13 Coordinate Geometry

Exercise 13.1

Question 1. Find the distance between the following pairs of points

(i) (2, 3), (4, 1) (ii) (– 5, 7), (– 1, 3)

(iii) (a, b), $(-a, -b)$

Solution (i) Let $A(2, 3)$ and $B(4, 1)$ be the given points.

Here, $x_1 = 2, y_1 = 3$ and $x_2 = 4, y_2 = 1$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-2)^2 + (1-3)^2}$$
$$= \sqrt{(2)^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8} = 2\sqrt{2}$$

(ii) Let $A(-5, 7)$ and $B(-1, 3)$ be the given points.

Here, $x_1 = -5, y_1 = 7$ and $x_2 = -1, y_2 = 3$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1+5)^2 + (3-7)^2}$$
$$= \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

(iii) Let $A(a, b)$ and $B(-a, -b)$ be the given points.

Here, $x_1 = a, y_1 = b$ and $x_2 = -a, y_2 = -b$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$
$$= \sqrt{(-2a)^2 + (-2b)^2}$$
$$= \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$$

Question 2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B.

Solution Let $M(0, 0)$ and $N(36, 15)$ be the given points.

Here, $x_1 = 0, y_1 = 0$ and $x_2 = 36, y_2 = 15$

$$\therefore \quad MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(36-0)^2 + (15-0)^2}$$
$$= \sqrt{1296 + 225} = \sqrt{1521} = 39$$

Since, the position of towns A and B are given (0, 0) and (36, 15), respectively and so, the distance between them is 39 km.

Question 3. Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.

Solution Let $A(1, 5)$, $B(2, 3)$ and $C(-2, -11)$ be the given points.

Then, we have $AB = \sqrt{(2-1)^2 + (3-5)^2}$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}\right]$$

$$= \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$$

$$= \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$$

$$= \sqrt{4 \times 53} = 2\sqrt{53}$$

$$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2}$$

$$= \sqrt{9+256} = \sqrt{265}$$

Now, $AB + AC = \sqrt{5} + \sqrt{265} \neq \sqrt{212}$

$\therefore$ $AB + AC \neq BC$

Similarly, $AB \neq BC + AC$

and $AC \neq AB + BC$

Hence, A, B and C are non-collinear.

Question 4. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Solution Let $P(5, -2)$, $Q(6, 4)$ and $R(7, -2)$ are the given points.

Then, $PQ = \sqrt{(6-5)^2 + (4+2)^2}$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{1+36} = \sqrt{37}$$

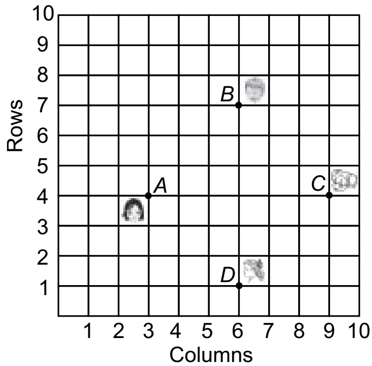
$$QR = \sqrt{(7-6)^2 + (-2-4)^2}$$

$$= \sqrt{1+36} = \sqrt{37}$$

Since, $PQ = QR$

$\therefore \Delta PQR$ is an isosceles triangle.

Question 5. In a classroom, 4 friends are seated at the points *A*, *B*, *C* and *D* as shown in figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think *ABCD* is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Solution From the given figure, the coordinates of points *A*, *B*, *C* and *D* are (3, 4), (6, 7), (9, 4) and (6, 1).

$\therefore$ $$AB = \sqrt{(6-3)^2 + (7-4)^2}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$= \sqrt{3^2 + 3^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2}$$

$$= \sqrt{3^2 + (-3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2}$$

$$= \sqrt{(-3)^2 + (-3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

and $$DA = \sqrt{(3-6)^2 + (4-1)^2}$$

$$= \sqrt{(-3)^2 + 3^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$\therefore$ $$AB = BC = CD = DA$$

Now, $$AC = \sqrt{(9-3)^2 + (4-4)^2}$$

$$= \sqrt{36+0} = 6$$

and $BD = \sqrt{(6-6)^2 + (1-7)^2}$

$= \sqrt{0 + 36} = \sqrt{36} = 6$

$\therefore$ $AC = BD = 6$

Since, the four sides and diagonals are equal. Hence, $ABCD$ is a square. So, Champa is correct.

Question 6. Name the type of quadrilateral formed, if any, by the following points and give reasons for your answer

(i) (– 1, – 2), (1, 0), (– 1, 2), (– 3, 0)

(ii) (– 3, 5), (3, 1), (0, 3), (– 1, – 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Solution (i) Let $A(-1, -2)$, $B(1, 0)$, $C(-1, 2)$ and $D(-3, 0)$ be the given points. Then,

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{2^2 + 2^2}$$

$$\left[\because \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{(-2)^2 + 2^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$\therefore$ $AB = BC = CD = DA$

Now, $AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0 + 4^2} = 4$

and $BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{(-4)^2 + 0} = 4$

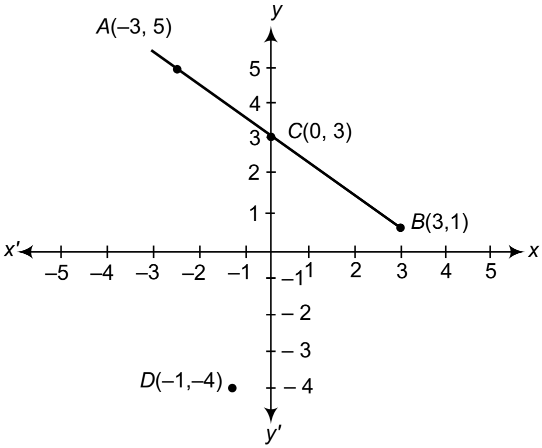
$\therefore$ $AC = BD$

Since, the four sides AB, BC, CD and DA are equal and also diagonals AC and BD are equal.

$\therefore$ The quadrilateral $ABCD$ is a square.

(ii) Let $A(-3, 5)$, $B(3, 1)$, $C(0, 3)$ and $D(-1, -4)$ be the given points.

From the figure, the points A, C and B are collinear. So, no quadrilateral is formed by given points.



(iii) Let $A(4, 5)$, $B(7, 6)$, $C(4, 3)$ and $D(1, 2)$ be the given points.

Then, $$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$

$$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$$

$$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0 + (-2)^2} = -2$$

and $$BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{(-6)^4 + (-4)^2}$$

$$= \sqrt{36 + 16} = \sqrt{52}$$

Since, $AB = CD$, $BC = DA$

and $AC \neq BD$

$\therefore$ The quadrilateral $ABCD$ is a parallelogram.

Question 7. Find the point on the X-axis which is equidistant from $(2, -5)$ and $(-2, 9)$.

Solution Since, the point on the X-axis.

$\therefore$ It's ordinate = 0

So, $A(x, 0)$ is any point on the X-axis.

Since, $A(x, 0)$ is equidistant from $B(2, -5)$ and $C(-2, 9)$.

$\therefore$ $$AB = AC$$

$\Rightarrow$ $$AB^2 = AC^2$$

$\Rightarrow$ $$(x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$$

$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$\Rightarrow \quad x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$
$$\Rightarrow \quad -4x - 4x = 81 - 25$$
$$\Rightarrow \quad -8x = 56$$
$$\Rightarrow \quad x = -\frac{56}{8} = -7$$

So, the point equidistant from given points on the x-axis is $(-7, 0)$.

Question 8. Find the values of y for which the distance between the points $P(2, -3)$ and $Q(10, y)$ is 10 units.

Solution According to question,

$$PQ = 10$$
$$\sqrt{(10-2)^2 + (y+3)^2} = 10$$
$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$
$$\sqrt{(8)^2 + (y+3)^2} = 10$$
$$\sqrt{64 + y^2 + 9 + 6y} = 10$$
$$\sqrt{y^2 + 6y + 73} = 10$$

Squaring both sides, we get

$$y^2 + 6y + 73 = 100$$
$$\Rightarrow \quad y^2 + 6y - 27 = 0$$
$$\Rightarrow \quad y^2 + 9y - 3y - 27 = 0$$
$$\Rightarrow \quad y(y+9) - 3(y+9) = 0$$
$$\Rightarrow \quad (y+9)(y-3) = 0$$
$$\Rightarrow \quad y = -9 \text{ or } y = 3$$
$$\Rightarrow \quad y = -9 \text{ or } 3$$

Question 9. If $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$, find the values of x. Also, find the distance QR and PR.

Solution Since, the point $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$.

$$\therefore \quad QP = QR$$
$$\Rightarrow \quad QP^2 = QR^2$$
$$\Rightarrow \quad (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$$
$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$
$$\Rightarrow \quad 5^2 + 4^2 = x^2 + 5^2$$
$$\Rightarrow \quad 25 + 16 = x^2 + 25$$
$$\Rightarrow \quad x^2 = 16$$
$$\Rightarrow \quad x = \pm 4$$

Thus, R is (4, 6) or (– 4, 6).

Now, QR = Distance between $Q(0, 1)$ and $R(4, 6)$

$$= \sqrt{(4-0)^2 + (6-1)^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{16 + 25} = \sqrt{41}$$

Also, QR = Distance between $Q(0, 1)$ and $R(-4, 6)$

$$= \sqrt{(-4-0)^2 + (6-1)^2}$$
$$= \sqrt{(-4)^2 + 5^2}$$
$$= \sqrt{16 + 25} = \sqrt{41}$$

and PR = Distance between $P(5, -3)$ and $R(4, 6)$

$$= \sqrt{(4-5)^2 + (6+3)^2}$$
$$= \sqrt{(-1)^2 + 9^2}$$
$$= \sqrt{1 + 81} = \sqrt{82}$$

Also, PR = Distance between $P(5, -3)$ and $R(-4, 6)$

$$= \sqrt{(-4-5)^2 + (6+3)^2}$$
$$= \sqrt{(-9)^2 + 9^2} = 9\sqrt{1+1}$$
$$= 9\sqrt{2}$$

Question 10. Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (– 3, 4).

Solution Let the point $A(x, y)$ be equidistant from the points $B(3, 6)$ and $C(-3, 4)$.

$$\therefore \quad AB = AC$$
$$\Rightarrow \quad AB^2 = AC^2$$
$$\Rightarrow \quad (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$
$$\left[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\,\right]$$
$$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$
$$\Rightarrow \quad -6x - 6x - 12y + 8y + 36 - 16 = 0$$
$$\Rightarrow \quad -12x - 4y + 20 = 0$$
$$\Rightarrow \quad -4(3x + y - 5) = 0$$
$$\Rightarrow \quad 3x + y - 5 = 0 \qquad (\because -4 \neq 0)$$

13 Coordinate Geometry

Exercise 13.2

Question 1. Find the coordinates of the point which divides the join of (– 1, 7) and (4, – 3) in the ratio 2 : 3.

Solution Let the coordinates of a point are (x, y).

We have, $x_1 = -1,\ y_1 = 7;$

$x_2 = 4,\ y_2 = -3$

and $m_1 = 2, m_2 = 3$

$\therefore$ By using section formula,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2(4) + 3(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

and

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2(-3) + 3(7)}{2+3}$$

$$= \frac{-6+21}{5} = \frac{15}{5} = 3$$

Hence, coordinates of the point are $(1, 3)$.

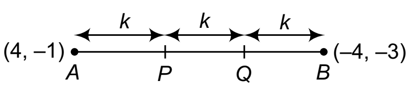
Question 2. Find the coordinates of the points of trisection of the line segment joining (4, – 1) and (– 2, – 3).

Solution Let $A(4, -1)$ and $B(-2, -3)$ be the line segments and points of trisection of the line segment be P and Q. Then,

$$AP = PQ = BQ = k \qquad \text{(Say)}$$

$\therefore$ $PB = PQ + QB = 2k$

and $AQ = AP + PQ = 2k$



$\Rightarrow$ $AP : PB = k : 2k = 1 : 2$

and $AQ : QB = 2k : k = 2 : 1$

Since, P divides AB internally in the ratio 1 : 2. So, the coordinate of P are

$$\left(\text{By using } \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \text{ and } \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1 \times (-2) + 2 \times 4}{1+2}, \frac{1 \times (-3) + 2 \times (-1)}{1+2}\right)$$

$$= \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right)$$

$$= \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, -\frac{5}{3}\right)$$

and Q divides AB internally in the ratio 2 : 1.

So, the coordinates of Q are

$$= \left(\frac{2 \times (-2) + 1 \times 4}{2+1}, \frac{2 \times (-3) + 1 \times (-1)}{2+1} \right)$$

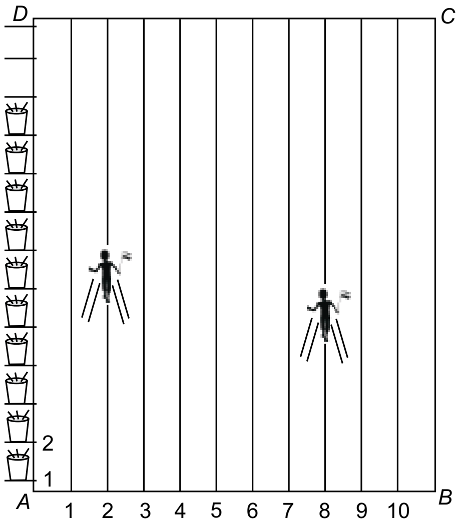
$$\left(\text{By using } \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \text{ and } \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6-1}{3} \right)$$

$$= \left(0, -\frac{7}{3} \right)$$

So, the two points of trisection are $\left(2, -\frac{5}{3}\right)$ and $\left(0, -\frac{7}{3}\right)$.

Question 3. To conduct Sports Day activities, in your rectangular shaped school ground *ABCD*, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along *AD*, as shown in figure. Niharika runs $\frac{1}{4}$th the distance *AD* on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance *AD* on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

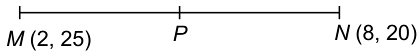


Solution From the above figure, the position of green flag posted by Niharika is $M\left(2, \frac{1}{4} \times 100\right)$ *i.e.*, M (2, 25) and red flag posted by Preet is $N\left(8, \frac{1}{5} \times 100\right)$ *i.e.*, N (8, 20).

Now,
$$MN = \sqrt{(8-2)^2 + (20-25)^2}$$
$$= \sqrt{(6)^2 + (-5)^2}$$
$$= \sqrt{36 + 25} = \sqrt{61}$$

Hence, the distance between flags $= \sqrt{61}$ m

Let P be the position of the blue flag posted by Rashmi in the halfway of line segment MN.



$\therefore$
$$P \text{ is given by } = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+8}{2}, \frac{25+20}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{45}{2}\right) = (5, 22.5)$$

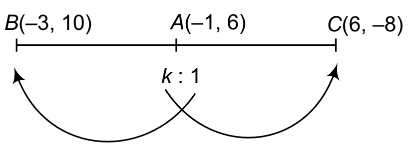
Hence, the blue flag is on the fifth line at a distance of 22.5 m above it.

Question 4. Find the ratio in which the line segment joining the points (–3, 10) and (6, – 8) is divided by (– 1, 6).

Solution Let the point $A(-1, 6)$ divide the line joining $B(-3, 10)$ and $C(6, -8)$ in the ratio $k : 1$. Then, the coordinates of A are $\left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$.

$$\left[\because \text{Internally ratio} \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)\right]$$

But, the coordinates of A are given by (– 1, 6).



On comparing coordinates, we get
$$\frac{6k-3}{k+1} = -1 \text{ and } \frac{-8k+10}{k+1} = 6$$
$$\Rightarrow \quad 6k - 3 = -k - 1 \text{ and } -8k + 10 = 6k + 6$$
$$\Rightarrow \quad 6k + k = -1 + 3 \text{ and } -8k - 6k = 6 - 10$$
$$\Rightarrow \quad 7k = 2 \text{ and } -14k = -4 \Rightarrow k = \frac{2}{7}$$

So, the point A divides BC in the ratio 2 : 7.

Question 5. Find the ratio in which the line segment joining $A(1, -5)$ and $B(-4, 5)$ is divided by the x-axis. Also, find the coordinates of the point of division.

Solution Let the required ratio be $k : 1$. So, the coordinates of the point M of division $A(1, -5)$ and $B(-4, 5)$ are $\left(\frac{kx_2 + 1 \cdot x_1}{k+1}, \frac{ky_2 + 1 \cdot y_1}{k+1}\right)$ *i.e.*, $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$.

But according to question, line segment joining $A(1, -5)$ and $B(-4, 5)$ is divided by the x-axis. So, y-coordinates must be zero.

$$\therefore \quad \frac{5k-5}{k+1} = 0$$

$$\Rightarrow \quad 5k - 5 = 0$$

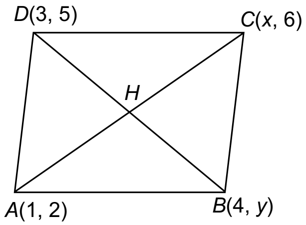
$$\Rightarrow \quad 5k = 5$$

$$\Rightarrow \quad k = 1$$

So, the required ratio is 1 : 1 and the point of division M is $\left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right)$ *i.e.*, $\left(\frac{-4+1}{2}, 0\right)$ *i.e.*, $\left(-\frac{3}{2}, 0\right)$.

Question 6. If $(1, 2)$, $(4, y)$, $(x, 6)$ and $(3, 5)$ are the vertices of a parallelogram taken in order, find x and y.

Solution Let $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$ are the vertices of a parallelogram.



Since, $ABCD$ is a parallelogram.

$\therefore$ AC and BD will bisect each other. Hence, mid-point of AC and mid-point of BD are same point.

$\therefore$ Mid-point of AC is $\left(\frac{1+x}{2}, \frac{2+6}{2}\right)$ = Mid-point of BD is $\left(\frac{4+3}{2}, \frac{y+5}{2}\right)$

$$\left[\because \text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

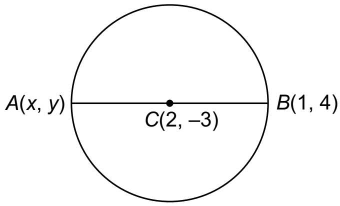
$$\therefore \quad \frac{1+x}{2} = \frac{4+3}{2} \text{ and } \frac{2+6}{2} = \frac{5+y}{2}$$

$$\Rightarrow \quad 1 + x = 7 \text{ and } 8 = 5 + y$$

$$\therefore \quad x = 6 \text{ and } y = 3$$

Question 7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

Solution Suppose, AB be a diameter of the circle having its centre at $C(2, -3)$ and coordinates of end point B are (1, 4).



Let the coordinates of A be (x, y).

Since, AB is diameter.

$\therefore C$ is the mid-point of AB.

The coordinates of C are $\left(\frac{x+1}{2}, \frac{y+4}{2}\right)$.

$$\left[\because \text{Coordinate of mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$$

But it is given that the coordinates of C are (2, – 3).

$\therefore$ $$\frac{x+1}{2} = 2$$

$\Rightarrow$ $$x + 1 = 4 \Rightarrow x = 3$$

and $$\frac{y+4}{2} = -3$$

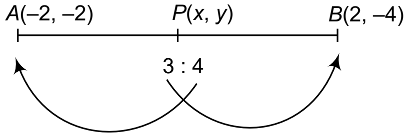
$\Rightarrow$ $$y + 4 = -6$$

$\Rightarrow$ $$y = -10$$

So, the required coordinates of A are (3, – 10).

Question 8. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that $AP = \frac{3}{7} AB$ and P lies on the line segment AB.

Solution According to the question,



$$AP = \frac{3}{7} AB$$

$\Rightarrow$ $$\frac{AB}{AP} = \frac{7}{3}$$

$$\Rightarrow \quad \frac{AP + PB}{AP} = \frac{3 + 4}{3}$$

$$\Rightarrow \quad 1 + \frac{PB}{AP} = 1 + \frac{4}{3}$$

$$\Rightarrow \quad \frac{PB}{AP} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{AP}{PB} = \frac{3}{4}$$

$$\Rightarrow \quad AP : PB = 3 : 4$$

Suppose, $P(x, y)$ be the point which divides the line segment joining the points $A(-2, -2)$ and $B(2, -4)$ in the ratio 3 : 4.

$$\therefore \quad x = \frac{3 \times 2 + 4 \times (-2)}{3 + 4} = \frac{6 - 8}{7} = -\frac{2}{7} \quad \left(\because x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}\right)$$

and

$$y = \frac{3 \times (-4) + 4 \times (-2)}{3 + 4} \quad \left(\because y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$= \frac{-12 - 8}{7}$$

$$= -\frac{20}{7}$$

Hence, the required coordinates of the point P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

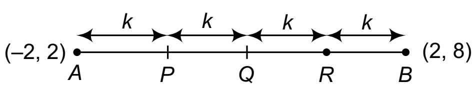
Question 9. Find the coordinates of the points which divide the line segment joining $A(-2, 2)$ and $B(2, 8)$ into four equal parts.

Solution Let P, Q and R be the points on line segment AB such that

$$AP = PQ = QR = RB$$

Let

$$AP = PQ = QR = RB = k$$



Now,

$$\frac{AP}{PB} = \frac{k}{3k} = \frac{1}{3}$$

Therefore, P divides AB internally in the ratio 1 : 3.

$$\because \quad \text{Internally ratio} = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$\therefore \quad P = \left(\frac{1 \times 2 + 3(-2)}{1 + 3}, \frac{1 \times 8 + 3 \times 2}{1 + 3}\right)$$

$$= \left(\frac{2 - 6}{4}, \frac{8 + 6}{4}\right)$$

$$= \left(\frac{-4}{4}, \frac{14}{4}\right) = \left(-1, \frac{7}{2}\right)$$

Again, $$\frac{AR}{RB} = \frac{3k}{k} = \frac{3}{1}$$

Therefore, R divides AB internally in the ratio 3 : 1.

$$\therefore \quad R = \left(\frac{3 \times 2 + 1 \times (-2)}{3+1}, \frac{3 \times 8 + 1 \times 2}{3+1}\right)$$
$$= \left(\frac{6-2}{4}, \frac{24+2}{4}\right)$$
$$= \left(\frac{4}{4}, \frac{26}{4}\right) = \left(1, \frac{13}{2}\right)$$

Also, $$\frac{AQ}{QB} = \frac{2k}{2k} = \frac{1}{1}$$

$\therefore Q$ is the mid-point of AB.

$$\therefore \quad Q = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) \quad i.e., \quad Q = \left(\frac{0}{2}, \frac{10}{2}\right) = (0, 5)$$

$$\left[\because \text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$$

So, required points are $\left(-1, \frac{7}{2}\right), \left(1, \frac{13}{2}\right)$ and (0, 5).

Question 10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (−1, 4) and (−2, −1) taken in order.

[**Hint** Area of a rhombus = $\frac{1}{2}$ (Product of its diagonals)]

Solution Let $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$ and $D(-2, -1)$ be the vertices of the rhombus $ABCD$.

$$\therefore \quad \text{Diagonal, } AC = \sqrt{(-1-3)^2 + (4-0)^2}$$

$$[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\ ]$$

$$= \sqrt{(-4)^2 + 4^2}$$
$$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$$

$$\text{Diagonal, } BD = \sqrt{(-2-4)^2 + (-1-5)^2}$$
$$= \sqrt{(-6)^2 + (-6)^2}$$
$$= \sqrt{36 + 36}$$
$$= \sqrt{72} = 6\sqrt{2}$$

$$\therefore \quad \text{Area of the rhombus } ABCD = \frac{1}{2} \times AC \times BD$$
$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$
$$= 2 \times 6 \times \sqrt{2} \times \sqrt{2} = 12 \times 2$$
$$= 24 \text{ sq units}$$

13 Coordinate Geometry

Exercise 13.3

Question 1. Find the area of the triangle whose vertices are

(i) (2, 3), (−1, 0), (2, − 4) (ii) (−5, − 1), (3, − 5), (5, 2)

Solution (i) Let $A = (x_1, y_1) = (2, 3)$, $B = (x_2, y_2) = (-1, 0)$, $C = (x_3, y_3) = (2, -4)$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[2(0 + 4) + (-1)(-4 - 3) + 2(3 - 0)]$$

$$= \frac{1}{2}(8 + 7 + 6) = \frac{21}{2} \text{ sq units}$$

(ii) Let $A = (x_1, y_1) = (-5, -1)$, $B = (x_2, y_2) = (3, -5)$ and $C = (x_3, y_3) = (5, 2)$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-5(-5 - 2) + 3(2 + 1) + 5(-1 + 5)]$$

$$= \frac{1}{2}[-5(-7) + 3(3) + 5(4)]$$

$$= \frac{1}{2}(35 + 9 + 20) = \frac{1}{2} \times 64 = 32 \text{ sq units}$$

Question 2. In each of the following, find the value of *k*, for which the points are collinear

(i) (7, −2), (5, 1), (3, *k*) (ii) (8, 1), (*k*, − 4), (2, − 5)

Solution (i) Let $A = (x_1, y_1) = (7, -2)$, $B = (x_2, y_2) = (5, 1)$ and $C = (x_3, y_3) = (3, k)$

Since, the points are collinear.

$$\therefore \text{Area of } \Delta ABC = 0$$

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow 7(1 - k) + 5(k + 2) + 3(-2 - 1) = 0 \quad \text{(Multiply by 2)}$$

$$\Rightarrow 7 - 7k + 5k + 10 - 9 = 0$$

$$\Rightarrow -2k + 8 = 0$$

$$\Rightarrow 2k = 8$$

$$\Rightarrow k = 4$$

(ii) Let $A = (x_1, y_1) = (8, 1)$, $B = (x_2, y_2) = (k, -4)$ and $C = (x_3, y_3) = (2, -5)$

Since, the points are collinear.

$$\therefore \text{Area of } \Delta ABC = 0$$

$$\therefore \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow 8(-4 + 5) + k(-5 - 1) + 2(1 + 4) = 0 \quad \text{(Multiply by 2)}$$

$$8\,(1) + k\,(-6) + 2\,(5) = 0$$

$$\Rightarrow \quad 8 - 6k + 10 = 0$$

$$\Rightarrow \quad -6k = -18$$

$$\therefore \quad k = \frac{18}{6} = 3$$

Question 3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Solution $\therefore$ Mid-point of $M = \left(\frac{0+0}{2}, \frac{3-1}{2}\right)$ $\left[\because \text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$

$$= (0, 1)$$

$$\text{Mid-point of } N = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

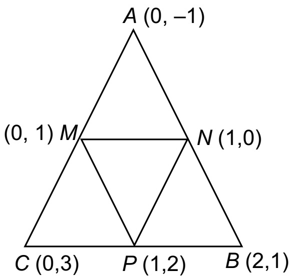
and $\quad \text{Mid-point of } P = \left(\frac{0+2}{2}, \frac{3+1}{2}\right) = (1, 2)$

Let $N\,(1, 0) = (x_1, y_1)$ and $P\,(1, 2) = (x_2, y_2)$ and $M\,(0, 1) = (x_3, y_3)$

$$\therefore \quad \text{Area of } \Delta NPM = \frac{1}{2}[x_1(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)]$$

$$= \frac{1}{2}[1\,(2-1) + 1\,(1-0) + 0\,(0-2)]$$

$$= \frac{1}{2}[1\,(1) + 1 + 0] = \frac{2}{2} = 1 = 1$$



Let $A = (x_1, y_1) = (0, -1)$, $B = (x_2, y_2) = (2, 1)$ and $C = (x_3, y_3) = (0, 3)$

$$\therefore \quad \text{Area of } \Delta\, ABC = \frac{1}{2}[x_1\,(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)]$$

$$= \frac{1}{2}[0\,(1-3) + 2\,(3+1) + 0\,(-1-1)]$$

$$= \frac{1}{2}(0 + 8 + 0)$$

$$= 4 \text{ sq units}$$

$$\therefore \quad \text{Required ratio} = \frac{\text{Area of } \Delta\, NPM}{\text{Area of } \Delta\, ABC} = \frac{1}{4}$$

Question 4. Find the area of the quadrilateral whose vertices, taken in order, are (–4, –2), (–3, – 5), (3, – 2) and (2, 3).

Solution Let A (–4, –2), B (–3, –5), C (3, – 2) and D (2, 3) be the vertices of the quadrilateral $ABCD$.

$\therefore$ Area of quadrilateral $ABCD$ = Area of ΔACD + Area of ΔABC

$$= \frac{1}{2}[-4(-2-3)+3(3+2)+2(-2+2)]$$

$$+\frac{1}{2}[-4(-5+2)-3(-2+2)+3(-2+5)]$$

$$\left(\because \text{Area of triangle} = \frac{1}{2}[x_1(y_1-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right)$$

$$= \frac{1}{2}[-4(-5)+3(5)+2(0)]$$

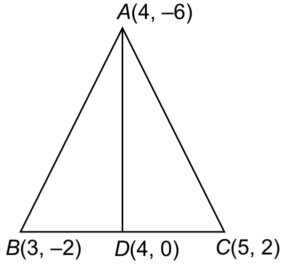
$$+\frac{1}{2}[-4(-3)-3(0)+3(3)]$$

$$= \frac{1}{2}(20+15+0)+\frac{1}{2}(12-0+9)$$

$$= \frac{1}{2}(35+21) = \frac{1}{2}\times 56 = 28 \text{ sq units}$$

Question 5. You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, – 6), B (3, – 2) and C (5, 2).

Solution According to the question, AD is the median of ΔABC, therefore D is the mid-point of BC.



$\therefore$ Coordinates of D are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$ *i.e.*, (4, 0)

$$\left[\because \text{Mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

$\therefore$ Area of $\Delta ADC = \frac{1}{2}[4(0-2)+4(2+6)+5(-6-0)]$

$$\left(\because \text{Area} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right)$$

Here, $A(4, 0) = (x_1, y_1)$

$D(4, 0) = (x_2, y_2)$ and $C(5, 2) = (x_3, y_3)$

$$= \frac{1}{2}(-8 + 32 - 30)$$

$$= \frac{1}{2} \times (-6) = -3$$

$= 3$ sq units (∵ Area of triangle is positive)

and Area of $\Delta ABD = \frac{1}{2}[4(-2 - 0) + 3(0 + 6) + 4(-6 + 2)]$

[Let $A(4, -6) = (x_1, y_1)$, $B(3, -2) = (x_2, y_2)$ and $D(4, 0) = (x_3, y_3)$]

$$= \frac{1}{2}(-8 + 18 - 16)$$

$$= \frac{1}{2}(-6) = -3$$

$= 3$ sq units (∵ Area of triangle is positive)

∵ Area of ΔADC = Area of ΔABD

Hence, the median of the triangle divides it into two triangle of equal areas.

Exercise 13.4 (Optional)

Question 1. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points $A\ (2, -2)$ and $B\ (3, 7)$.

Solution Let the line $2x + y - 4 = 0$ divides the line segment joining the points $A\ (2, -2)$ and $B\ (3, 7)$ in the ratio $k : 1$ at the point P.

$\therefore$ The coordinates of P are $\left(\dfrac{3k + 2}{k + 1}, \dfrac{7k - 2}{k + 1}\right)$.

$$\left[\because \text{Internally ratio} = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)\right]$$

But, P lies on $2x + y - 4 = 0$

$$\therefore \quad 2\left(\frac{3k + 2}{k + 1}\right) + \frac{7k - 2}{k + 1} - 4 = 0$$

$$\Rightarrow \quad 6k + 4 + 7k - 2 - 4k - 4 = 0$$

$$\Rightarrow \quad 9k - 2 = 0$$

$$\Rightarrow \quad 9k = 2$$

$$\Rightarrow \quad k = \frac{2}{9}$$

$\therefore$ Point P divides the line in the ratio $2 : 9$.

Question 2. Find a relation between x and y if the points (x, y), $(1, 2)$ and $(7, 0)$ are collinear.

Solution Since, the points $A\ (x, y)$, $B\ (1, 2)$ and $C\ (7, 0)$ are collinear.

$$\therefore \quad \text{Area of } \Delta\ ABC = 0$$

$$\therefore \quad \frac{1}{2}[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[x (2 - 0) + 1 (0 - y) + 7 (y - 2)] = 0$$

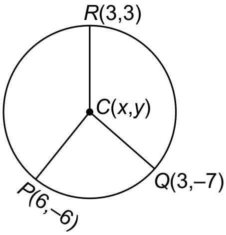
$$\Rightarrow \quad 2x - y + 7y - 14 = 0 \qquad \text{(Multiply by 2)}$$

$$\Rightarrow \quad 2x + 6y - 14 = 0$$

$$\Rightarrow \quad x + 3y - 7 = 0 \qquad \text{(Divide by 2)}$$

Question 3. Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).

Solution Let $C(x, y)$ be the centre of the circle passing through the points $P(6, -6)$, $Q(3, -7)$ and $R(3, 3)$.



Then, $PC = QC = CR$ (Radius of circle)

Now, $PC = QC$

$\Rightarrow PC^2 = QC^2$

$\Rightarrow (x-6)^2 + (y+6)^2 = (x-3)^2 + (y+7)^2$

$[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}]$

$\Rightarrow x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$

$\Rightarrow -12x + 6x + 12y - 14y + 72 - 58 = 0$

$\Rightarrow -6x - 2y + 14 = 0$

$3x + y - 7 = 0$ (Divide by – 2) ...(i)

and $QC = CR$

$\Rightarrow QC^2 = CR^2$

$\Rightarrow (x-3)^2 + (y+7)^2 = (x-3)^2 + (y-3)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 + 14y + 49 = x^2 - 6x + 9 + y^2 - 6y + 9$

$\Rightarrow -6x + 6x + 14y + 6y + 58 - 18 = 0$

$\Rightarrow 20y + 40 = 0$

$\Rightarrow y = -\dfrac{40}{20} = -2$ ...(ii)

Putting $y = -2$ in Eq. (i), we get

$3x - 2 - 7 = 0$

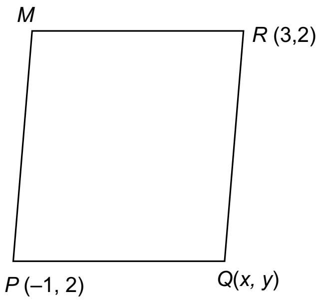
$\Rightarrow 3x = 9$

$\Rightarrow x = 3$

Hence, centre is (3, – 2).

Question 4. The two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of the other two vertices.

Solution Let $PQRM$ be a square and let $P(-1, 2)$ and $R(3, 2)$ be the vertices. Let the coordinates of Q be (x, y).



$\because PQ = MR$

$\Rightarrow PQ^2 = MR^2$

$\Rightarrow \quad (x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$[\because \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}]$

$\Rightarrow \quad x^2 + 1 + 2x + y^2 + 4 - 4y = x^2 + 9 - 6x + y^2 + 4 - 4y$

$\Rightarrow \quad 2x + 1 = -6x + 9$

$\Rightarrow \quad 8x = 8$

$\Rightarrow \quad x = 1 \quad \ldots\text{(i)}$

In ΔPQR, we have

$$PQ^2 + QR^2 = PR^2$$

$$(x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2 = (3+1)^2 + (2-2)^2$$

$\Rightarrow \quad x^2 + 1 + 2x + y^2 + 4 - 4y + x^2 + 9 - 6x + y^2 + 4 - 4y = 4^2 + 0^2$

$\Rightarrow \quad 2x^2 + 2y^2 + 2x - 4y - 6x - 4y + 1 + 4 + 9 + 4 = 16$

$\Rightarrow \quad 2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\Rightarrow \quad x^2 + y^2 - 2x - 4y + 1 = 0 \quad \ldots\text{(ii)}$

Putting $x = 1$ from Eq. (i) in Eq. (ii), we get

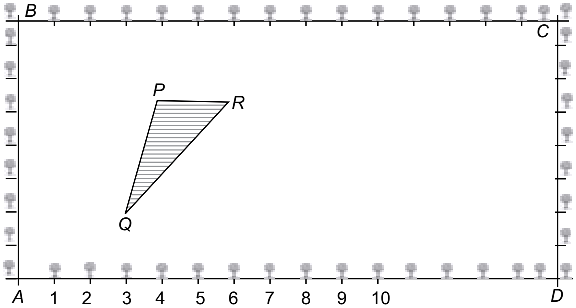
$$1 + y^2 - 2 - 4y + 1 = 0$$

$\Rightarrow \quad y^2 - 4y = 0$

$\Rightarrow \quad y(y-4) = 0 \Rightarrow y = 0 \text{ or } 4$

Hence, the required vertices of square are (1, 0) and (1, 4).

Question 5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of ΔPQR, if C is the origin?

Also, calculate the areas of the triangles in these cases. What do you observe?

Solution (i) When A is taken as origin, AD and AB as coordinate axes, *i.e.*, X-axis and Y-axis, respectively. Coordinates of P, Q and R are respectively (4, 6), (3, 2) and (6, 5).

(ii) When C is taken as origin and CB as X-axis and CD as Y-axis.

$\therefore$ Coordinates of P, Q and R are respectively (12, 2), (13, 6) and (10, 3)

Areas of triangles according to both conditions

Condition (i)

When A is taken as origin and AD and AB as coordinates axes.

$$\therefore \quad \text{Area of } \Delta PQR = \frac{1}{2}[4(2-5) + 3(5-6) + 6(6-2)]$$

$$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}[4(-3) + 3(-1) + 6 \times 4]$$

$$= \frac{1}{2}(-12 - 3 + 24) = \frac{9}{2} \text{ sq units}$$

Condition (ii), When C is taken as origin and CB and CD as axes

$$\therefore \quad \text{Area of } \Delta PQR = \frac{1}{2}[12(6-3) + 13(3-2) + 10(2-6)]$$

$$= \frac{1}{2}[12 \times 3 + 13 \times 1 + 10 \times (-4)] = \frac{1}{2}[36 + 13 - 40]$$

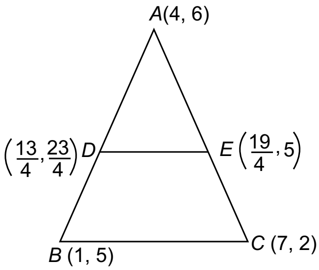
$$= \frac{9}{2} \text{ sq units}$$

Hence, we observe that the areas of both triangles are same.

Question 6. The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC.

Solution Given, $\frac{AD}{AB} = \frac{1}{4}$

$\therefore$ $\frac{AD}{DB} = \frac{1}{3}$



Hence, D divides AB internally in the ratio 1 : 3.

$\therefore$ The coordinates of D are

$$\left(\frac{1 \times 1 + 3 \times 4}{1+3}, \frac{1 \times 5 + 3 \times 6}{1+3}\right) \text{ i.e.,} \left(\frac{1+12}{4}, \frac{5+18}{4}\right) \text{ or } \left(\frac{13}{4}, \frac{23}{4}\right)$$

$$\left[\because \text{Internally ratio} = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)\right]$$

Again, $\frac{AE}{AC} = \frac{1}{4}$

$\therefore$ $\frac{AE}{EC} = \frac{1}{3}$

Hence, E divides AC internally in the ratio 1 : 3.

$\therefore$ The coordinates of E are $\left(\frac{1\times 7+3\times 4}{1+3}, \frac{1\times 2+3\times 6}{1+3}\right)$ *i.e.,* $\left(\frac{7+12}{y}, \frac{2+18}{4}\right)$ or $\left(\frac{19}{4}, 5\right)$.

$$\left[\because \text{Internally ratio} = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)\right]$$

Now, area of $\Delta ABC = \frac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$

$$\left(\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right)$$

$$= \frac{1}{2}[12-4+7]$$

$$= \frac{15}{2} \text{ sq units}$$

and area of $\Delta ADE = \frac{1}{2}\left[4\left(\frac{23}{4}-5\right)+\frac{13}{4}(5-6)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$

$$= \frac{1}{2}\left[4\times\frac{3}{4}-\frac{13}{4}+\frac{19}{4}\times\frac{1}{4}\right]$$

$$= \frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right]$$

$$= \frac{1}{2}\left[\frac{48-52+19}{16}\right] = \frac{15}{32} \text{ sq units}$$

$$\therefore \quad \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{15/32}{15/2} = 1:16$$

Question 7. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that $AP : PD = 2 : 1$.

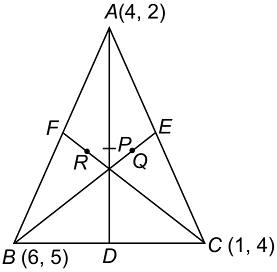
(iii) Find the coordinates of points Q and R on medians BE and CF respectively, such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$.

(iv) What do you observe?

Note *The point which is common to all the three medians is called the centroid and this point divides each median in the ratio 2 : 1.*

(v) If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Solution Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.



(i) Since, AD is the median of ΔABC.

$\therefore$ D is the mid-point of BC.

$\therefore$ The coordinates of D are $\left(\frac{6+1}{2}, \frac{5+4}{2}\right)$ *i.e.*, $\left(\frac{7}{2}, \frac{9}{2}\right)$

$$\left[\because \text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$$

(ii) Since, P divides AD is the ratio 2 : 1, so its coordinates are

$$\left(\frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}, \frac{2 \times \frac{9}{2} + 1 \times 2}{2+1}\right) \text{ or } \left(\frac{7+4}{3}, \frac{9+2}{3}\right) \textit{ i.e.}, \left(\frac{11}{3}, \frac{11}{3}\right)$$

$$\left[\because \text{Internally ratio} = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)\right]$$

(iii) Since, BE is the median of ΔABC, so E is the mid-point of AC and its coordinates are $E\left(\frac{4+1}{2}, \frac{2+4}{2}\right)$ *i.e.*, $E\left(\frac{5}{2}, 3\right)$.

Since, Q divides BE in the ratio 2 : 1 so, its coordinates are

$$Q\left(\frac{2 \times \frac{5}{2} + 1 \times 6}{2+1}, \frac{2 \times 3 + 1 \times 5}{2+1}\right) \text{ or } Q\left(\frac{5+6}{3}, \frac{6+5}{3}\right) \text{ or } Q\left(\frac{11}{3}, \frac{11}{3}\right)$$

Since, CF is the median of ΔABC, so F is the mid-point of AB. Therefore, its coordinates are $F\left(\frac{4+6}{2}, \frac{2+5}{2}\right)$ *i.e.*, $F\left(5, \frac{7}{2}\right)$.

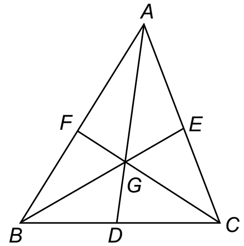
Since, R divides in the ratio 2 : 1, so, its coordinates are

$$R\left(\frac{2 \times 5 + 1 \times 1}{2+1}, \frac{2 \times \frac{7}{2} + 1 \times 4}{2+1}\right) \text{ or } R\left(\frac{10+1}{3}, \frac{7+4}{3}\right) \text{ or } R\left(\frac{11}{3}, \frac{11}{3}\right).$$

(iv) We find that the points P, Q and R coincide at the point $\left(\frac{11}{3}, \frac{11}{3}\right)$. This point is known as the centroid of the triangle.

(v) Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC whose medians are AD, BE and CF respectively, then D, E and F are respectively the mid-points of BC, CA and AB.

$\therefore$ Coordinates of D are $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$.



Coordinates of a point G dividing AD in the ratio 2 : 1 are

$$\left(\frac{1(x_1) + 2\frac{(x_2 + x_3)}{2}}{1+2}, \frac{1(y_1) + 2\frac{(y_2 + y_3)}{2}}{1+2}\right)$$

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_3}{2}\right)$. The coordinates of a point dividing ***BE*** in the ratio 2 : 1 are

$$\left(\frac{1(x_2) + 2\frac{(x_1 + x_3)}{2}}{1+2}, \frac{1(y_2) + 2\frac{(y_1 + y_3)}{2}}{1+2}\right) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Question 8. *ABCD* is a rectangle formed by the points $A\,(-1, -1)$, $B\,(-1, 4)$, $C\,(5, 4)$ and $D\,(5, -1)$. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral $PQRS$ a square? a rectangle? or a rhombus? Justify your answer.

Solution Given, vertices of a rectangle are $A\,(-1, -1)$, $B\,(-1, 4)$, $C\,(5, 4)$ and $D\,(5, -1)$.

$\therefore$ Mid-point of AB is $P\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right)$ or $P\left(-1, \frac{3}{2}\right)$.

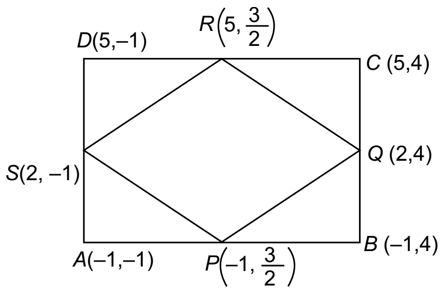
$$\left[\because \text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$$

Mid-point of BC is $Q\left(\frac{5-1}{2}, \frac{4+4}{2}\right)$ or $Q\,(2, 4)$.

Mid-point of CD is $R\left(\frac{5+5}{2}, \frac{4-1}{2}\right)$ or $R\left(5, \frac{3}{2}\right)$.

Mid-point of AD is $S\left(\frac{-1+5}{2}, \frac{-1-1}{2}\right)$ or $S\,(2, -1)$.

Now, $$PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2}$$



$$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36 + 25}{4}} = \sqrt{\frac{61}{4}}$$

$$QR = \sqrt{(5 - 2)^2 + \left(\frac{3}{2} - 4\right)^2}$$

$$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2} = \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36 + 25}{4}} = \sqrt{\frac{61}{4}}$$

$$RS = \sqrt{(2 - 5)^2 + \left(-1 - \frac{3}{2}\right)^2}$$

$$= \sqrt{(-3)^2 + \left(-\frac{5}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{36 + 25}{4}} = \sqrt{\frac{61}{4}}$$

$$SP = \sqrt{(-1 - 2)^2 + \left(\frac{3}{2} + 1\right)^2}$$

$$= \sqrt{(-3)^3 + \left(\frac{5}{2}\right)^2} = \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36 + 25}{4}}$$

$$= \sqrt{\frac{61}{4}}$$

$\because$ $PQ = QR = RS = SP$

Now, $PR = \sqrt{(5 + 1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$

$$= \sqrt{6^2 + 0} = 6$$

and $SQ = \sqrt{(2 - 2)^2 + (4 + 1)^2}$

$$= \sqrt{0 + 5^2} = 5$$

$\Rightarrow$ $PR \neq SQ$

Since, all the sides are equal but diagonals are not equal.

$\therefore$ *PQRS* is a rhombus.