, ...

Solution Here, $a_1 = 11, a_2 = 8, a_3 = 5, a_4 = 2$

$$a_2 - a_1 = 8 - 11 = -3$$
$$a_3 - a_2 = 5 - 8 = -3$$
$$a_4 - a_3 = 2 - 5 = -3$$

Clearly, the successive difference of terms is constant. So, the list of numbers are in AP. Hence, common difference $d = -3$.

Let – 150 be the nth term of the given AP.

We know that, the nth term of an AP is $a_n = a_1 + (n-1)d$

$$\Rightarrow \quad -150 = 11 + (n-1)(-3)$$
$$\Rightarrow \quad -3(n-1) = -150 - 11 = -161$$
$$\Rightarrow \quad n - 1 = \frac{161}{3}$$
$$\Rightarrow \quad n = \frac{161}{3} + 1 = \frac{164}{3}$$

But n should be positive integer. So, our assumption was wrong and so – 150 is not a term of the given AP.

Question 7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Solution Let a be the first term and d the common difference of an AP.

Now, the nth term of an AP is $a_n = a + (n-1)d$

$$a_{11} = a + 10d = 38 \quad [\because a_{11} = 38 \text{ (given)}] \quad \ldots\text{(i)}$$

and $$a_{16} = a + 15d = 73 \quad [\because a_{16} = 73 \text{ (given)}] \quad \ldots\text{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$5d = 35 \Rightarrow d = \frac{35}{5} = 7$$

From Eq. (i), $a + 10 \times 7 = 38$

$$\Rightarrow \quad a = 38 - 70 = -32$$

$\therefore$ The 31st term of an AP $a_{31} = a + 30d$

$$= -32 + 30 \times 7$$
$$= -32 + 210 = 178$$

Question 8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Solution Let a be the first term and d the common difference

Now, the nth term of an AP $a_n = a + (n-1)d$

$\therefore$ $$a_3 = a + 2d = 12 \quad [\because a_3 = 50 \text{ (given)}] \quad \ldots\text{(i)}$$

and $$a_{50} = a + 49d = 106 \quad [\because a_{50} = 106 \text{ (given)}] \quad \ldots\text{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$47\,d = 94$$

$$\Rightarrow \quad d = \frac{94}{47} = 2$$

$\therefore$ From Eq. (i) $\quad a + 2 \times 2 = 12$

$$\Rightarrow \quad a = 12 - 4 = 8$$

$\therefore$ The 29th term of an AP $\quad a_{29} = a + (29 - 1)\,d$

$$= 8 + 28 \times 2$$

$$= 8 + 56 = 64$$

Question 9. If the 3rd and 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Solution Let a be the first term and d the common difference of an AP.

$\because$ The n th term of an AP is

$$a_n = a + (n - 1)\,d$$

$$\therefore \quad a_3 = a + 2d = 4 \quad [\because a_3 = 4 \text{ (given)}] \quad \ldots\text{(i)}$$

$$\text{and} \quad a_9 = a + 8d = -8 \quad [\because a_9 = -8 \text{ (given)}] \quad \ldots\text{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$6d = -12 \Rightarrow d = \frac{-12}{6} = -2$$

$\therefore$ From Eq. (i), $\quad a + 2 \times (-2) = 4$

$$\Rightarrow \quad a - 4 = 4 \;\Rightarrow\; a = 4 + 4 = 8$$

Let the nth term of an AP is zero.

Then $\quad a_n = 0 \Rightarrow a + (n - 1)\,d = 0$

$$\Rightarrow \quad 8 + (n - 1)(-2) = 0$$

$$\Rightarrow \quad (n - 1)(-2) = -8$$

$$\Rightarrow \quad n - 1 = \frac{-8}{-2} = 4 \Rightarrow n = 4 + 1 = 5$$

Hence, 5th term of an AP is zero.

Question 10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solution Let a be the first term and d the common difference of an AP.

Given, $\quad a_{17} - a_{18} = 7$

$$(a + 16d) - (a + 9d) = 7 \quad [\because a_n = a + (n - 1)d]$$

$$\Rightarrow \quad 7d = 7 \;\Rightarrow\; d = 1$$

Hence, the common difference of an AP is 1.

Question 11. Which term of the AP 3, 15, 27, 39, ... will be 132 more than its 54th term?

Solution Here, first term $a = 3$, common difference $d = 15 - 3 = 12$

Then, $a_{54} = a + 53\,d = 3 + 53 \times 12$ $[\because a_n = a + (n-1)d]$

$= 3 + 636 = 639$

Let a_n be 132 more than its 54th term.

Then $a_n = a_{54} + 132$ (By given condition)

$a_n = 639 + 132$

$a_n = 771$

$\Rightarrow$ $a + (n-1)\,d = 771$ $[\because a_n = a + (n-1)d]$

$\Rightarrow$ $3 + (n-1)\,12 = 771$

$\Rightarrow$ $12\,(n-1) = 771 - 3$

$\Rightarrow$ $12\,(n-1) = 768$

$\Rightarrow$ $n - 1 = \dfrac{768}{12} = 64$

$\Rightarrow$ $n = 65$

Hence, 65th term is 132 more than its 54th term of an AP.

Question 12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Solution Let the two APs be $a_1, a_2, a_3, \ldots.. a_n$ and $b_1, b_2, b_3, \ldots. b_n$

Also, let d be the same common difference of two APs, then

the nth term of first AP $a_n = a_1 + (n-1)\,d$

and the nth term of second AP $b_n = b_1 + (n-1)\,d$

Now, $a_n - b_n = [a_1 + (n-1)\,d] - [b_1 + (n-1)\,d]$

$\Rightarrow$ $a_n - b_n = a_1 - b_1$ for all $n \in N$

$\Rightarrow$ $a_{100} - b_{100} = a_1 - b_1 = 100$ (Given)

$\therefore$ $a_{1000} - b_{1000} = a_1 - b_1$

$\Rightarrow$ $a_{1000} - b_{1000} = 100$ $[\because a_1 - b_1 = 100]$

Hence, the difference between their 1000th terms is also 100 for all $n \in N$.

Question 13. Have many three digit numbers are divisible by 7?

Solution We know that, 105 is the first and 994 is the last 3 digit number divisible by 7. Thus, we have to determine the number of terms in the list 105, 112, 119, ... ,994.

Clearly, the successive difference of terms is constant with common difference

$$d = 112 - 105 = 7$$

So, it forms an AP.

Let there be n terms in the AP, then nth term $= 994$

$$\because \quad a_n = a + (n-1)d$$

$$\Rightarrow \quad 105 + (n-1)7 = 994$$

$$\Rightarrow \quad 7(n-1) = 994 - 105$$

$$\Rightarrow \quad 7(n-1) = 889$$

$$\Rightarrow \quad n - 1 = \frac{889}{7}$$

$$= 127$$

$$\Rightarrow \quad n = 127 + 1 = 128$$

So, there are 128 numbers of three digit which are divisible by 7.

Question 14. How many multipliers of 4 lie between 10 and 250?

Solution We see that 12 is the first integer between 10 and 250, which is a multiple of 4. Also, when we divide 250 by 4, the remainder is 2. Therefore, 250 – 2 = 248 is the greatest integer divisible by 4 and lying between 10 and 250. Thus, we have to find the number of terms in an AP whose first term = 12, last term = 248 and common difference = 4.

Let the n term of the AP, is

$$a_n = 248$$

$$\Rightarrow \quad 12 + (n-1)4 = 248 \qquad [\because a_n = a + (n-1)d]$$

$$\Rightarrow \quad 4(n-1) = 248 - 12$$

$$\Rightarrow \quad 4(n-1) = 236$$

$$\Rightarrow \quad n - 1 = \frac{236}{4} = 59$$

$$\Rightarrow \quad n = 59 + 1 = 60$$

Hence, there are 60 multiples of 4 lie between 10 and 250.

Question 15. For what value of n, are the nth terms of the AP's 63, 65, 67, ... and 3, 10, 17 are equal?

Solution If nth terms of the AP's 63, 65, 67, ... and 3, 10, 17, ... are equal

Here, first term of first AP $(a_1) = 63$

Common difference of first AP $(d_1) = 65 - 63 = 2$

and first term of second AP $(b_1) = 3$

Common difference of second AP $(d_2) = 10 - 3 = 7$

Then by condition nth term of both AP's are equal.

$$\therefore \quad 63 + (n-1)2 = 3 + (n-1)7$$

$$\Rightarrow \quad 7(n-1) - 2(n-1) = 63 - 3 \qquad [\because a_n = a + (n-1)d]$$

$$\Rightarrow \quad (n-1)(7-2) = 60 \Rightarrow 5(n-1) = 60$$

$$\Rightarrow \quad (n-1) = \frac{60}{5} = 12$$

$$\Rightarrow \quad n = 12 + 1 = 13$$

Hence, the 13th terms of the two given AP's are same.

Question 16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution Let a be the first term and d the common difference of an AP.
Given that, the third term of the AP, $a_3 = 16$
and 7th term of an AP = 12 + 5th term of an AP

$$\Rightarrow \quad a_7 = 12 + a_5$$
$$\Rightarrow \quad a_7 - a_5 = 12$$
$$\Rightarrow \quad a + 2d = 16 \quad [\because a_n = a + (n-1)d] \quad \ldots(i)$$
$$\text{and} \quad (a + 6d) - (a + 4d) = 12$$
$$\Rightarrow \quad 2d = 12$$
$$\Rightarrow \quad d = 6 \quad \ldots(ii)$$

On putting $d = 6$ in Eq. (i), we get

$$a + 2 \times 6 = 16$$
$$\Rightarrow \quad a = 16 - 12 = 4$$

Since, the terms of an AP in the form $a, a + d, a + 2d, a + 3d, \ldots$
Then, the AP is $4, 4 + 6, 4 + 2 \times 6, 4 + 3 \times 6, \ldots$
i.e., 4, 10, 16, 22, ...

Question 17. Find the 20th term from the last term of the AP 3, 8, 13, ... 253.

Solution Given, l = last term = 253
d = common difference = 8 − 3 = 5

$$\therefore \text{20th term from the end} = l - (20-1)d = l - 19d = 253 - 19 \times 5 \quad (\because n = 20)$$
$$= 253 - 95 = 158$$

Question 18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Solution Let a be the first term and d the common difference of an AP.

$$\text{Given,} \quad a_4 + a_8 = 24 \quad \text{(By condition)}$$
$$\Rightarrow \quad (a + 3d) + (a + 7d) = 24 \quad [\because a_n = a + (n-1)d]$$
$$\Rightarrow \quad 2a + 10d = 24$$
$$\Rightarrow \quad a + 5d = 12 \quad \ldots(i)$$
$$\text{and} \quad a_6 + a_{10} = 44 \quad \text{(By condition)}$$
$$\Rightarrow \quad (a + 5d) + (a + 9d) = 44$$
$$\Rightarrow \quad 2a + 14d = 44$$
$$\Rightarrow \quad a + 7d = 22 \quad \ldots(ii)$$

On subtracting Eq. (i) from Eq. (ii), we get

$$2d = 10 \Rightarrow d = 5$$

$\therefore$ From Eq. (i), we get

$$a + 25 = 12$$
$$\Rightarrow \quad a = -13$$

Hence, the first three terms are $a, (a + d), (a + 2d)$
i.e., $-13, (-13 + 5)$ and $(-13 + 2 \times 5)$
i.e., $-13, -8$ and -3.

Question 19. Subha Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

Solution The annual salary received by Subha Rao in the years 1995, 1996, 1997 etc., is ₹ 5000, ₹ 5200, ₹ 5400, ..., ₹ 7000

Hence, the list of numbers 5000, 5200, 5400, ..., 7000 forms an AP

$\because \quad a_2 - a_1 = a_3 - a_2 = 200$

Let nth term of an AP $\quad a_n = 7000$

$\Rightarrow \quad 7000 = a + (n-1)\,d \quad [\because a_n = a + (n-1)\,d]$

$\Rightarrow \quad 7000 = 5000 + (n-1)\,(200)$

$\Rightarrow \quad 200\,(n-1) = 7000 - 5000 = 2000$

$\Rightarrow \quad n - 1 = \dfrac{2000}{200} = 10$

$\Rightarrow \quad n = 10 + 1 = 11$

Thus, 11th year of his service or in 2005 Subha Rao received an annual salary ₹ 7000.

Question 20. Ramkali saves ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly saving becoms ₹ 20.75. Find n.

Solution Ramkali's savings in the subsequent weeks are respectively ₹ 5, ₹ 5 + ₹ 1.75, ₹ 5 + 2 × ₹ 1.75, ₹ 5 + 3 × ₹ 1.75, ...

In nth week her saving will be ₹ 5 + $(n-1)$ × ₹ 1.75

$\Rightarrow \quad 5 + (n-1) \times 1.75 = 20.75 \quad$ (Given)

$\Rightarrow \quad (n-1) \times 1.75 = 20.75 - 5 = 15.75$

$\Rightarrow \quad n - 1 = \dfrac{15.75}{1.75} = 9$

$\Rightarrow \quad n = 9 + 1 = 10$

8 Arithmetic Progressions

Exercise 8.3

Question 1. Find the sum of the following AP's

(i) 2, 7, 12, ..., to 10 terms

(ii) – 37, – 33, – 29, ..., to 12 terms

(iii) 0.6, 1.7, 2.8, ... to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots,$ to 11 terms

Solution (i) Let a be the first term and d be the common difference of the given AP.

Then, we have $a = 2$ and $d = 7 - 2 = 5$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

Putting $a = 2, d = 5, n = 10$, we get

$$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5] = 5(4 + 9 \times 5)$$
$$= 5(4 + 45) = 5 \times 49 = 245$$

(ii) Let a be the first term and d be the common difference of the given AP.

Then, we have $a = -37, d = -33 - (-37) = -33 + 37 = 4$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$S_{12} = \frac{12}{2}[2 \times (-37) + (12-1)4]$$
$$= 6(-74 + 11 \times 4)$$
$$= 6(-74 + 44)$$
$$= 6 \times (-30)$$
$$= -180$$

(iii) Let a be the first term and d be the common difference of the given AP.

Then, we have $a = 0.6, d = 1.7 - 0.6 = 1.1$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$S_{100} = \frac{100}{2}[2 \times 0.6 + (100-1)1.1]$$
$$= 50(1.2 + 99 \times 1.1)$$
$$= 50(1.2 + 108.9)$$
$$= 50 \times 110.1$$
$$= 5505$$

(iv) Let a be the first term and d be the common difference of the given AP.

Then, we have $a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

Putting, $a = \frac{1}{15}, d = \frac{1}{60}, n = 11$ in $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

Sum of 11 terms of an AP, $S_{11} = \frac{11}{2}\left[2 \times \frac{1}{15} + (11-1)\frac{1}{60}\right]$

$$= \frac{11}{2}\left(\frac{2}{15} + 10 \times \frac{1}{60}\right)$$

$$= \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right)$$

$$= \frac{11}{2} \times \frac{4+5}{30} = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$$

Question 2. Find the sums given below

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$ (ii) $34 + 32 + 30 + \ldots. + 10$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

Solution (i) $7 + 10\frac{1}{2} + 14 + \ldots. + 84$ since, the last term is given.

Here, first term $a = 7$, common difference $d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$

and last term $l = a_n = 84$

$$84 = a + (n-1)d \qquad [\because a_n = a + (n-1)d]$$

$$\Rightarrow \quad 84 = 7 + (n-1)\frac{7}{2}$$

$$\Rightarrow \quad \frac{7}{2}(n-1) = 84 - 7$$

$$\Rightarrow \quad \frac{7}{2}(n-1) = 77$$

$$\Rightarrow \quad n - 1 = 77 \times \frac{2}{7}$$

$$\Rightarrow \quad n - 1 = 22$$

$$\Rightarrow \quad n = 23$$

$\because$ Sum of n terms of an AP, $S_n = \frac{n}{2}(a + l)$

$$\Rightarrow \quad S_{23} = \frac{23}{2}(7 + 84)$$

$$= \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$$

(ii) $34 + 32 + 30 + ... + 10$

Since, the last term is given.

Here, first term $a = 34$, common difference $d = 32 - 34 = -2$,

last term $l = a_n = 10$

$\therefore \quad 10 = a + (n-1)d \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 10 = 34 + (n-1)(-2)$

$\Rightarrow \quad (-2)(n-1) = 10 - 34$

$\Rightarrow \quad (-2)(n-1) = -24$

$\Rightarrow \quad n - 1 = 12$

$\Rightarrow \quad n = 12 + 1 = 13$

By sum of n terms of an AP, $S_n = \frac{n}{2}(a + l)$, we get

$$S_{13} = \frac{13}{2}(34 + 10)$$

$$= \frac{13}{2} \times 44$$

$$= 13 \times 22 = 286$$

(iii) $-5 + (-8) + (-11) + ... + (-230)$

Since, the last term is given,

Here, first term $a = -5$, common difference $d = -8 - (-5) = -8 + 5 = -3$,
last term $l = a_n = -230$

$\therefore \quad -230 = a + (n-1)d \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad -230 = -5 + (n-1)(-3)$

$\Rightarrow \quad (-3)(n-1) = -230 + 5$

$\Rightarrow \quad (-3)(n-1) = -225$

$\Rightarrow \quad n - 1 = \frac{-225}{-3}$

$\Rightarrow \quad n - 1 = 75$

$\Rightarrow \quad n = 75 + 1 = 76$

$\therefore$ By sum of n terms of an AP, $S_n = \frac{n}{2}(a + l)$, we get

$$S_{76} = \frac{76}{2}(-5 - 230)$$

$$= 38 \times -235 = -8930$$

Question 3. In an AP

(i) given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(ii) given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) given $a = 2$, $d = 8$, $S_n = 90$, find n and a_5.

(vii) given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) given $a = 3$, $n = 8$, $S = 192$, find d.

(x) given $l = 28$, $S = 144$, and there are total 9 terms, find a.

Solution (i) Here, $a = 5$, $d = 3$ and $a_n = 50$

$\Rightarrow \quad a + (n-1)d = 50 \qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 5 + (n-1)3 = 50$

$\Rightarrow \quad 3(n-1) = 50 - 5$

$\Rightarrow \quad (n-1) = \frac{45}{3} = 15$

$\Rightarrow \quad n = 15 + 1 = 16$

Putting, $n = 16$, $a = 5$ and $l = a_n = 50$ in $S_n = \frac{n}{2}(a + l)$, we get

$$S_{16} = \frac{16}{2}(5 + 50)$$

$$= 8 \times 55 = 440$$

So, $n = 16$ and $S_{16} = 440$

(ii) Here, $a = 7$ and $a_{13} = 35$

Let d be the common difference of the given AP.

Then, $a_{13} = 35 \Rightarrow a + 12d = 35 \qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 7 + 12d = 35 \qquad (\because a = 7)$

$\Rightarrow \quad 12d = 35 - 7 = 28$

$\Rightarrow \quad d = \frac{28}{12} = \frac{7}{3}$

Putting, $n = 13$, $a = 7$ and $l = a_{13} = 35$ in $S_n = \frac{n}{2}(a + l)$, we get

$$S_{13} = \frac{13}{2}(7 + 35) = \frac{13}{2} \times 42$$

$$= 13 \times 21 = 273$$

Hence, $d = \frac{7}{3}$ and $S_{13} = 273$

(iii) Here, $a_{12} = 37$, $d = 3$

Let a be the first term of given AP, then

$$a_{12} = 37 \qquad [\because a_n = a + (n-1)d]$$

$$\Rightarrow \quad a + 11d = 37 \qquad (\because d = 3)$$

$$\Rightarrow \quad a + 11(3) = 37$$

$$\Rightarrow \quad a = 37 - 33 = 4$$

Putting, $n = 12, a = 4$ and $l = a_{12} = 37$ in $S_n = \frac{n}{2}(a + l)$, we get

$$S_{12} = \frac{12}{2}(4 + 37) = 6 \times 41 = 246$$

Hence, $a = 4$

and $S_{12} = 246$

(iv) Here, $a_3 = 15$, $S_{10} = 125$

Let a be the first term and d the common difference of the given AP.

Then, $a_3 = 15$

and $S_{10} = 125$

$$\Rightarrow \quad a + 2d = 15 \qquad [\because a_n = a + (n-1)d] \quad \ldots(i)$$

$$\text{and} \quad \frac{10}{2}[2a + (10-1)d] = 125 \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d\right]$$

$$\Rightarrow \quad 5(2a + 9d) = 125$$

$$\Rightarrow \quad 2a + 9d = 25 \qquad \ldots(ii)$$

On multiplying Eq. (i) by 2 and subtracting Eq. (ii), we get

$$2(a + 2d) - (2a + 9d) = 2 \times 15 - 25$$

$$\Rightarrow \quad 4d - 9d = 30 - 25$$

$$\Rightarrow \quad -5d = 5$$

$$\Rightarrow \quad d = -\frac{5}{5} = -1$$

$$\therefore \quad a_{10} = a + 9d = (a + 2d) + 7d$$

$$= 15 + 7(-1) \qquad \text{[From Eq. (i)]}$$

$$= 15 - 7 = 8$$

Hence, $d = -1, a_{10} = 8$

(v) Here, $d = 5$, $S_9 = 75$

Let a be the first term of the given AP. Then,

$$S_9 = 75$$

$$\Rightarrow \quad \frac{9}{2}[2a + (9-1)5] = 75 \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$\Rightarrow \quad \frac{9}{2}(2a + 40) = 75$$

$$\Rightarrow \quad 9a + 180 = 75 \Rightarrow 9a = 75 - 180$$

$$\Rightarrow \quad 9a = -105$$

$$\Rightarrow \quad a = \frac{-105}{9} = \frac{-35}{3}$$

So, $$a_9 = a + 8d$$

$$= \frac{-35}{3} + 8 \times 5 \qquad [\because a_n = a + (n-1)d]$$

$$= \frac{-35 + 120}{3} = \frac{85}{3}$$

Hence, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

(vi) Here, first term $a = 2$, common difference $d = 8$, sum of n terms $S_n = 90$

$$S_n = 90$$

$$\Rightarrow \quad \frac{n}{2}[2 \times 2 + (n-1)8] = 90 \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$\Rightarrow \quad \frac{n}{2}(4 + 8n - 8) = 90$$

$$\Rightarrow \quad \frac{n}{2}(8n - 4) = 90$$

$$\Rightarrow \quad n(4n - 2) = 90$$

$$\Rightarrow \quad 4n^2 - 2n - 90 = 0$$

$$\therefore \quad n = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 4 \times (-90)}}{2 \times 4}$$

$$\left(\because \text{By quadratice formula } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{2 \pm \sqrt{4 + 1440}}{8} = \frac{2 \pm \sqrt{1444}}{8}$$

$$= \frac{2 \pm 38}{8} = \frac{40}{8}, \frac{-36}{8} = 5, \frac{-9}{2}$$

Since, n cannot be negative

$$\therefore \quad n = 5$$

So, $$a_n = a + (n-1)d$$

$$\Rightarrow \quad a_5 = 2 + (5-1)8$$

$$= 2 + 32 = 34$$

Hence, $n = 5$ and $a_5 = 34$

(vii) Here, $a = 8$, $l = a_n = 62$, $S_n = 210$

Let d be the common difference, n be the number of terms of the given AP.

Since, $S_n = 210$

$\Rightarrow \quad \frac{n}{2}(a + l) = 210 \quad \left[\because S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow \quad \frac{n}{2}(8 + 62) = 210 \quad (\because a = 8, l = a_n = 62)$

$\Rightarrow \quad \frac{n}{2} \times 70 = 210$

$\Rightarrow \quad n = 210 \times \frac{2}{70} = 3 \times 2 = 6$

and $\quad a_n = 62 \Rightarrow a_6 = 62$

$\Rightarrow \quad a + 5d = 62 \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 8 + 5d = 62 \quad (\because a = 8)$

$\Rightarrow \quad 5d = 62 - 8 = 54$

$\Rightarrow \quad d = \frac{54}{5}$

Hence, $\quad d = \frac{54}{5}$ and $n = 6$

(viii) Here, $l = a_n = 4$, $d = 2$, $S_n = -14$

Let a be the first term and n be the number of terms of the given AP. Then,

$a_n = 4$

$\Rightarrow \quad a + (n-1)2 = 4 \quad (\because d = 2) \quad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad a = 4 - 2(n-1) \quad \ldots(i)$

and $\quad S_n = -14$

$\Rightarrow \quad \frac{n}{2}(a + l) = -14 \quad \left[\because S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow \quad n(a + 4) = -28 \quad (\because l = a_n)$

$\Rightarrow \quad n[4 - 2(n-1) + 4] = -28 \quad$ [From Eq. (i)]

$\Rightarrow \quad n(4 - 2n + 2 + 4) = -28$

$\Rightarrow \quad n(-2n + 10) = -28$

$\Rightarrow \quad n(-n + 5) = -14$

$\Rightarrow \quad -n^2 + 5n = -14$

$\Rightarrow \quad n^2 - 5n - 14 = 0$

$\Rightarrow \quad (n-7)(n+2) = 0 \quad$ (By factorisation method)

$\Rightarrow \quad n = 7$ or $n = -2$

Since, n cannot be negative.

$\therefore \quad n = 7$

Putting $n = 7$ in Eq. (i), we get

$$a = 4 - 2(7-1) = 4 - 2 \times 6$$
$$= 4 - 12 = -8$$

Hence, $n = 7$ and $a = -8$

(ix) Here, $a = 3, n = 8, S_n = 192$

Let d be the common difference of the given AP.

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad 192 = \frac{8}{2}[2 \times 3 + (8-1)d]$

$\Rightarrow \quad 192 = 4(6 + 7d)$

$\Rightarrow \quad 48 = 6 + 7d$

$\Rightarrow \quad 7d = 48 - 6 = 42$

$\Rightarrow \quad d = \frac{42}{7} = 6$

Hence, $d = 6$

(x) Here, last term $l = 28$, sum of n terms $S_n = 144$, total number of terms $n = 9$

Let a be the first term of given AP.

$\because \quad S_n = 144$

$\Rightarrow \quad \frac{n}{2}(a + l) = 144 \left[\because S_n = \frac{n}{2}(\text{First term} + \text{Last term})\right]$

$\Rightarrow \quad \frac{9}{2}(a + 28) = 144$

$\Rightarrow \quad a + 28 = 144 \times \frac{2}{9}$

$\Rightarrow \quad a + 28 = 32$

$\Rightarrow \quad a = 32 - 28 = 4$

Hence, $a = 4$

Question 4. How many term of the AP 9, 17, 25 ... must be taken to give a sum of 636?

Solution Let the first term be $a = 9$, common difference $d = 17 - 9 = 8$ and the number of terms is n.

Given, $S_n = 636$

$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 636 \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow \quad \frac{n}{2}[2 \times 9 + (n-1)8] = 636$

$\Rightarrow \quad \frac{n}{2}(18 + 8n - 8) = 636$

$\Rightarrow \quad \frac{n}{2}(8n + 10) = 636$

$\Rightarrow \quad n(4n + 5) = 636$

$\Rightarrow \quad 4n^2 + 5n - 636 = 0$

$\therefore \quad n = \dfrac{-5 \pm \sqrt{(5)^2 - 4 \times 4 \times (-636)}}{2(4)}$

$$\left(\text{By quadratic formula, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{-5 \pm \sqrt{25 + 10176}}{8}$$

$$= \frac{-5 \pm \sqrt{10201}}{8} = \frac{-5 \pm 101}{8}$$

$$= \frac{96}{8}, \frac{-106}{8} = 12, \frac{-53}{4}$$

Since, n cannot be negative.

$\therefore \quad n = 12$

Hence, the sum of 12 terms is 636.

Question 5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution Let a be the first term and d be the common difference of an AP.

Given, first term $a = 5$, last term $l = 45$ and sum of n terms $S_n = 400$

$\because \quad S_n = \dfrac{n}{2}(a + l)$

$\Rightarrow \quad 400 = \dfrac{n}{2}(5 + 45)$

$\Rightarrow \quad 400 \times 2 = 50n$

$\Rightarrow \quad n = \dfrac{400 \times 2}{50} = 8 \times 2 = 16$

and $\quad l = 45$

$\Rightarrow \quad a + (n-1)d = 45 \quad [\because l = a_n = a + (n-1)d]$

$\Rightarrow \quad 5 + (16-1)d = 45$

$\Rightarrow \quad 15d = 45 - 5 = 40$

$\Rightarrow \quad d = \dfrac{40}{15} = \dfrac{8}{3}$

$\therefore$ The number of terms is 16 and the common difference is $\dfrac{8}{3}$.

Question 6. The first and the last terms of an AP are 17 and 350, respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution Let a be the first term and d be the common difference.

Given, first term $a = 17$, last term $l = a_n = 350$, common difference $d = 9$

$\because \quad l = a_n = 350$

$\Rightarrow \quad a + (n-1)d = 350 \quad [\because l = a_n = a + (n-1)d]$

$\Rightarrow \quad 17 + (n-1)\,9 = 350$

$\Rightarrow \quad 9\,(n-1) = 350 - 17 = 333$

$\Rightarrow \quad n - 1 = \frac{333}{9} = 37$

$\Rightarrow \quad n = 37 + 1 = 38$

Putting $a = 17$, $l = 350$, $n = 38$

$\because$ Sum of n terms, $S_n = \frac{n}{2}(a + l)$, we get

$$S_{38} = \frac{38}{2}(17 + 350)$$

$$= 19\,(367) = 6973$$

So, there are 38 terms in the AP having their sum as 6973.

Question 7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Solution Let a be the first term and d the common difference of the given AP.
Then, $d = 7$ and $a_{22} = 149$

$\Rightarrow \quad a + (22-1)\,d = 149 \qquad [\because a_n = a + (n-1)\,d]$

$\Rightarrow \quad a + 21 \times 7 = 149$

$\Rightarrow \quad a = 149 - 147 = 2$

Put $a = 2$, $n = 22$ and $d = 7$ in

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)\,d]$, we get

$$S_{22} = \frac{22}{2}[2 \times 2 + (22-1)\,7]$$

$$= 11\,(4 + 21 \times 7)$$

$$= 11\,(4 + 147)$$

$$= 11 \times 151 = 1661$$

Hence, the sum of first 22 term is 1661.

Question 8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18, respectively.

Solution Let a be the first term and d be the common difference of the given AP.

Given that, second term $a_2 = 14$ and third term $a_3 = 18$

$\Rightarrow \quad a + d = 14 \qquad [\because a_n = a + (n-1)\,d] \ldots$(i)

and $\quad a + 2d = 18 \qquad \ldots$(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$d = 4$$

Put $d = 4$ in Eq. (i), we get

$$a + 4 = 14$$

$\Rightarrow \quad a = 14 - 4 = 10$

$\because$ Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$\therefore$
$$S_{51} = \frac{51}{2}[2 \times 10 + (51-1) \times 4]$$
$$= \frac{51}{2}[20 + 50 \times 4]$$
$$= \frac{51}{2}(20 + 200)$$
$$= \frac{51}{2} \times 220$$
$$= 51 \times 110 = 5610$$

Question 9. If the sum of 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of n terms.

Solution Let a be the first term and d be the common difference of the given AP.

Then, sum of 7 terms $S_7 = 49$ and sum of 17 terms $S_{17} = 289$ (Given)

$\Rightarrow$ $\frac{7}{2}[2a + (7-1)d] = 49$ $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow$ $\frac{7}{2}(2a + 6d) = 49$

$\Rightarrow$ $a + 3d = 7$ ...(i)

and $\frac{17}{2}[2a + (17-1)d] = 289$

$\Rightarrow$ $\frac{17}{2}(2a + 16d) = 289$

$\Rightarrow$ $a + 8d = 17$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get
$$5d = 10 \Rightarrow d = 2$$

Put, $d = 2$ in Eq. (i), we get
$$a + 3(2) = 7$$

$\Rightarrow$ $a + 6 = 7 \Rightarrow a = 1$

$\therefore$ Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$
$$= \frac{n}{2}[2 \times 1 + (n-1)2]$$
$$= \frac{n}{2}(2 + 2n - 2) = \frac{n}{2} \times 2n = n^2$$

Question 10. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP, where a_n is defined as below (i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$. Also, find the sum of the first 15 terms in each case.

Solution Here, $a_n = 3 + 4n$ (Given)

Put $n = 1, 2, 3, 4, \ldots$, we get

7, 11, 15, 19, ... $(3 + 4n)$ which is an AP with common difference 4.

Here, first term $a = 7$, common difference $d = 4$ and number of terms $n = 15$

$$\text{Sum of } n \text{ terms, } S_n = \frac{n}{2}[2a + (n-1)d], \text{ we get}$$

$$S_{15} = \frac{15}{2}[2 \times 7 + (15-1) 4]$$

$$= \frac{15}{2}(14 + 14 \times 4) = \frac{15}{2}(14 + 56)$$

$$= \frac{15}{2} \times 70 = 15 \times 35 = 525$$

(ii) Here, $a_n = 9 - 5n$ (Given)

Putting, $n = 1, 2, 3, 4, \ldots$, we get,

the sequence $4, -1, -6, -11, \ldots, (9 - 5n)$ which is an AP with common difference -5.

Putting, first term $a = 4$, common difference $d = -5$

and number of terms $n = 15$ in sum of n terms,

$$S_n = \frac{n}{2}[2a + (n-1)d], \text{ we get}$$

$$\Rightarrow \quad S_{15} = \frac{15}{2}[2 \times 4 + (15-1)(-5)]$$

$$= \frac{15}{2}[(8 + 14(-5)]$$

$$= \frac{15}{2}(8 - 70)$$

$$= \frac{15}{2} \times (-62) = 15 \times (-31) = -465$$

Question 11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Solution Given, the sum of first n terms, $S_n = 4n - n^2$

Put $n = 1$, $\quad S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$

$\Rightarrow$ $\quad$ First term $= 3$

Put, $n = 2$, we get

$$S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$$

$\because$ The nth term of an AP, $\quad a_n = S_{n+1} - S_n$

$\therefore$ $\quad$ Second term $= S_2 - S_1 = 4 - 3 = 1$

Put $n = 3$, $\quad S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

$\therefore$ Third term $= S_3 - S_2 = 3 - 4 = -1$

Put $n = 9$, $S_9 = 4 \times 9 - 9^2 = 36 - 81 = -45$

and put $n = 10$, $S_{10} = 4 \times 10 - 10^2$

$= 40 - 100 = -60$

$\therefore$ Tenth term $= S_{10} - S_9$

$= -60 - (-45) = -60 + 45 = -15$

Also, $S_n = 4n - n^2$ (Given)

Put $n = n - 1$, $S_{n-1} = 4(n-1) - (n-1)^2$

$= 4n - 4 - n^2 + 2n - 1$

$= -n^2 + 6n - 5$

$\therefore$ nth term $= S_n - S_{n-1} = (4n - n^2) - 4(n-1) + (n-1)^2$

$= 4n - n^2 - (-n^2 + 6n - 5)$

$= 4n - n^2 + n^2 - 6n + 5$

$= 5 - 2n$

Question 12. Find the sum of the first 40 positive integers divisible by 6.

Solution The first 40 positive integers divisible by 6 are 6, 12, 18, ...

Clearly, it is an AP with first term $a = 6$ and common difference $d = 6$

$\because$ Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{40} = \frac{40}{2}[2 \times 6 + (40-1)6]$

$= 20(12 + 39 \times 6)$

$= 20(12 + 234) = 20 \times 246 = 4920$

Question 13. Find the sum of the first 15 multiples of 8.

Solution The first 15 multiples of 8 are $8 \times 1, 8 \times 2, 8 \times 3, ..., 8 \times 15$.

i.e., 8, 16, 24, ..., 120 are in AP.

Here, first term $a = 8$, last term, $l = 120$ and number of term $n = 15$

$\because$ Sum of n terms, $S_n = \frac{n}{2}(a + l)$

$\therefore$ $S_{15} = \frac{15}{2}(8 + 120)$

$= \frac{15}{2} \times 128 = 15 \times 64 = 960$

Question 14. Find the sum of the odd numbers between 0 and 50.

Solution The odd numbers between 0 and 50 are 1, 3, 5,..., 49.

Here, first term $a = 1$ and last term, $l = 49$

Let n be the number of term and the common difference $d = 3 - 1 = 2$

$\because$ nth term $a_n = a + (n-1)d = l$

$\therefore \quad 1 + (n-1)(2) = 49$

$\Rightarrow \quad 2(n-1) = 48$

$\Rightarrow \quad n - 1 = 24$

$\Rightarrow \quad n = 25$

Now, sum of n terms, $\quad S_n = \frac{n}{2}(a + l)$

$\therefore \quad S_{25} = \frac{25}{2}(1 + 49)$

$= \frac{25}{2} \times 50$

$= 25 \times 25 = 625$

Question 15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows : ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Solution Since, the penalty for each succeeding day is ₹ 50 more than the preceding day, therefore the penalties for the first day, the second day, the third day, etc. will form an AP.

Let us denote the penalty for the nth day by a_n, then

$a_1 = ₹\ 200, \quad a_2 = ₹\ 250, \quad a_3 = ₹\ 300$

Here, $a = 200$, $d = ₹\ 250 - ₹\ 200 = ₹\ 50$ and $n = 30$

$\therefore$ The money the contractor has to pay penalty, if he delayed the work by 30 days.

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{30}{2}[2 \times 200 + (30 - 1)50]$

$= 15(400 + 29 \times 50)$

$= 15(400 + 1450) = 15 \times 1850 = 27750$

So, a delay of 30 days costs is ₹ 27750.

Question 16. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Solution Suppose, the respective prizes are $a + 60$, $a + 40$, $a + 20$, a, $a - 20$, $a - 40$, $a - 60$

According to question,

$a + 60 + a + 40 + a + 20 + a + a - 20 + a - 40 + a - 60 = 700$

$$\Rightarrow \quad 7a = 700 \Rightarrow a = \frac{700}{7} = 100$$

Hence, the seven prizes are $100 + 60$, $100 + 40$, $100 + 20$, 100, $100 - 20$, $100 - 40$, $100 - 60$

i.e., 160, 140, 120, 100, 80, 60, 40

Question 17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, *e.g.*, a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Solution According to question, there are three sections of each class, so the number of trees that each section planted *i.e.*, Class I, Class II, Class III,..., Class XII are $1\times 3, 2\times 3, 3\times 3, \ldots, 12\times 3$, respectively.

3, 6, 9, ... 36. Clearly, it form an AP.

Here, the first term, $a = 3$, common difference, $d = 6 - 3 = 3$

and the last term, $l = 36$

Now, the nth term of an AP, $a_n = a + (n-1)d = l$

$$\Rightarrow \quad 3 + (n-1)(3) = 36$$

$$\Rightarrow \quad (n-1)3 = 33$$

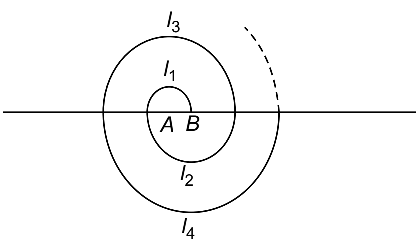
$$\Rightarrow \quad n - 1 = 11 \Rightarrow n = 12$$

Hence, the number of the tre es planted by the students

$$= \frac{12}{2}(3 + 36) \qquad \left[\because S_n = \frac{n}{2}(a + l)\right]$$

$$= 6 \times 39 = 234$$

Question 18. A spiral is made up of successive semi-circles with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semi-circles? $\left(\text{Take, } \pi = \frac{22}{7}\right)$



[**Hint** Length of successive semi-circles is $l_1, l_2, l_3, l_4 \ldots.$ with centres at A, B, A, B, respectively.]

Solution Length of spiral made up of thirteen consecutive semi-circles

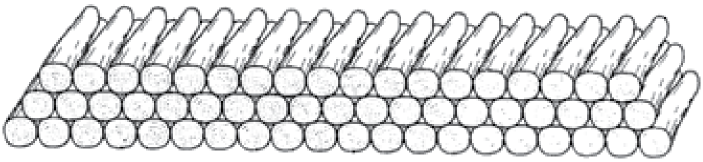
$$= (\pi \times 0.5 + \pi \times 1.0 + \pi \times 1.5 + \pi \times 2.0 + \ldots + \pi \times 6.5)$$
$$= \pi \times 0.5\,(1 + 2 + 3 + \ldots + 13)$$

which form an AP with first term, $a = 1$, common difference, $d = 2 - 1 = 1$ and number of term, $n = 13$

$\because$ Sum of n terms $= \frac{n}{2}[2a + (n-1)d]$

$$= \pi \times 0.5 \times \frac{13}{2}[2 \times 1 + (13-1) \times 1]$$
$$= \frac{22}{7} \times \frac{5}{10} \times \frac{13}{2} \times 14$$
$$= 143 \text{ cm}$$

Question 19. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?



Solution Since, logs are stacked in each row form a series $20 + 19 + 18 + 17 + \ldots$. Clearly, it is an AP with first term, $a = 20$ and common difference, $d = 19 - 20 = -1$

Suppose, $S_n = 200$

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ $200 = \frac{n}{2}[2 \times 20 + (n-1)(-1)]$

$\Rightarrow$ $400 = n(40 - n + 1)$

$\Rightarrow$ $n^2 - 41n + 400 = 0$

$\Rightarrow$ $n^2 - 25n - 16n + 400 = 0$ (By factorisation method)

$\Rightarrow$ $n(n-25) - 16(n-25) = 0$

$\Rightarrow$ $(n-25)(n-16) = 0$

$\Rightarrow$ $n = 16$

or $n = 25$

Hence, the number of rows is either 25 or 16.

When $n = 16$, $\qquad t_n = a + (n-1)d$

$= 20 + (16-1)(-1)$

$= 20 - 15$

$= 5$

When $n = 25$, $\qquad t_n = a + (n-1)d$

$= 20 + (25-1)(-1)$

$= 20 - 24$

$= -4$ (Not possible)

Hence, the number of row is 16 and number of logs in the top row = 5.

Question 20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in lines (see figure)

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[**Hint** To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Solution According to question, a competitor pick up the Ist potato, second potato, third potato, fourth potato ....

The distances sum by competitor are 2×5, $2 \times (5 + 3)$, $2 \times (5 + 3 + 3)$, $2 \times (5 + 3 + 3 + 3)$ *i.e.*, 10, 16, 22, 28, ...

Clearly, it is an AP with first term, $a = 10$ and common difference, $d = 16 - 10 = 6$

$\because$ The sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ The sum of 10 terms, $S_{10} = \frac{10}{2}[2 \times 10 + (10-1) \times 6]$ $\qquad [\because n = 10 \text{ (given)}]$

$= 5(20 + 54)$

$= 5 \times 74$

$= 370$

Hence, the total distance the competitor has to run = 370 m

8 Arithmetic Progressions

Exercise 8.4

Question 1. Which term of the AP 121, 117, 113, ... is its first negative term?

[**Hint** Find n for $a_n < 0$]

Solution Given, first term, $2\,a = 121$, common difference, $d = 117 - 121 = -4$

$\because n$ th term of an AP, $a_n = a + (n-1)\,d$

$= 121 + (n-1) \times (-4)$

$= 121 - 4n + 4 = 125 - 4n$

For first negative term, $a_n < 0$

$\Rightarrow \quad 125 - 4n < 0$

$\Rightarrow \quad 125 < 4n$

$\Rightarrow \quad 4n > 125$

$\Rightarrow \quad n > \dfrac{125}{4}$

$\Rightarrow \quad n > 31\dfrac{1}{4}$

Least integral value of $n = 32$,

Hence, 32nd term of the given AP is the first negative term.

Question 2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Solution Let the first term and the common difference of the AP be a and d, respectively.

According to the question,

Third term + Seventh term = 6

$\Rightarrow \quad [a + (3-1)d] + [a + (7-1)\,d] = 6 \qquad [\because a_n = a + (n-1)\,d]$

$\Rightarrow \quad (a + 2d) + (a + 6d) = 6$

$\Rightarrow \quad 2a + 8d = 6$

$\Rightarrow \quad a + 4d = 3 \qquad \ldots\text{(i)}$

and (third term) (seventh term) = 8

$\Rightarrow \quad (a + 2d)\,(a + 6d) = 8$

$\Rightarrow \quad \{(a + 4d) - 2d\}\,\{(a + 4d) + 2d\} = 8$

$\Rightarrow \quad (3 - 2d)\,(3 + 2d) = 8 \qquad$ [Using Eq. (i)]

$\Rightarrow \quad 9 - 4d^2 = 8 \qquad [\because (a-b)\,(a+b) = a^2 - b^2]$

$\Rightarrow \quad 4d^2 = 9 - 8$

$\Rightarrow \quad d^2 = \dfrac{1}{4}$

$\Rightarrow \quad d = \pm\,\dfrac{1}{2}$

Taking $d = \frac{1}{2}$, from Eq. (i), we get

$$a + 4\left(\frac{1}{2}\right) = 3$$

$$\Rightarrow \quad a + 2 = 3$$

$$\Rightarrow \quad a = 3 - 2 \quad \Rightarrow \quad a = 1$$

$\therefore$ Sum of first sixteen terms of the AP = S_{16}

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{16} = \frac{16}{2}[2a + (16-1)d]$$

$$= 8[2a + 15d]$$

$$= 8\left[2(1) + 15\left(\frac{1}{2}\right)\right]$$

$$= 8\left(2 + \frac{15}{2}\right)$$

$$= 8\left(\frac{19}{2}\right) = 76$$

Taking $d = -\frac{1}{2}$ from Eq. (i), we get

$$a + 4\left(-\frac{1}{2}\right) = 3$$

$$\Rightarrow \quad a - 2 = 3$$

$$\Rightarrow \quad a = 5$$

$\therefore$ Sum of first sixteen terms of the AP = S_{16}

$$s_{16} = \frac{16}{2}[2a + (16-1)d] \qquad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$$

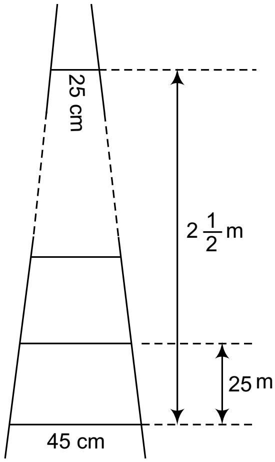
$$= 8[2a + 15d]$$

$$= 8\left[2(5) + 15\left(-\frac{1}{2}\right)\right] = 8\left(10 - \frac{15}{2}\right)$$

$$= 8\left(\frac{5}{2}\right) = 20$$

So, $\quad S_{16} = 20, 76$

Question 3. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



[**Hint** Number of rungs = $\frac{250}{25} + 1$]

Solution According to the question,

$$\text{Number of rungs} = \frac{2\frac{1}{2}\text{m}}{25 \text{ cm}} = \frac{250 \text{ cm}}{25 \text{ cm}} = 10$$

Hence, there are 10 rungs.

The length of the wood required for the rungs = Sum of 10 rungs

$$= \frac{10}{2}(25 + 45) \quad \left[\because S_n = \frac{n}{2}(a + l)\right]$$

$$= 5 \times 70 = 350 \text{ cm}$$

Question 4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

[**Hint** $S_{x-1} = S_{49} - S_x$]

Solution The consecutive numbers on the houses of a row are 1, 2, 3, ...49. Clearly, this list of number forming an AP.

Here, $a = 1, d = 2 - 1 = 1$

$$\because \quad S_x = \frac{x}{2}[2a + (x - 1)d]$$

$$\therefore \quad S_{x-1} = \frac{x-1}{2}[2 \times 1 + (x - 1 - 1) \times 1]$$

$$= \frac{x-1}{2}(2 + x - 2) = \frac{(x-1)x}{2} = \frac{x^2 - x}{2}$$

$$S_x = \frac{x}{2}[2 \times 1 + (x-1) \times 1]$$

$$= \frac{x}{2}(x+1) = \frac{x^2+x}{2}$$

and $$S_{49} = \frac{49}{2}[2 \times 1 + (49-1) \times 1]$$

$$= \frac{49}{2}[2+48]$$

$$= \frac{49}{2} \times 50 = 49 \times 25$$

Given condition, $$S_{x-1} = S_{49} - S_x$$

$$\frac{x^2-x}{2} = 49 \times 25 - \frac{x^2+x}{2}$$

$\Rightarrow$ $$\frac{x^2-x}{2} + \frac{x^2+x}{2} = 49 \times 25$$

$\Rightarrow$ $$\frac{x^2-x+x^2+x}{2} = 49 \times 25$$

$\Rightarrow$ $$x^2 = 49 \times 25$$

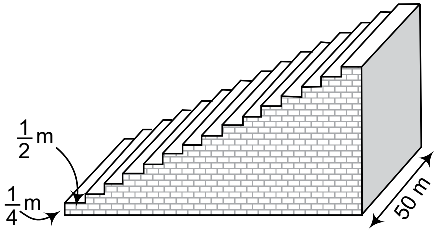
$\Rightarrow$ $$x = \pm 7 \times 5 = \pm 35$$

Since, x is a counting number.

$\therefore$ Taking positive sign, we get

$$x = 35$$

Question 5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.



[**Hint** Volume of concrete required to build the first step $= \frac{1}{4} \times \frac{1}{2} \times 50 \text{ m}^3$]

Solution Since, volume of concrete required to build the first step, IInd step, IIIrd step,... are $\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \ldots$

i.e., $\frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \ldots$

So, total volume of concrete required

$$= \frac{50}{8} + 2 \times \frac{50}{8} + 3 \times \frac{50}{8} + \ldots$$

$$= \frac{50}{8}[1 + 2 + 3 + \ldots]$$

which forms an AP with first term, $a = 1$, common difference, $d = 2 - 1 = 1$ and number of term, $n = 15$

$$= \frac{50}{8} \times \frac{15}{2}[2 \times 1 + (15 - 1) \times 1]$$

$$\left[\because S_n = \frac{n}{2}\{2a + (n - 1)\}\right]$$

$$= \frac{50}{8} \times \frac{15}{2} \times (2 + 14)$$

$$= \frac{25 \times 15}{8} \times 16$$

$$= 750\,\text{m}^3$$