11 Circles

Exercise 11.1

Question 1. Fill in the blanks.

(i) The centre of a circle lies in _______ of the circle. (exterior/interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in _______ of the circle. (exterior/interior)

(iii) The longest chord of a circle is a _______ of the circle.

(iv) An arc is a _______ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and _______ of the circle.

(vi) A circle divides the plane, on which it lies, in _______ parts.

Solution

(i) The centre of a circle lies in **interior** of the circle.

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in **exterior** of the circle.

(iii) The longest chord of a circle is a **diameter** of the circle.

(iv) An arc is a **semi-circle** when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and **chord** of the circle.

(vi) A circle divides the plane, on which it lies, in **three** parts.

Question 2. Write True or False. Give reason for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Solution

(i) True. Because all points are equidistant from the centre to the circle.

(ii) False. Because circle has infinitely may equal chords can be drawn.

(iii) False. Because all three arcs are equal, so their is no difference between the major and minor arcs.

(iv) True. By the definition of diameter, that diameter is twice the radius.

(v) False. Because the sector is the region between two radii and an arc.

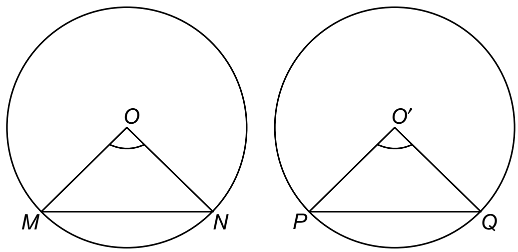
(vi) True. Because circle is a part of the plane figure.

Exercise 11.2

Question 1. Recall that two circles are congruent, if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres

Solution **Given** MN and PQ are two equal chords of two congruent circles with centre at O and O'.

To prove $\angle MON = \angle PO'Q$



Proof In ΔMON and $\Delta PO'Q$, we have

$MO = PO'$ (Radii of congruent circles)

$NO = QO'$ (Radii of congruent circles)

and $MN = PQ$ (Given)

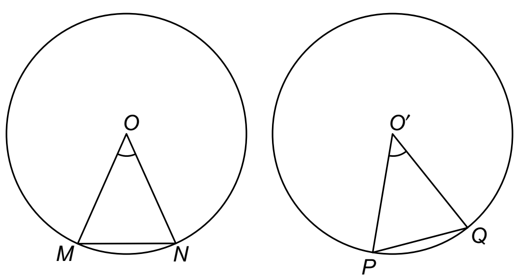
$\therefore$ By SSS criterion, we get

$$\Delta MON \cong \Delta PO'Q$$

Hence, $\angle MON = \angle PO'Q$ (By CPCT)

Question 2. Prove that, if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Solution **Given** MN and PQ are two chords of congruent circles such that angles subtended by these chords at the centres O and O' of the circles are equal.



i.e., $\angle MON = \angle PO'Q$

To prove $MN = PQ$

Proof In $\Delta\ MON$ and $\Delta\ PO'Q$, we get

$MO = PO'$ (Radii of congruent circles)

$NO = QO'$ (Radii of congruent circles)

and $\angle MON = \angle PO'Q$ (Given)

$\therefore$ By SAS criteria, we get

$$\Delta\ MON \cong \Delta\ PO'Q$$

Hence, $MN = PQ$ (By CPCT)

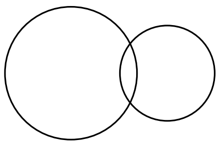
11 Circles

Exercise 11.3

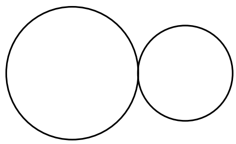
Question 1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Solution Different pairs of circles are

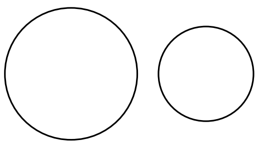
(i) Two points common



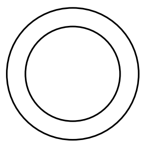
(ii) One point is common



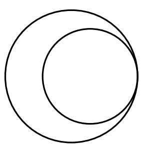
(iii) No point is common



(iv) No point is common



(v) One point is common

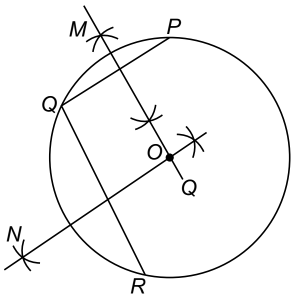


From figures, it is obvious that these pairs many have 0 or 1 or 2 points in common.

Hence, a pair of circles cannot intersect each other at more than two points.

Question 2. Suppose you are given a circle. Give a construction to find its centre.

Solution **Steps of construction**



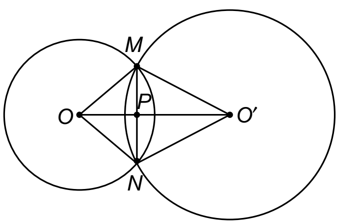
1. Taking three points P,Q and R on the circle.
2. Join PQ and QR.
3. Draw MQ and NS, respectively the perpendicular bisectors of PQ and RQ, which intersect each other at O.

   Hence, O is the centre of the circle.

Question 3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Solution **Given** Two circles with centres O and O' intersect at two points M and N so that MN is the common chord of the two circles and OO' is the line segment joining the centres of the two circles. Let OO' intersect MN at P.

To prove OO' is the perpendicular bisector of MN.



Construction Draw line segments OM, ON, $O'M$ and $O'N$.

Proof In $\Delta\ OMO'$ and ONO', we get

$OM = ON$ (Radii of the same circle)

$O'M = O'N$ (Radii of the same circle)

$OO' = OO'$ (Common)

$\therefore$ By SSS criterion, we get

$$\Delta\ OMO' \cong ONO'$$

So, $\angle MOO' = \angle NOO'$ (By CPCT)

$\therefore$ $\angle MOP = \angle NOP$ ...(i)

($\because \angle MOO' = \angle MOP$ and $\angle NOO' = \angle NOP$)

In ΔMOP and ΔNOP, we get

$OM = ON$ (Radii of the same circle)

$\angle MOP = \angle NOP$ [From Eq. (i)]

and $OM = OM$ (Common)

$\therefore$ By SAS criterion, we get

$$\Delta MOP \cong \Delta NOP$$

So, $MP = NP$ (By CPCT)

and $\angle MPO = \angle NPO$

But $\angle MPO + \angle NPO = 180°$ ($\because MPN$ is a straight line)

$\therefore$ $2\angle MPO = 180°$ ($\because \angle MPO = \angle NPO$)

$\Rightarrow$ $\angle MPO = 90°$

So, $MP = PN$

and $\angle MPO = \angle NPO = 90°$

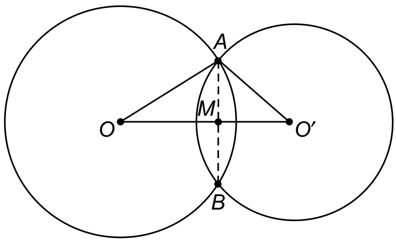
Hence, OO' is the perpendicular bisector of MN.

11 Circles

Exercise 11.4

Question 1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution Let O and O' be the centres of the circles of radii 5 cm and 3 cm, respectively.
Let AB be their common chord.



Given, $OA = 5\text{ cm}, O'A = 3\text{ cm and } OO' = 4\text{ cm}$

$\therefore$ $AO'^2 + OO'^2 = 3^2 + 4^2 = 9 + 16 = 25$

$= OA^2$

$\therefore OO'A$ is a right angled triangle and right angled at O'.

$$\text{Area of } \Delta OO'A = \frac{1}{2} \times O'A \times OO'$$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ sq units} \quad \ldots(i)$$

Also, $$\text{area of } \Delta OO'A = \frac{1}{2} \times OO' \times AM$$

$$= \frac{1}{2} \times 4 \times AM = 2\,AM \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

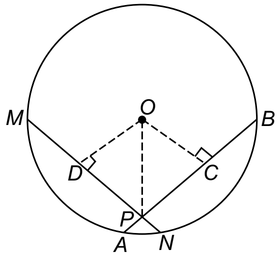
$$2AM = 6 \Rightarrow AM = 3$$

Since, when two circles intersect at two points, then their centre lie on the perpendicular bisector of the common chord.

$\therefore$ $AB = 2 \times AM = 2 \times 3 = 6\text{ cm}$

Question 2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution **Given** MN and AB are two chords of a circle with centre O, AB and MN intersect at P and $MN = AB$



To prove $MP = PB$ and $PN = AP$

Construction Draw $OD \perp MN$ and $OC \perp AB$.

Join OP.

Proof $\because$ $DM = DN = \frac{1}{2} MN$

(Perpendicular from centre bisects the chord)

and $AC = CB = \frac{1}{2} AB$

(Perpendicular from centre bisects the chord)

$MD = BC$ and $DN = AC$ ($\because MN = AB$) ...(i)

In ΔODP and ΔOPC

$OD = OC$

(Equal chords of a circle are equidistant from the centre)

$\angle ODP = \angle OCP$

$OP = OP$ (Common)

$\therefore$ RHS criterion of congruence,

$\Delta ODP \cong \Delta OCP$

$\therefore$ $DP = PC$ (By CPCT)...(ii)

On adding Eqs. (i) and (ii), we get

$MD + DP = BC + PC$

$MP = PB$

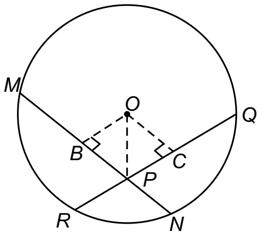
On subtracting Eq. (ii) from Eq. (i), we get

$DN - DP = AC - PC$

$PN = AP$

Hence, $MP = PB$ and $PN = AP$ are proved.

Question 3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.



Solution **Given** RQ and MN are chords of a circle with centre O. MN and RQ intersect at P and $MN = RQ$

To prove $\angle OPC = \angle OPB$

Construction Draw $OC \perp RQ$ and $OB \perp MN$. Join OP.

Proof In ΔOCP and ΔOBP, we get

$$\angle OCP = \angle OBP \quad \text{(Each = 90°)}$$

$$OP = OP \quad \text{(Common)}$$

$$OC = OB$$

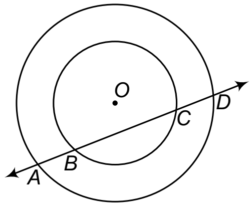
(Equal chords of a circle are equidistant from the centre)

$\therefore$ By RHS criterion of congruence, we get

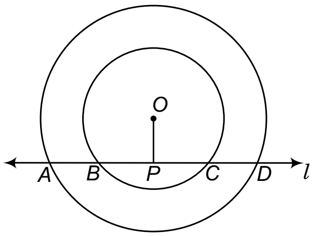
$$\Delta OCP \cong \Delta OBP$$

$$\therefore \quad \angle OPC = \angle OPB \quad \text{(By CPCT)}$$

Question 4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see figure).



Solution Let OP be the perpendicular from O on line l. Since, the perpendicular from the centre of a circle to a chord bisects the chords.



Now, BC is the chord of the smaller circle and $OP \perp BC$.

$$\therefore \quad BP = PC \quad \text{...(i)}$$

Since, AD is a chord of the larger circle and $OP \perp AD$.

$$\therefore \quad AP = PD \quad \text{...(ii)}$$

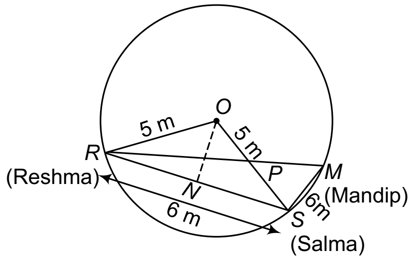
On subtracting Eq. (i) from Eq. (ii), we get

$$AP - BP = PD - PC \Rightarrow AB = CD$$

Hence proved.

Question 5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Solution Let O be the centre of the circle and Reshma, Salma and Mandip are represented by the points R, S and M, respectively. Let $RP = x$ m.



$$\text{Area of } \Delta ORS = \frac{1}{2} \times x \times 5 = \frac{5x}{2} \quad \ldots\text{(i)}$$

($\because$ In ORM, RM is a chord therefore $OP \perp RM$)

Again, $$\text{Area of } \Delta\, ORS = \frac{1}{2} \times RS \times ON = \frac{1}{2} \times 6 \times 4 = 12 \quad \ldots\text{(ii)}$$

$$\left[\begin{array}{r} \because RS \text{ is a chord, therefore } ON \perp RS. \\ \text{In right } \Delta\, RON, OR^2 = RN^2 + NO^2 \Rightarrow 5^2 = 3^2 + NO^2 \\ \Rightarrow NO^2 = 25 - 9 = 16 \Rightarrow NO = 4 \text{ cm} \end{array}\right]$$

From Eqs. (i) and (ii), we get

$$\frac{5x}{2} = 12$$

$\Rightarrow$ $$x = \frac{24}{5}$$

Since, P is the mid-point of RM

$\therefore$ $$RM = 2RP = 2 \times \frac{24}{5}$$

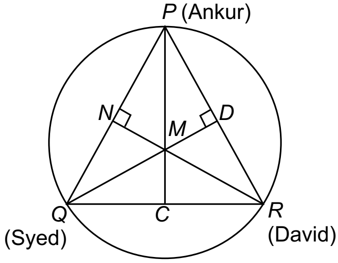
$$= \frac{48}{5} = 9.6 \text{ m}$$

Hence, the distance between Reshma and Mandip is 9.6 m.

Question 6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution Let Ankur, Syed and David standing on the point P, Q and R.

Let $PQ = QR = PR = x$



Therefore, ΔPQR is an equilateral triangle. Drawn altitudes PC, QD and RN from vertices to the sides of a triangle and intersect these altitudes at the centre of a circle M.

As PQR is an equilateral, therefore these altitudes bisects their sides.

In ΔPQC,

$$PQ^2 = PC^2 + QC^2 \quad \text{(By Pythagoras theorem)}$$

$$x^2 = PC^2 + \left(\frac{x}{2}\right)^2$$

$$PC^2 = x^2 - \frac{x^2}{4} = \frac{3x^2}{4} \quad \left(\because QC = \frac{1}{2}QR = \frac{x}{2}\right)$$

$$\therefore \quad PC = \frac{\sqrt{3}x}{2}$$

Now, $$MC = PC - PM = \frac{\sqrt{3}x}{2} - 20 \quad (\because PM = \text{radius} = 20\,\text{m})$$

In ΔQCM,

$$QM^2 = QC^2 + MC^2$$

$$\therefore \quad (20)^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{\sqrt{3}x}{2} - 20\right)^2 \quad (\because QM = \text{radius} = 20\,\text{m})$$

$$400 = \frac{x^2}{4} + \frac{3x^2}{4} - 20\sqrt{3}x + 400$$

$$0 = x^2 - 20\sqrt{3}x$$

$$x^2 = 20\sqrt{3}x$$

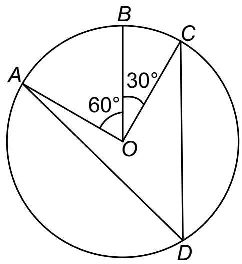
$$x = 20\sqrt{3} \quad \text{(Divide by } x\text{)}$$

Hence, $$PQ = QR = PR = 20\sqrt{3}\text{ m}$$

11 Circles

Exercise 11.5

Question 1. In figure A, B and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.



Solution $\therefore \angle AOC = \angle AOB + \angle BOC = 60^\circ + 30^\circ = 90^\circ$

$\therefore$ Arc ABC makes 90° at the centre of the circle.

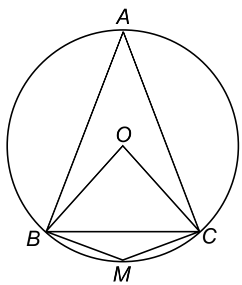
$\therefore$ $$\angle ADC = \frac{1}{2} \angle AOC$$

($\because$ The angle subtended by an arc at the centre is double the angle subtended by it any part of the circle.)

$$= \frac{1}{2} \times 90^\circ = 45^\circ$$

Question 2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution Let BC be chord, which is equal to the radius. Join OB and OC.



Given, $$BC = OB = OC$$

$\therefore \Delta OBC$ is an equilateral triangle.

$$\angle BOC = 60^\circ$$

$\therefore \quad BAC = \frac{1}{2} \angle BOC$

$= \frac{1}{2} \times 60° = 30°$

($\because$ The angle subtended by an arc at the centre is double the angle subtended by it any part of the circle.)

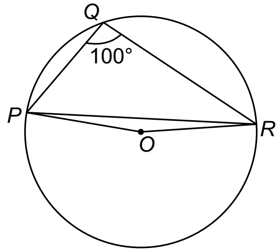
Here, *ABMC* is a cyclic quadrilateral.

$\therefore \quad \angle BAC + \angle BMC = 180°$

($\because$ In a cyclic quadrilateral the sum of opposite angles is 180°)

$\Rightarrow \quad \angle BMC = 180° - 30° = 150°$

Question 3. In figure, $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Solution $\therefore \angle POR = 2\angle PQR = 2 \times 100° = 200°$

(Since, the angle subtended by the centre is double the angle subtended by circumference.)

Since, in ΔOPR, $\quad \angle POR = 360° - 200° = 160°$ ...(i)

Again, ΔOPR, $\quad OP = OR$ (Radii of the circle)

$\therefore \quad \angle OPR = \angle ORP$ (By property of isosceles triangle)

In ΔPOR, $\quad \angle OPR + \angle ORP + \angle POR = 180°$ ...(ii)

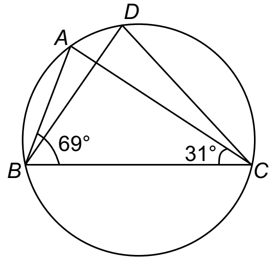
From Eqs. (i) and (ii), we get

$\angle OPR + \angle OPR + 160° = 180°$

$\therefore \quad 2\angle OPR = 180° - 160° = 20°$

$\therefore \quad \angle OPR = \frac{20°}{2} = 10°$

Question 4. In figure, $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



Solution $\because$ $\angle BDC = \angle BAC$ ...(i)

(Since, the angles in the same segment are equal)

Now, in ΔABC,

$\therefore$ $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow$ $\angle A + 69^\circ + 31^\circ = 180^\circ$

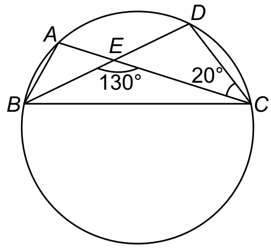
$\Rightarrow$ $\angle A + 100^\circ = 180^\circ$

$\therefore$ $\angle A = 180^\circ - 100^\circ = 80^\circ$

$\Rightarrow$ $\angle BAC = 80^\circ$

$\therefore$ From Eq. (i), $\angle BDC = 80^\circ$

Question 5. In figure, A, B and C are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.



Solution $\therefore$ $\angle AEB = 180^\circ - 130^\circ = 50^\circ$ (Linear pair)...(i)

$\Rightarrow$ $\angle CED = \angle AEB = 50^\circ$ (Vertically opposite)

Again, $\angle ABD = \angle ACD$

(Since, the angles in the same segment are equal)

$\angle ABE = \angle ECD$

$\Rightarrow$ $\angle ABE = 180^\circ$ ...(ii)

$\therefore$ In ΔCDE $\angle A + 20^\circ + 50^\circ = 180^\circ$ [From Eqs. (i) and (ii)]

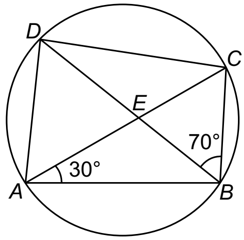
$\angle A + 70^\circ = 180^\circ$

$\therefore$ $\angle A = 180^\circ - 70^\circ = 110^\circ$

Hence, $\angle BAC = 110^\circ$

Question 6. $ABCD$ is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Solution $\because$ Angles in the same segment are equal.



$\therefore \quad \angle BDC = \angle BAC$

$\therefore \quad \angle BDC = 30^\circ$

In ΔBCD, we have

$\therefore \quad \angle BDC + \angle DBC + \angle BCD = 180^\circ$

(Given, $\angle DBC = 70^\circ$ and $\angle BDC = 30^\circ$)

$\therefore \quad 30^\circ + 70^\circ + \angle BCD = 180^\circ$

$\therefore \quad \angle BCD = 180^\circ - 30^\circ - 70^\circ = 80^\circ$

If $AB = BC$, then $\quad \angle BCA = \angle BAC = 30^\circ$

(Angles opposite to equal sides in a triangle are equal)

Now, $\quad \angle ECD = \angle BCD - \angle BCA = 80^\circ - 30^\circ = 50^\circ$

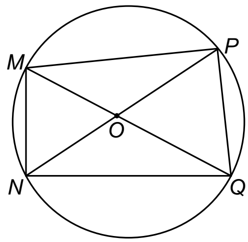
($\because \angle BCD = 80^\circ$ and $\angle BCA = 30^\circ$)

Hence, $\quad \angle BCD = 80^\circ$

and $\quad \angle ECD = 50^\circ$

Question 7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution **Given** Diagonals NP and QM of a cyclic quadrilateral are diameters of the circle through the vertices M, P, Q and N of the quadrilateral $NQPM$.



To prove Quadrilateral $NQPM$ is a rectangle.

Proof $\because \quad ON = OP = OQ = OM \quad$ (Radii of circle)

Now, $\quad ON = OP = \frac{1}{2}NP$

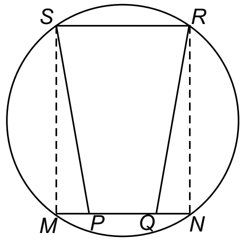
and $\quad OM = OQ = \frac{1}{2}MQ$

$\therefore \quad NP = MQ$

Hence, the diagonals of the quadrilateral $MPQN$ are equal and bisect each other. So, quadrilateral $NQPM$ is a rectangle.

Question 8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution **Given** Non-parallel sides *PS* and *QR* of a trapezium *PQRS* are equal.



To prove *ABCD* is a cyclic trapezium.

Construction Draw $SM \perp PQ$ and $RN \perp PQ$.

Proof In ΔSMP and ΔRNQ, we get

$$SP = RQ \quad \text{(Given)}$$

$$\angle SMP = \angle RNQ \quad (\text{Each} = 90^\circ)$$

and $$SM = RN$$

($\because$ Distance between two parallel lines is always equal)

$\therefore$ By RHS criterion, we get

$$\Delta SMP \cong \Delta RNQ$$

So, $$\angle P = \angle Q \quad \text{(By CPCT)}$$

and $$\angle PSM = \angle QRN$$

Now, $$\angle PSM = \angle QRN$$

$\therefore$ $$90^\circ + \angle PSM = 90^\circ + \angle QRN \quad \text{(Adding both sides } 90^\circ)$$

$\therefore$ $$\angle MSR + \angle PSM = \angle NRS + \angle QRN \quad (\because \angle MSR = \angle NRS = 90^\circ)$$

So, $$\angle PSR = \angle QRS$$

i.e., $$\angle S = \angle R$$

Thus, $$\angle P = \angle Q \quad \text{and} \quad \angle R = \angle S \quad \text{...(i)}$$

$\therefore$ $$\angle P + \angle Q + \angle R + \angle S = 360^\circ$$

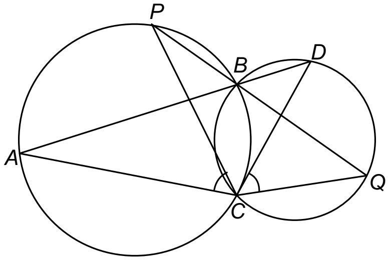
($\because$ Sum of the angles of a quadrilateral is 360°)

$\therefore$ $$2\angle S + \angle Q = 360^\circ \quad \text{[From Eq. (i)]}$$

$$\angle S + \angle Q = 180^\circ$$

Hence, *PQRS* is a cyclic trapezium.

Question 9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see figure). Prove that $\angle ACP = \angle QCD$.



Solution **Given** Two circles intersect at two points B and C. Through B two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q, respectively.

To prove $\angle ACP = \angle QCD$

Proof In circle I, $\angle ACP = \angle ABP$ (Angles in the same segment) ...(i)

In circle II, $\angle QCD = \angle QBD$ (Angles in the same segment)...(ii)

$\angle ABP = \angle QBD$ (Vertically opposite angles)

From Eqs. (i) and (ii), we get $\angle ACP = \angle QCD$

Question 10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution **Given** Two circles are drawn with sides AC and AB of ΔABC as diameters . Both circles intersect each other at D.

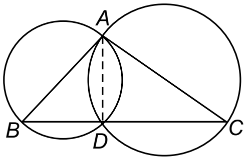
To prove D lies on BC.

Construction Join AD.

Proof Since, AC and AB are the diameters of the two circles.

$\angle ADB = 90^\circ$ ($\therefore$ Angles in a semi-circle) ...(i)

and $\angle ADC = 90^\circ$ (Angles in a semi-circle) ...(ii)



On adding Eqs. (i) and (ii), we get

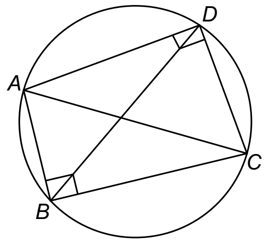
$$\angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$$

Hence, BCD is a straight line.

So, D lies on BC.

Question 11. ABC and ADC are two right angled triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Solution Since, ΔADC and ΔABC are right angled triangles with common hypotenuse.



Draw a circle with AC as diameter passing through B and D. Join BD.

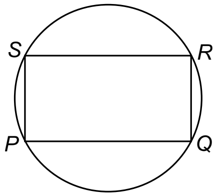
$\because$ Angles in the same segment are equal.

$\therefore$ $$\angle CBD = \angle CAD$$

Question 12. Prove that a cyclic parallelogram is a rectangle.

Solution **Given** $PQRS$ is a parallelogram inscribed in a circle.

To prove $PQRS$ is a rectangle.



Proof Since, $PQRS$ is a cyclic quadrilateral.

$\therefore$ $$\angle P + \angle R = 180^\circ$$

($\because$ Sum of opposite angles in a cyclic quadrilateral is 180°) ...(i)

But $\angle P = \angle R$ ($\because$ In a || gm opposite angles are equal) ...(ii)

From Eqs. (i) and (ii), we get

$$\angle P = \angle R = 90^\circ$$

Similarly, $\angle Q = \angle S = 90^\circ$

$\therefore$ Each angle of $PQRS$ is 90°.

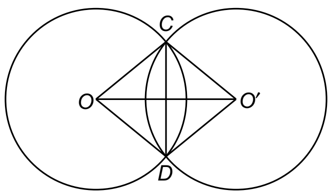
Hence, $PQRS$ is a rectangle.

11 Circles

Exercise 11.6 (Optional)

Question 1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution **Given** Two circles with centres O and O' which intersect each other at C and D.



To prove $\angle OCO' = \angle ODO'$

Construction Join OC, OD, $O'C$ and $O'D$

Proof In $\Delta\, OCO'$ and $\Delta ODO'$, we have

$OC = OD$ (Radii of the same circle)

$O'C = O'D$ (Radii of the same circle)

$OO' = OO'$ (Common)

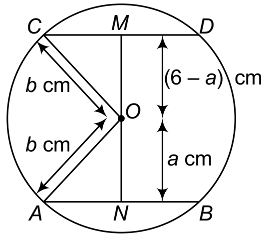
$\therefore$ By SSS criterion, we get

$$\Delta OCO' \cong \Delta ODO'$$

Hence, $\angle OCO' = \angle ODO'$ (By CPCT)

Question 2. Two chords AB and CD of lengths 5 cm and 11 cm, respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution Let O be the centre of the given circle and let its radius be b cm. Draw $ON \perp AB$ and $OM \perp CD$ since, $ON \perp AB$, $OM \perp CD$ and $AB \parallel CD$, therefore points N, O, M are collinear.



Let $ON = a$ cm

$\therefore$ $OM = (6 - a)$ cm

Join OA and OC.

Then, $$OA = OC = b \text{ cm}$$

Since, the perpendicular from the centre to a chord of the circle bisects the chord.

Therefore, $AN = NB = 2.5$ cm and $OM = MD = 5.5$ cm

In ΔOAN and ΔOCM, we get

$$OA^2 = ON^2 + AN^2$$

and $$OC^2 = OM^2 + CM^2$$

$\Rightarrow$ $$b^2 = a^2 + (2.5)^2$$

and $$b^2 = (6 - a)^2 + (5.5)^2 \quad \ldots(i)$$

So, $$a^2 + (2.5)^2 = (6 - a)^2 + (5.5)^2$$

$\Rightarrow$ $$a^2 + 6.25 = 36 - 12a + a^2 + 30.25$$

$\Rightarrow$ $$12a = 60 \Rightarrow a = 5$$

On putting $a = 5$ in Eq. (i), we get

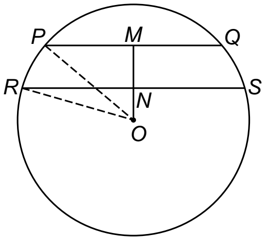
$$b^2 = (5)^2 + (2.5)^2$$

$$= 25 + 6.25 = 31.25$$

So, $$r = \sqrt{31.25} = 5.6 \text{ cm (Approx.)}$$

Question 3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution Let PQ and RS be two parallel chords of a circle with centre O such that $PQ = 6$ cm and $RS = 8$ cm.



Let a be the radius of circle.

Draw $ON \perp RS$, $OM \perp PQ$. Since, $PQ \parallel RS$ and $ON \perp RS$, $OM \perp PQ$, therefore points O, N, M are collinear.

$\because$ $OM = 4$ cm and M and N are the mid-points of PQ and RS respectively.

$\therefore$ $$PM = MQ = \frac{1}{2} PQ = \frac{6}{2} = 3 \text{ cm}$$

and $$RN = NS = \frac{1}{2} RS = \frac{8}{2} = 4 \text{ cm}$$

In ΔOPM, we have

$$OP^2 = OM^2 + PM^2$$

$\Rightarrow$ $$a^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$\Rightarrow$ $$a = 5$$

In ΔORN, we have

$$OR^2 = ON^2 + RN^2$$

$$\Rightarrow \quad a^2 = ON^2 + (4)^2$$

$$\Rightarrow \quad 25 = ON^2 + 16$$

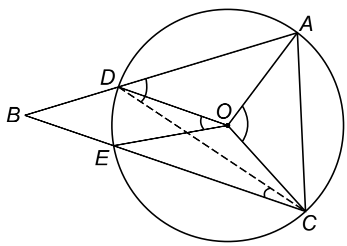
$$\Rightarrow \quad ON^2 = 9$$

$$\Rightarrow \quad ON = 3\text{cm}$$

Hence, the distance of the chord RS from the centre is 3 cm.

Question 4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Solution Since, an exterior angle of a triangle is equal to the sum of the interior opposite angles.



$\therefore$ In ΔBDC, we get

$$\angle ADC = \angle DBC + \angle DCB \qquad \ldots\text{(i)}$$

Since, angle at the centre is twice at a point on the remaining part of circle.

$$\therefore \quad \angle DCE = \frac{1}{2}\angle DOE$$

$$\Rightarrow \quad \angle DCB = \frac{1}{2}\angle DOE \qquad (\because \angle DCE = \angle DCB)$$

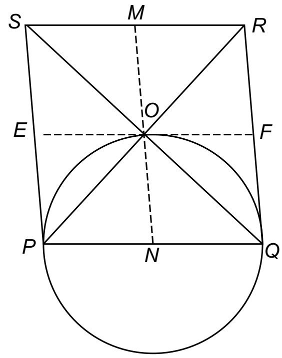
and
$$\angle ADC = \frac{1}{2}\angle AOC$$

$$\therefore \quad \frac{1}{2}\angle AOC = \angle ABC + \frac{1}{2}\angle DOE \qquad (\because \angle DBC = \angle ABC)$$

$$\therefore \quad \angle ABC = \frac{1}{2}(\angle AOC - \angle DOE)$$

Hence, $\angle ABC$ is equal to half the difference of angles subtended by the chords AC and DE at the centre.

Question 5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.



Solution **Given** $PQRS$ is a rhombus. PR and SQ are its two diagonals which bisect each other at right angles.

To prove A circle drawn on PQ as diameter will pass through O.

Construction Through O, draw $MN \parallel PS$ and $EF \parallel PQ$.

Proof $\because$ $PQ = SR \Rightarrow \frac{1}{2} PQ = \frac{1}{2} SR$

So, $PN = SM$

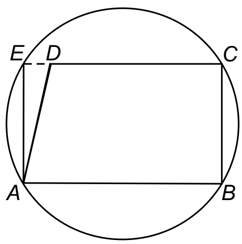
Similarly, $PE = ON$

So, $PN = ON = NQ$

Therefore, a circle drawn with N as centre and radius PN passes through P, O, Q.

Question 6. $ABCD$ is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that $AE = AD$.

Solution Since, $ABCE$ is a cyclic quadrilateral, therefore



$\angle AED + \angle ABC = 180°$

($\because$ Sum of opposite angle of a cyclic quadrilateral is 180°) ...(i)

$\because$ $\angle ADE + \angle ADC = 180°$ (EDC is a straight line)

So, $\angle ADE + \angle ABC = 180°$

($\because \angle ADC = \angle ABC$ opposite angle of a || gm)...(ii)

From Eqs. (i) and (ii), we get

$\angle AED + \angle ABC = \angle ADE + \angle ABC$

$\Rightarrow$ $\angle AED = \angle ADE$

$\therefore$ In ΔAED, we have

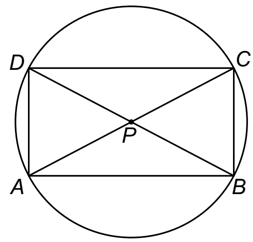
$\angle AED = \angle ADE$

So, $AD = AE$

($\because$ Sides opposite to equal angles of a triangle are equal)

Question 7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) $ABCD$ is a rectangle.

Solution (i) Let BD and AC be two chords of a circle bisect at P.



In ΔAPB and ΔCPD, we get

$$PA = PC \qquad (\because P \text{ is the mid-point of } AC)$$

$$\angle APB = \angle CPD \qquad \text{(Vertically opposite angles)}$$

and $$PB = PD \qquad (\because P \text{ is the mid-point of } BD)$$

$\therefore$ By SAS criterion

$$\Delta CPD \cong \Delta APB$$

$\therefore$ $$CD = AB \qquad \text{(By CPCT)}$$

$\Rightarrow$ $$\overset{\frown}{CD} = \overset{\frown}{AB} \qquad \ldots\text{(i)}$$

Similarly, in ΔAPD and ΔCPB, we get

$$\overset{\frown}{CB} = \overset{\frown}{AD} \qquad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$\overset{\frown}{CD} + \overset{\frown}{CB} = \overset{\frown}{AB} + \overset{\frown}{AD} \Rightarrow \overset{\frown}{BCD} = \overset{\frown}{BAD}$$

$\therefore$ BD divides the circle into two equal parts. So, BD is a diameter.

Similarly, AC is a diameter.

(ii) Now, BD and AC bisect each other.

So, $ABCD$ is a parallelogram.

Also, $$AC = BD$$

$\therefore$ $ABCD$ is a rectangle.

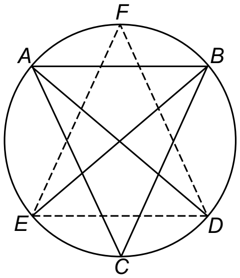
Question 8. Bisectors of angles A, B and C of a ΔABC intersect its circumcircle at D, E and F, respectively. Prove that the angles of the ΔDEF are $90^\circ - \frac{1}{2}A$, $90^\circ - \frac{1}{2}B$ and $90^\circ - \frac{1}{2}C$.

Solution $\because \angle EDF = \angle EDA + \angle ADF \qquad \ldots\text{(i)}$

$\because \angle EDA$ and $\angle EBA$ are the angles in the same segment of the circle.

$\therefore$ $$\angle EDA = \angle EBA$$

and similarly $\angle ADF$ and $\angle FCA$ are the angles in the same segment and hence



$$\angle ADF = \angle FCA$$

$\therefore$ From Eq (i) $\quad \angle EDF = \frac{1}{2}\angle B + \frac{1}{2}\angle C$

$\Rightarrow \quad \angle D = \frac{\angle B + \angle C}{2}$

Similarly, $\quad \angle F = \frac{\angle A + \angle B}{2}$

$$\angle E = \frac{\angle C + \angle A}{2}$$

So, $\quad \angle D = \frac{\angle B + \angle C}{2}$

$$= \frac{180^\circ - \angle A}{2} \quad (\because \angle A + \angle B + \angle C = 180^\circ)$$

$$\angle E = \frac{180^\circ - \angle B}{2} \quad (\because \angle A + \angle B + \angle C = 180^\circ)$$

and $\quad \angle F = \frac{180^\circ - \angle C}{2} \quad (\because \angle A + \angle B + \angle C = 180^\circ)$

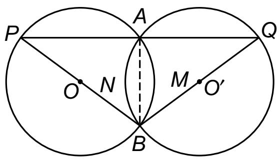
$\Rightarrow \quad \angle D = 90^\circ - \frac{\angle A}{2}$

$\Rightarrow \quad \angle E = 90^\circ - \frac{\angle B}{2}$

and $\quad \angle F = 90^\circ - \frac{\angle C}{2}$

Question 9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that $BP = BQ$.

Solution Let O' and O be the centres of two congruent circles.



Since, AB is a common chord of these circles.

$\therefore \quad \angle BPA = \angle BQA$

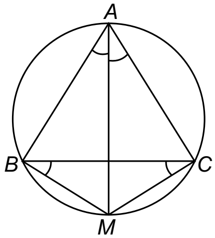
(∵ Angle subtended by equal chords are equal)

$\Rightarrow \quad BP = BQ$

Question 10. In any ΔABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the ΔABC.

Solution (i) Let bisector of $\angle A$ meet the circumcircle of ΔABC at M.

Join BM and CM.



$\therefore$ $\angle MBC = \angle MAC$ (Angles in same segment)

and $\angle BCM = \angle BAM$ (Angles in same segment)

But $\angle BAM = \angle CAM$ ($\because$ AM is bisector of $\angle A$) ...(i)

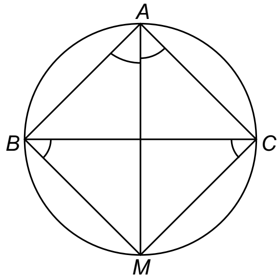
$\therefore$ $\angle MBC = \angle BCM$

So, $MB = MC$ (Sides opposite to equal angles are equal)

So, M must lie on the perpendicular bisector of BC

(ii) Let M be a point on the perpendicular bisector of BC which lies on circumcircle of ΔABC.

Join AM.



Since, M lies on perpendicular bisector of BC.

$\therefore$ $BM = CM$

$\angle MBC = \angle MCB$

But $\angle MBC = \angle MAC$ (Angles in same segment)

and $\angle MCB = \angle BAM$ (Angles in same segment)

So, from Eq. (i),

$$\angle BAM = \angle CAM$$

AM is the bisector of A.

Hence, bisector of $\angle A$ and perpendicular bisector of BC at M which lies on circumcircle of ΔABC.