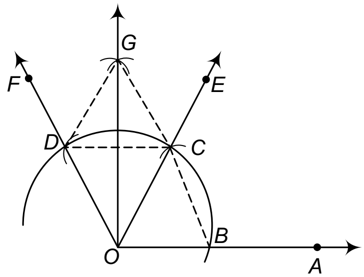
12 **Constructions**

Exercise 12.1

Question 1. Construct an angle of 90° at the initial point of a given ray and justify the construction.

Solution Steps of construction



1. Taking O as centre and some radius, draw an arc of a circle which intersects OA, say at a point B.
2. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
3. Taking C as centre and with the same radius as before, draw an arc intersecting the arc drawn in step 1, say at D.
4. Draw the ray OE passing through C.

   Then, $\angle EOA = 60°$
5. Draw the ray of passing through D. Then, $\angle FOE = 60°$.
6. Next, taking C and D as centres and with the radius more than $\frac{1}{2}CD$, draw arcs to intersect each other, say at G.
7. Draw the ray OG. This ray OG is the bisector of the $\angle FOE$ *i.e.*,

$$\angle FOG = \angle EOG = \frac{1}{2}\angle FOE = \frac{1}{2}(60°) = 30°$$

Thus, $\angle GOA = \angle GOE + \angle EOA$

$= 30° + 60° = 90°$

Justification

(i) Join BC.

Then, $OC = OB = BC$ (By construction)

$\therefore \Delta\ COB$ is an equilateral triangle.

$\therefore$ $\angle COB = 60°$

$\therefore$ $\angle EOA = 60°$

(ii) Join CD.

Then, $OD = OC = CD$ (By construction)

$\therefore \Delta\ DOC$ is an equilateral triangle.

$\therefore$ $\angle DOC = 60°$

$\therefore$ $\angle FOE = 60°$

(iii) Join CG and DG.

In ΔODG and ΔOCG,

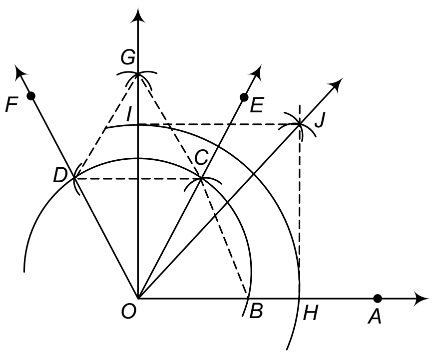
$$OD = OC \quad \text{(Radii of the same arc)}$$
$$DG = CG \quad \text{(Arcs of equal radii)}$$
$$OG = OG \quad \text{(Common)}$$
$$\therefore \quad \Delta ODG \cong \Delta OCG \quad \text{(SSS rule)}$$
$$\therefore \quad \angle DOG = \angle COG \quad \text{(CPCT)}$$
$$\therefore \quad \angle FOG = \angle EOG = \frac{1}{2} \angle FOE$$
$$= \frac{1}{2}(60^\circ) = 30^\circ$$

Thus, $\angle GOA = \angle GOE + \angle EOA = 30^\circ + 60^\circ = 90^\circ$

Question 2. Construct an angle of 45° at the initial point of a given ray and justify the construction.

Solution **Steps of construction**

1. Taking O as centre and some radius, draw an arc of a circle which intersects OA, say at a point B.
2. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.



3. Taking C as centre and with the same radius as before, draw an arc intersecting the arc drawn in step 1, say at D.
4. Draw the ray OE passing through C. Then, $\angle EOA = 60^\circ$.
5. Draw the ray OF passing through D. Then, $\angle FOE = 60^\circ$.
6. Next, taking C and D as centres and with radius more than $\frac{1}{2}CD$, draw arcs to intersect each other, say at G.
7. Draw the ray OG. This ray OG is the bisector of the $\angle FOE$, *i.e.*, $\angle FOG = \angle EOG = \frac{1}{2}\angle FOE = \frac{1}{2}(60^\circ) = 30^\circ$.

Thus, $\angle GOA = \angle GOE + \angle EOA$
$= 30^\circ + 60^\circ = 90^\circ$

8. Now taking O as centre and any radius, draw an arc to intersect the rays OA and OG, say at H and I, respectively.
9. Next, taking H and I as centres and with the radius more than $\frac{1}{2}HI$, draw arcs to intersect each other, say at J.
10. Draw the ray OJ. This ray OJ is the required bisector of the $\angle GOA$.

Thus, $$\angle GOJ = \angle AOJ = \frac{1}{2}\angle GOA$$
$$= \frac{1}{2}(90°) = 45°$$

Justification

(i) Join BC. (By construction)

Then, $$OC = OB = BC$$

$\therefore \Delta\, COB$ is an equilateral triangle.

$\therefore$ $$\angle COB = 60°$$

$\therefore$ $$\angle EOA = 60°$$

(ii) Join CD.

Then, $$OD = OC = CD \quad \text{(By construction)}$$

$\therefore \Delta\, DOC$ is an equilateral triangle.

$\therefore$ $$\angle DOC = 60°$$

$\therefore$ $$\angle FOE = 60°$$

(iii) Join CG and DG.

In $\Delta\, ODG$ and $\Delta\, OCG$,

$$OD = OC \quad \text{(Radii of the same arc)}$$
$$DG = CG \quad \text{(Arcs of equal radii)}$$
$$OG = OG \quad \text{(Common)}$$

$\therefore$ $$\Delta\, ODG \cong \Delta\, OCG \quad \text{(SSS rule)}$$

$\therefore$ $$\angle DOG = \angle COG \quad \text{(CPCT)}$$

$\therefore$ $$\angle FOG = \angle EOG = \frac{1}{2}\angle FOE$$
$$= \frac{1}{2}(60°) = 30°$$

Thus, $$\angle GOA = \angle GOE + \angle EOA$$
$$= 30° + 60° = 90°$$

(iv) Join HJ and IJ.

In $\Delta\, OIJ$ and $\Delta\, OHJ$,

$$OI = OH \quad \text{(Radii of the same arc)}$$
$$IJ = HJ \quad \text{(Arcs of equal radii)}$$
$$OJ = OJ \quad \text{(Common)}$$

$\therefore$ $$\Delta\, OIJ \cong \Delta\, OHJ \quad \text{(SSS rule)}$$

$\therefore \quad \angle IOJ = \angle HOJ$ (CPCT)

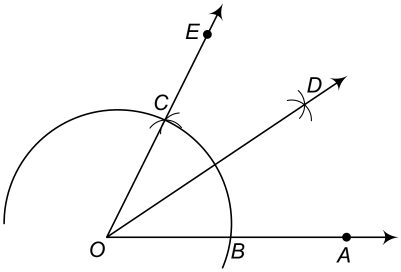
$\therefore \quad \angle AOJ = \angle GOJ = \frac{1}{2} \angle GOA$

$= \frac{1}{2} (90°) = 45°$

Question 3. Construct the angles of the following measurements

(i) 30° (ii) $22\frac{1}{2}°$ (iii) 15°

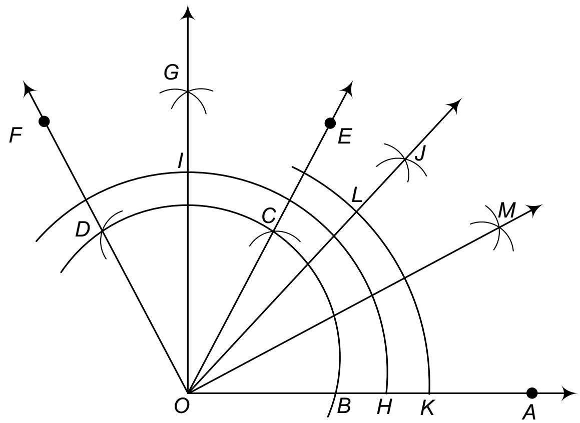
Solution (i) **Steps of construction**



1. Taking O as centre and some radius, draw an arc of a circle which intersects OA, say at a point B.
2. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
3. Draw the ray OE passing through C. Then, $\angle EOA = 60°$.
4. Taking B and C as centres and with the radius more than $\frac{1}{2}BC$, draw arcs to intersect each other, say at D.
5. Draw the ray OD, this ray OD is the bisector of the $\angle EOA$, *i.e.*,

$$\angle EOD = \angle AOD = \frac{1}{2}\angle EOA = \frac{1}{2}(60°) = 30°$$

(ii) **Steps of construction**



1. Taking O as centre and some radius, draw an arc of a circle which intersects OA, say at a point B.

2. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
3. Taking C as centre and with the same radius as before, drawn an arc intersecting the arc drawn in step 1, say at D.
4. Draw the ray OE passing through C. Then, $\angle EOA = 60°$.
5. Draw the ray OF passing through D. Then, $\angle FOE = 60°$.
6. Next, taking C and D as centres and with radius more than $\frac{1}{2}CD$, draw arcs to intersect each other, say at G.
7. Draw the ray OG. This ray OG is the bisector of the $\angle FOE$,

   i.e., $\angle FOG = \angle EOG = \frac{1}{2}\angle FOE$

   $= \frac{1}{2}(60°) = 30°$

   Thus, $\angle GOA = \angle GOE + \angle EOA$

   $= 30° + 60° = 90°$
8. Now, taking O as centre and any radius, draw an arc to intersect the rays OA and OG, say at H and I, respectively.
9. Next, taking H and I as centres and with the radius more than $\frac{1}{2}HI$, draw arcs to intersect each other, say at J.
10. Draw the ray OJ. This ray OJ is the bisector of the $\angle GOA$.

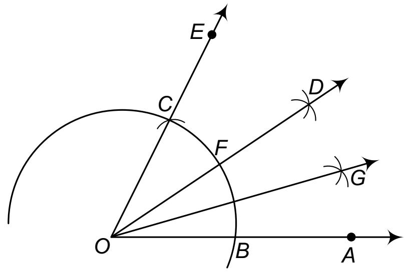
    i.e., $\angle GOJ = \angle AOJ = \frac{1}{2}\angle GOA$

    $= \frac{1}{2}(90°) = 45°$
11. Now, taking O as centre and any radius, drawn an arc to intersect the rays OA and OJ, say at K and L, respectively.
12. Next, taking K and L as centres and with the radius more than $\frac{1}{2}KL$, draw arcs to intersect each other, say at H.
13. Draw the ray OM. This ray OM is the bisector of the $\angle AOJ$, *i.e.*, $\angle JOM = \angle AOM$

    $$= \frac{1}{2}\angle AOJ = \frac{1}{2}(45°) = 22\frac{1}{2}°$$

(iii) **Steps of construction**

1. Taking O as centre and some radius, draw an arc of a circle which intersects OA, say at a point B.
2. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
3. Draw the ray OE passing through C. Then, $\angle EOA = 60°$.



4. Now, taking B and C as centres and with the radius more than $\frac{1}{2}BC$, draw arcs to intersect each other, say at D.
5. Draw the ray OD intersecting the arc drawn in step 1 at F. This ray OD is the bisector of the $\angle EOA$, *i.e.*,

$$\angle EOD = \angle AOD = \frac{1}{2}\angle EOA = \frac{1}{2}(60°) = 30°$$

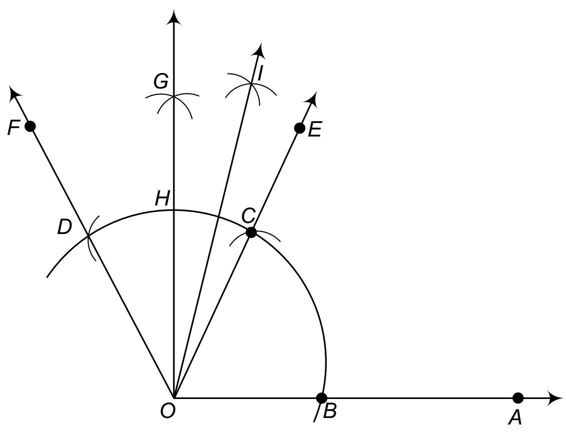
6. Now, taking B and F as centres and with the radius more than $\frac{1}{2}BF$, draw arcs to intersect each other, say at G.
7. Draw the ray OG. This ray OG is the bisector of the $\angle AOD$,

i.e., $$\angle DOG = \angle AOG = \frac{1}{2}\angle AOD = \frac{1}{2}(30°) = 15°$$

Question 4. Construct the following angles and verify by measuring them by a protractor

(i) 75° (ii) 105° (iii) 135°

Solution (i) **Steps of construction**



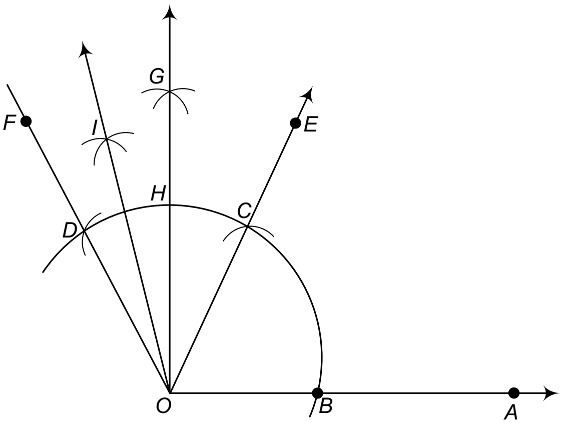
1. Taking O as centre and some radius, draw an arc of a circle which intersects OA, say at a point B.
2. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
3. Taking C as centre and with the same radius as before, draw an arc intersecting the arc drawn in step 1, say at D.
4. Join the ray OE passing through C. Then, $\angle EOA = 60°$.
5. Draw the ray of passing through D. Then, $\angle FOE = 60°$.
6. Next, taking C and D as centres and with the radius more than $\frac{1}{2}CD$, draw arcs to intersect each other, say at G.
7. Draw the ray OG intersecting the arc of step 1 at H. This ray OG is the bisector of the $\angle FOE$, *i.e.*, $\angle FOG = \angle EOG$
$$= \frac{1}{2}\angle FOE = \frac{1}{2}(60°) = 30°$$
8. Next, taking C and H as centres and with the radius more than $\frac{1}{2}CH$, draw arcs to intersect each other, say at I.
9. Draw the ray OI. This ray OI is the bisector of the $\angle GOE$,
i.e., $\angle GOI = \angle EOI = \frac{1}{2}\angle GOE = \frac{1}{2}(30°) = 15°$

Thus, $\angle IOA = \angle IOE + \angle EOA$
$= 15° + 60° = 75°$

On measuring the $\angle IOA$ by protractor, we find that $\angle IOA = 15°$

Thus, the construction is verified.

(ii) **Steps of construction**



1. Taking O as centre and some radius, draw an arc of a circle which intersects OA, say at a point B.

2. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
3. Taking C as centre and with the same radius as before, draw an arc intersecting the arc drawn in step 1, say at a point D.
4. Draw the ray OE passing through C. Then, $\angle EOA = 60°$.
5. Draw the ray OF passing through D. Then, $\angle FOE = 60°$.
6. Next, taking C and D as centres and with the radius more than $\frac{1}{2}CD$, draw arcs to intersect each other, say at G.
7. Draw the ray OG intersecting the arc drawn in step 1 at H. This ray OG is the bisector of the $\angle FOE$, *i.e.*,

$$\angle FOG = \angle EOG = \frac{1}{2}\angle FOE$$

$$= \frac{1}{2}(60°) = 30°.$$

Thus, $\angle GOA = \angle GOE + \angle EOA$

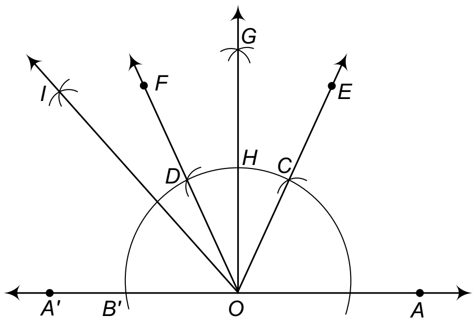
$$= 30° + 60° = 90°$$

8. Next, taking H and D as centres and with the radius more than $\frac{1}{2}HD$, draw arcs to intersect each other, say at I.
9. Draw the ray OI. This ray OI is the bisector of the $\angle FOG$, *i.e.*, $\angle FOI = \angle GOI = \frac{1}{2}\angle FOG = \frac{1}{2}(30°) = 15°$

Thus, $\angle IOA = \angle IOG + \angle GOA = 15° + 90° = 105°$. On measuring the $\angle IOA$ by protractor, we find that $\angle FOA = 105°$.

Thus, the construction is verified.

(iii) **Steps of construction**



1. Produce AO to A' to form ray OA'.
2. Taking O as centre and some radius, draw an arc of a circle which intersects OA at a point B and OA' at a point B'.
3. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc at a point C.

4. Taking C as centre and with the same radius as before, draw an arc intersecting the arc drawn in step 1, say at D.
5. Draw the ray OE passing through C, then $\angle EOA = 60°$.
6. Draw the ray OF passing through D, then $\angle FOE = 60°$.
7. Next, taking C and D as centres and with the radius more than $\frac{1}{2}CD$, draw arcs to intersect each other, say at G.
8. Draw the ray OG intersecting the arc drawn in step 1 at H. This ray OG is the bisector of the $\angle FOE$ *i.e.*,

$$\angle FOG = \angle EOG = \frac{1}{2}\angle FOE = \frac{1}{2}(60°) = 30°$$

Thus, $$\angle GOA = \angle GOE + \angle EOA = 30° + 60° = 90°$$

$\therefore$ $$\angle B'OH = 90°$$

9. Next, taking B' and H as centres and with the radius more than $\frac{1}{2}B'H$, drawn arcs to intersect each other, say at I.
10. Draw the ray OI. This ray OI is the bisector of the $\angle B'OG$ *i.e.*, $\angle B'OI = \angle GOI = \frac{1}{2}\angle B'OG = \frac{1}{2}(90°) = 45°$

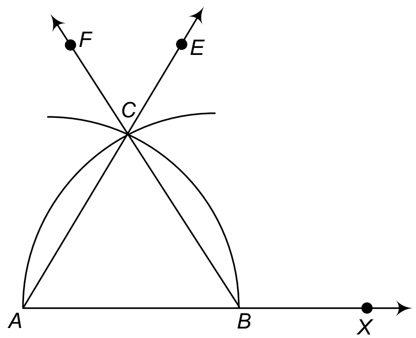
Thus, $$\angle IOA = \angle IOG + \angle GOA = 45° + 90° = 135°$$

On measuring the $\angle IOA$ by protractor, we find that $\angle IOA = 135°$.

Thus, the construction is verified.

Question 5. Construct an equilateral triangle, given its side and justify the construction.

Solution **Steps of construction**



1. Take a ray AX with initial point A. From AX, cut off $AB = 4$ cm.
2. Taking A as centre and radius (= 4 cm), draw an arc of a circle, which intersects AX, say at a point B.

3. Taking B as centre and with the same radius as before, draw an arc intersecting the previously drawn arc, say at a point C.
4. Draw the ray AE passing through C.
5. Next, taking B as centre and radius (= 4 cm), draw an arc of a circle, which intersects AX, say at a point A.
6. Taking A as centre and with the same radius as in step 5, draw an arc intersecting the previously drawn arc, say at a point C.
7. Draw the ray BF passing through C.

   Then, Δ ABC is the required triangle with gives side 4 cm.

   Justification

   $AB = BC$ (By construction)

   $AB = AC$ (By construction)

   $\therefore$ $AB = BC = CA$

   $\therefore \Delta$ ABC is an equilateral triangle.

   $\therefore$ The construction is justified.

12 Constructions

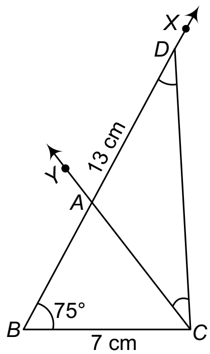
Exercise 12.2

Question 1. Construct a ΔABC in which $BC = 7$ cm, $\angle B = 75°$ and $AB + AC = 13$ cm.

Solution Given that, in ΔABC, $BC = 7$ cm, $\angle B = 75°$ and $AB + AC = 13$ cm

Steps of construction

1. Draw the base $BC = 7$ cm.



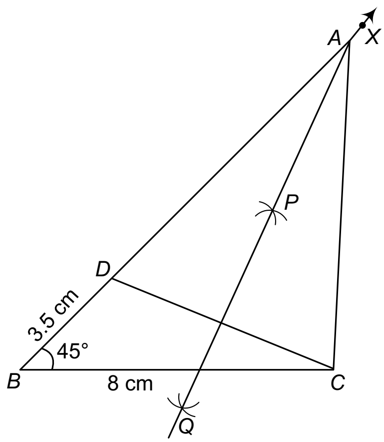
2. At the point B make an $\angle XBC = 75°$.
3. Cut a line segment BD equal to $AB + AC = 13$ cm from the ray BX.
4. Join DC.
5. Make an $\angle DCY = \angle BDC$.
6. Let CY intersect BX at A.

Then, ABC is the required triangle.

Question 2. Construct a ΔABC in which $BC = 8$ cm, $\angle B = 45°$ and $AB - AC = 35$ cm.

Solution Given that, in ΔABC,

$$BC = 8 \text{ cm}, \angle B = 45° \text{ and } AB - AC = 3.5 \text{ cm}$$



Steps of construction

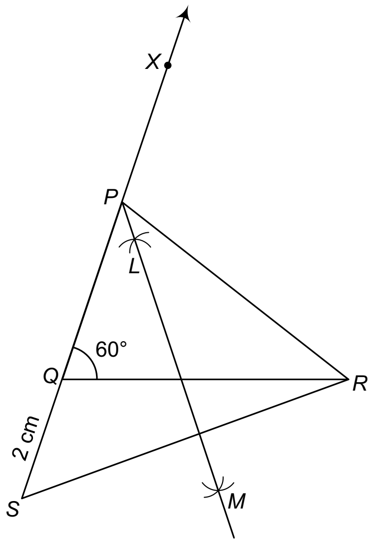
1. Draw the base $BC = 8$ cm
2. At the point B make an $\angle XBC = 45°$.
3. Cut the line segment BD equal to $AB - AC = 3.5$ cm from the ray BX.
4. Join DC.
5. Draw the perpendicular bisector, say PQ of DC.
6. Let it intersect BX at a point A.
7. Join AC.

Question 3. Construct a $\Delta\, ABC$ in which $QR = 6$ cm, $\angle Q = 60°$ and $PR - PQ = 2$ cm.

Solution Given that, in $\Delta\, ABC$, $QR = 6$ cm, $\angle Q = 60°$ and $PR - PQ = 2$ cm

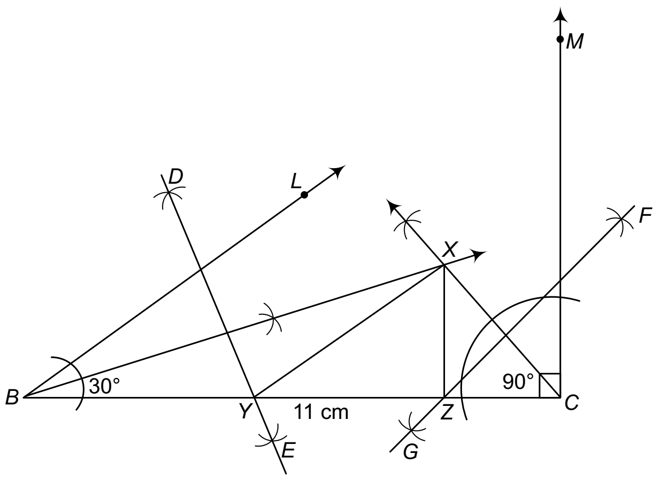
Steps of construction

1. Draw the base $QR = 6$ cm.
2. At the point Q make an $\angle XQR = 60°$.
3. Cut line segment $QS = PR - PQ$ (= 2 cm) from the line QX extended on opposite side of line segment QR.
4. Join SR.
5. Draw the perpendicular bisector LM of SR.
6. Let LM intersect QX at P.
7. Join PR.



Question 4. Construct a ΔXYZ in which $\angle Y = 30^\circ$, $\angle Z = 90^\circ$ and $XY + YZ + ZX = 11$ cm.

Solution Given that, in ΔXYZ $\angle Y = 30^\circ$, $\angle Z = 90^\circ$ and $XY + YZ + ZX = 11$ cm.



Steps of construction

1. Draw a line segment $BC = XY + YZ + ZX = 11$ cm
2. Make $\angle LBC = \angle Y = 30^\circ$ and $\angle MCB = \angle Z = 90^\circ$.
3. Bisect $\angle LBC$ and $\angle MCB$. Let these bisectors meet at a point X.

4. Draw perpendicular bisectors *DE* of *XB* and *FG* of *XC*.
5. Let *DE* intersect *BC* at *Y* and *FC* intersect *BC* at *Z*.
6. Join *XY* and *XZ*.

Then, *XYZ* is the required triangle.

Question 5. Construct a right triangle whose base is 12 cm and sum of its hypotenuse and other side is 18 cm.

Solution Given that, in ΔABC, base $BC = 12$ cm, $\angle B = 90°$ and $AB + BC = 18$ cm.

Steps of construction

1. Draw the base $BC = 12$ cm.
2. At the point *B*, make an $\angle XBC = 90°$.
3. Cut a line segment $BD = AB + AC = 18$ cm from the ray *BX*.
4. Join *DC*.
5. Draw the perpendicular bisector *PQ* of *CD* to intersect *BD* at a point *A*.
6. Join *AC*.

Then, *ABC* is the required right triangle.