# 9 Circles

## Exercise 9.1

**Question 1.** How many tangents can a circle have?

**Solution** For every point of a circle, we can draw a tangent. Therefore, infinite tangents can be drawn.

**Question 2.** Fill in the blanks.

(i) A tangent to a circle intersects it in ......... point(s).

(ii) A line intersecting a circle in two points is called a ......... .

(iii) A circle can have ........ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ......... .

**Solution** (i) One. A tangent line touch or intersect the circle only at one point.

(ii) Secant. Any line intersecting the circle at two points is called a secant.

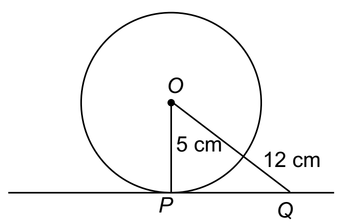
(iii) Two. Maximum a circle an have two parallel tangents which can be drawn to the opposite side of the centre.

(iv) Point of contact.

**Question 3.** A tangent $PQ$ at a point $P$ of a circle of radius 5 cm meets a line through the centre $O$ at a point $Q$ so that $OQ = 12$ cm. Length $PQ$ is

(a) 12 cm (b) 13 cm (c) 8.5 cm (d) $\sqrt{119}$ cm

**Solution** (d) Here, $PQ$ is a tangent, which touches a point $P$ of the circle. And $OP = 5$ cm is a radius of circle. Join $OQ = 12$ cm. Since, radius of circle $OP$ is perpendicular to the tangent $PQ$.




$\therefore$ In right $\Delta OPQ$, By Pythagoras theorem

$$OQ^2 = OP^2 + PQ^2$$

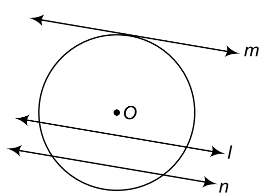
$$\Rightarrow \quad (12)^2 = (5)^2 + PQ^2$$

$$\Rightarrow \quad PQ^2 = 144 - 25 = 119$$

$$\Rightarrow \quad PQ = \sqrt{119} \text{ cm}$$

**Question 4.** Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

**Solution** Firstly draw a circle with centre $O$ and draw a line $l$. Now, we draw a two parallel lines to $l$, such that one line $m$ is tangent to the circle and another $n$ is secant to the circle.



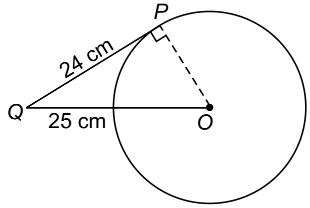
## Exercise 9.2

- *Choose the correct option and give justification.*

**Question 1.** From a point $Q$, the length of the tangent to a circle is 24 cm and the distance of $Q$ from the centre is 25 cm. The radius of the circle is

(a) 7 cm (b) 12 cm
(c) 15 cm (d) 24.5 cm

**Solution** (a) Given that, the length of the tangent $QP = 24$ cm and $QO = 25$ cm. Join $OP$.




We know, radius $OP$ is perpendicular to the tangent $PQ$.

In right $\Delta OPQ$

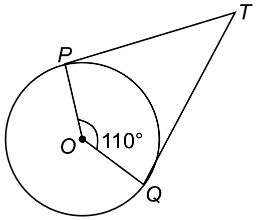
$$OQ^2 = OP^2 + PQ^2 \quad \text{(By Pythagoras theorem)}$$

$\Rightarrow$ $25^2 = OP^2 + 24^2$

$\Rightarrow$ $OP^2 = 625 - 576 = 49$

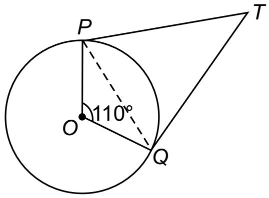
$\Rightarrow$ $OP = 7$ cm

**Question 2.** In figure, if $TP$ and $TQ$ are the two tangents to a circle with centre $O$ so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to




(a) 60° (b) 70° (c) 80° (d) 90°

**Solution** (b) Join $PQ$,



In $\Delta OPQ$

$$OP = OQ = \text{radius of circle}$$

$\Rightarrow$ $$\angle OPQ = \angle OQP$$

(Equal sides have corresponding equal angles)

$\Rightarrow$ $$\angle OPQ = \angle OQP = \frac{180° - 110°}{2} = 35°$$

Since, $OP \perp PT$ and $OQ \perp TQ$ (Radius of circle is perpendicular to the tangent)

$\therefore$ $$\angle OPT = 90° \text{ and } \angle OQT = 90°$$

$\Rightarrow$ $$\angle OPQ + \angle QPT = 90°$$

and $$\angle OQP + \angle PQT = 90°$$

$\Rightarrow$ $$35° + \angle QPT = 90°$$

and $$35° + \angle PQT = 90°$$

$\Rightarrow \angle QPT = 55°$ and $\angle PQT = 55°$

In $\Delta PTQ$,

$$\angle QPT + \angle PQT + \angle PTQ = 180°$$

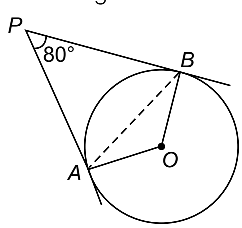
$\Rightarrow$ $$55° + 55° + \angle PTQ = 180°$$

$\Rightarrow$ $$\angle PTQ = 180° - (55° + 55°) = 70°$$

**Question 3.** If tangents $PA$ and $PB$ from a point $P$ to a circle with centre $O$ are inclined to each other at angle of 80°, then $\angle POA$ is equal to

(a) 50° (b) 60° (c) 70° (d) 80°

**Solution** (a) $PA$ and $PB$ are two tangents drawn from a point $P$.




*i.e.*, $$\angle APB = 80°$$

Join $AB$.

Since, tangents drawn from external point to the circle is equal in lengths.

*i.e.*, $$PA = PB$$

$$\angle PAB = \angle PBA \quad \ldots\text{(i)}$$

In $\Delta APB$,

$$\angle APB + \angle PAB + \angle PBA = 180°$$

$\Rightarrow$ $$80° + 2\angle PAB = 180° \quad \text{[From Eq. (i)]}$$

$\Rightarrow$ $$2\angle PAB = 100°$$

$\Rightarrow$ $$\angle PAB = 50°$$

$\therefore$ $$\angle PAB = \angle PBA = 50°$$

Since, $OA \perp PA$ and $OB \perp BP$

$\therefore$ $$\angle PAO = 90° \text{ and } \angle PBO = 90°$$

$\Rightarrow$ $$\angle PAB + \angle BAO = 90°$$

and $$\angle PBA + \angle ABO = 90°$$

$\Rightarrow \qquad 50° + \angle BAO = 90°$

and $\qquad 50° + \angle ABO = 90°$

$\Rightarrow \qquad \angle BAO = 40°$ and $\angle ABO = 40°$

Now, in $\Delta AOB$,

$$\angle OAB + \angle ABO + \angle AOB = 180°$$

$\Rightarrow \qquad 40° + 40° + \angle AOB = 180°$

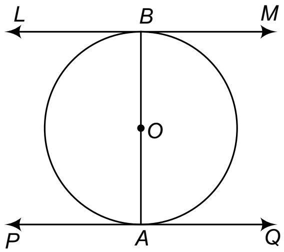
$\Rightarrow \qquad \angle AOB = 180° - 80° = 100°$

Since, angle bisector divides an angle into two equal parts.

$\therefore \qquad \angle PAO = \frac{1}{2} \angle AOB = \frac{1}{2} \times 100° = 50°$

**Question 4.** Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

**Solution** Let $AB$ be a diameter of a given circle and let $LM$ and $PQ$ be the tangent lines drawn to the circle at points $A$ and $B$, respectively. Since, the tangent at a point to a circle is perpendicular to the radius through the point.




$\therefore AB \perp PQ$ and $AB \perp LM$

$\Rightarrow \qquad \angle PAB = 90°$

and $\qquad \angle ABM = 90°$

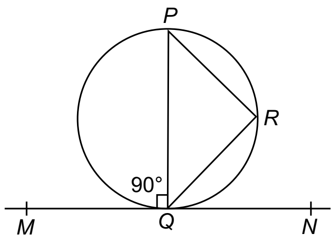
$\Rightarrow \qquad \angle PAB = \angle ABM$

$\Rightarrow \qquad PQ \parallel LM$

Hence proved.

**Question 5.** Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

**Solution** Tangent $MN$ is drawn from point $Q$ of the circle. A perpendicular line $PQ$ is drawn to the $MQN$ line.




*i.e.*, $\qquad \angle MQP = 90°$

Take any point $R$ on the circle.

Join $PR$ and $QR$.

By using alternate segment theorem

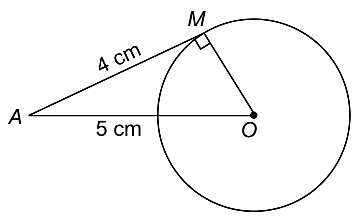
$$\angle MQP = \angle PRQ = 90°$$

We know, any chord $PQ$ subtends an angle 90° only when chord $PQ$ is a diameter of the circle.

Hence, perpendicular $PQ$ is always passes through the centre.

**Question 6.** The length of a tangent from a point $A$ at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

**Solution** Given, $OP = 5$ cm and $AM = 4$ cm




$\because$ $OM \perp AM$ ($\because$ Radius is perpendicular to $AM$)

In right $\Delta OMA$,

$$OP^2 = OM^2 + MA^2 \quad \text{(By Pythagoras theorem)}$$

$\Rightarrow$ $5^2 = OM^2 + 4^2$

$\Rightarrow$ $OM^2 = 25 - 16 = 9$

$\Rightarrow$ $OM = 3$ cm

Hence, radius of the circle is 3 cm.

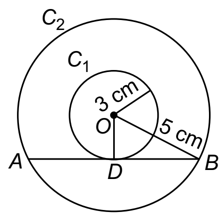
**Question 7.** Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

**Solution** Here, we draw two circles $C_1$ and $C_2$ of radii $r_1 = 3$ cm and $r_2 = 5$ cm

Now, we draw a chord $AB$ such that it touches the circle $C_1$ at point $D$.

The centre of concentric circle is $O$.

Now, we draw a perpendicular bisector from $O$ to $AB$ which meets $AB$ at $D$.



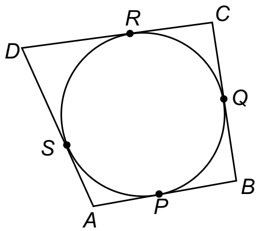

*i.e.,* $AD = BD$

In right $\Delta OBD$,

$$OB^2 = OD^2 + DB^2 \quad \text{(By Pythagoras theorem)}$$

$$\Rightarrow \quad 5^2 = 3^2 + DB^2$$
$$\Rightarrow \quad DB^2 = 25 - 9 = 16$$
$$\Rightarrow \quad DB = 4 \text{ cm}$$
$$\therefore \quad \text{Length of chord } = AB = 2\,AD = 2 \times 4 = 8 \text{ cm}$$

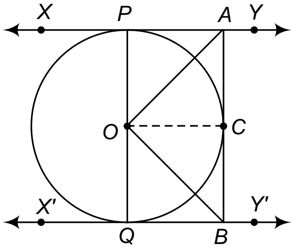
**Question 8.** A quadrilateral $ABCD$ is drawn to circumscribe a circle (see figure). Prove that $AB + CD = AD + BC$.




**Solution** Using theorem, the lengths of tangents drawn from an external point to a circle are equal.

Suppose, $A$ is an external point, then
$$AP = AS \quad \ldots\text{(i)}$$
Suppose, $B$ is an external point, then
$$BP = BQ \quad \ldots\text{(ii)}$$
Suppose, $C$ is an external point, then
$$CQ = RC \quad \ldots\text{(iii)}$$
Suppose, $D$ is an external point, then
$$SD = RD \quad \ldots\text{(iv)}$$
On adding Eqs. (i), (ii), (iii) and (iv), we get
$$(AP + BP) + (RC + RD) = (AS + BQ) + (CQ + SD)$$
$$\Rightarrow \quad AB + CD = (AS + SD) + (BQ + CQ)$$
$$\Rightarrow \quad AB + CD = AD + BC$$
Hence proved.

**Question 9.** In figure, $XY$ and $X'Y'$ are two parallel tangents to a circle with centre $O$ and another tangent $AB$ with point of contact $C$ intersecting $XY$ at $A$ and $X'Y'$ at $B$. Prove that $\angle AOB = 90°$.



**Solution** Since, tangents drawn from an external point to a circle are equal.

$\therefore \quad AP = AC$

Thus, in $\Delta APO$ and $\Delta ACO$,

$$AP = AC$$

$$AO = AO \quad \text{(Common)}$$

$$OP = OC \quad \text{(Radius of circle)}$$

$\therefore$ By SSS criterion of congruence, we have

$$\Delta APO \cong \Delta ACO$$

$\Rightarrow \quad \angle PAO = \angle OAC$

$\Rightarrow \quad \angle PAC = 2\angle CAO$

Similarly, we can prove that

$$\angle CBO = \angle OBQ$$

$\Rightarrow \quad \angle CBQ = 2\angle CBO$

Since, $XY \parallel X'Y'$

$\therefore \quad \angle PAC + \angle QBC = 180°$

(Sum of interior angles on the same side of transversal is 180°)

$\therefore \quad 2\angle CAO + 2\angle CBO = 180° \quad \text{...(i)}$

$\Rightarrow \quad \angle CAO + \angle CBO = 90°$

In $\Delta AOB$,

$\Rightarrow \quad \angle CAO + \angle CBO + \angle AOB = 180°$

$\Rightarrow \quad \angle CAO + \angle CBO = 180° - \angle AOB \quad \text{...(ii)}$

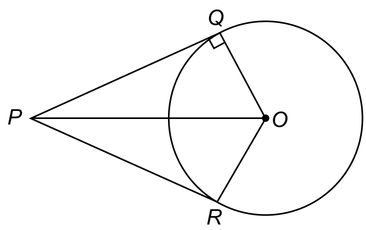
$\therefore$ From Eqs. (i) and (ii), we get

$$180° - \angle AOB = 90°$$

$\Rightarrow \quad \angle AOB = 90°$

**Question 10.** Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

**Solution** Let $PQ$ and $PR$ be two tangents drawn from an external point $P$ to a circle with centre $O$.




We have to prove that,

$$\angle QOR = 180° - \angle QPR$$

or $\quad \angle QOR + \angle QPR = 180°$

In right $\Delta OQP$ and $\Delta ORP$,

$$PQ = PR$$

(Tangents drawn from an external point are equal)

$$OQ = OR \quad \text{(Radius of circle)}$$

$$OP = OP \quad \text{(Common)}$$

Therefore, by SSS criterion of congruence,

$$\Delta OQP \cong ORP$$

$\Rightarrow$ $\angle QPO = \angle RPO$

and $\angle POQ = \angle POR$

$\Rightarrow$ $\angle QPR = 2\angle OPQ$

and $\angle QOR = 2\angle POQ$ ...(i)

In $\Delta OPQ$,

$$\angle QPO + \angle QOP = 90°$$

$\Rightarrow$ $\angle QOP = 90° - \angle QPO$

$\Rightarrow$ $2\angle QOP = 180° - 2\angle QPO$ (Multiplying both sides by 2)

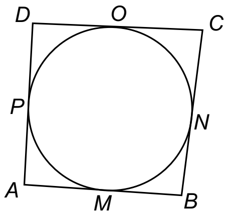
$\Rightarrow$ $\angle QOR = 180° - \angle QPR$ [From Eq. (i)]

$\Rightarrow$ $\angle QOR + \angle QPR = 180°$

Hence proved.

**Question 11.** Prove that the parallelogram circumscribing a circle is a rhombus.

**Solution** Let *ABCD* be a parallelogram circumscribing a circle. Using the property that the tangents to a circle from an exterior point are equal in length.




$\therefore$ $AM = AP$ and $BM = BN$

$CO = CN$ and $DO = DP$

On adding all equations, we get

$$(AM + BM) + (CO + DO) = AP + BN + CN + DP \quad \left[\begin{array}{l} \because AB = AM + MB, \\ BC = BN + NC, \\ CD = CO + OD, \\ AD = AP + PD \end{array}\right]$$

$\Rightarrow$ $AB + CD = (AP + PD) + (BN + NC) = AD + BC$

$\Rightarrow$ $2AB = 2BC$

($\because$ *ABCD* is a parallelogram. $\therefore AB = CD$ and $BC = AD$)

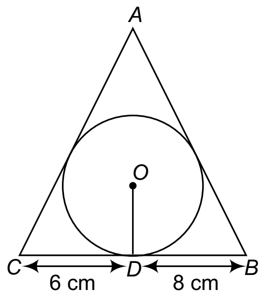
$\Rightarrow$ $AB = BC$

$\therefore$ $AB = BC = CD = DA$

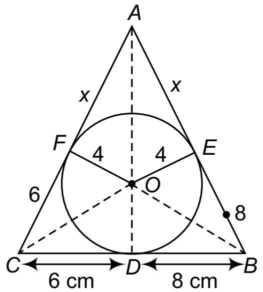
Hence, *ABCD* is a rhombus.

Hence proved.

**Question 12.** A $\Delta ABC$ is drawn to circumscribe a circle of radius 4 cm such that the segments $BD$ and $DC$ into which $BC$ is divided by the point of contact $D$ are of lengths 8 cm and 6 cm respectively (see figure). Find the sides $AB$ and $AC$.




**Solution** Given, $CD = 6$ cm, $BD = 8$ cm and radius $= 4$ cm




Join $OC$, $OA$ and $OB$.

By using the property, tangents drawn from external point equal in length.

$\therefore$ $\qquad CD = CF = 6$ cm

and $\qquad BD = BE = 8$ cm

Let $\qquad AF = AE = x$ cm

In $\Delta OCB$,

$$\text{Area of triangle, } A_1 = \frac{1}{2}\text{Base} \times \text{Height}$$
$$= \frac{1}{2} \times CB \times OD$$
$$= \frac{1}{2} \times 14 \times 4 = 28 \text{ cm}^2$$

In $\Delta OCA$,

$$\text{Area of triangle, } A_2 = \frac{1}{2} \times AC \times OF = \frac{1}{2}(6 + x) \times 4 = 12 + 2x$$

In $\Delta OBA$,

$$\text{Area of triangle, } A_3 = \frac{1}{2} \times AB \times OE = \frac{1}{2}(8 + x) \times 4$$

$$= 16 + 2x$$

Now, perimeter of triangle, $ABC = \frac{1}{2}(AB + BC + CA)$

$$S = \frac{1}{2}(x + 6 + 14 + 8 + x)$$

$$S = 14 + x$$

Now, area of $\Delta ABC = A_1 + A_2 + A_3$

$$= 28 + (12 + 2x) + (16 + 2x)$$

$$= 56 + 4x \quad \text{...(i)}$$

Using Heron's formula,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(14 + x)(14 + x - 14)(14 + x - x - 6)(14 + x - x - 8)}$$

$$= \sqrt{(14 + x)(x)(8)(6)}$$

$$= \sqrt{(14 + x)\, x\, 48} \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\sqrt{(14 + x)\, x\, 48} = 56 + 4x$$

On squaring both sides,

$$(14 + x)\, 48\, x = 4^2\, (14 + x)^2$$

$\Rightarrow$ $3x = 14 + x$

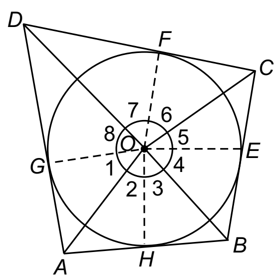
$\Rightarrow$ $2x = 14 \Rightarrow x = 7$

$\therefore$ Length $AC = 6 + x = 6 + 7 = 13$ cm

Length of $AB = 8 + x = 8 + 7 = 15$ cm

**Question 13.** Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Solution** Let $ABCD$ be a quadrilateral circumscribing a circle with centre $O$. A circle touches the sides of a quadrilateral at points $E$, $F$, $G$ and $H$.



**To prove** $\angle AOB + \angle COD = 180°$

and $\angle AOD + \angle BOC = 180°$

**Construction** Join *OH*, *OE*, *OF* and *OG*.

**Proof** Using the property, two tangents drawn from an external point to a circle subtend equal angles at the centre.

$$\therefore \quad \left.\begin{aligned} \angle 1 &= \angle 2 \\ \angle 3 &= \angle 4 \\ \angle 5 &= \angle 6 \\ \angle 7 &= \angle 8 \end{aligned}\right\} \quad \ldots(i)$$

We know the sum of all angles subtended at a point *O* is 360°.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \quad 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

and $\quad 2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$ [From Eq. (i)]

$\Rightarrow \quad (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$

and $\quad (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

$\Rightarrow \quad \angle AOB + \angle COD = 180°$

and $\quad \angle AOD + \angle BOC = 180°$

Hence proved.