7

Heron's Formula

Exercise 7.1

Question 1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side a. Find the area of the signal board, using Heron's formula.If its perimeter is 180 cm, what will be the area of the signal board?

Solution We know that, an equilateral triangle has equal sides. So, all sides are equal to a.

$$\text{Perimeter of triangle} = 180 \text{ cm} \qquad \text{(Given)}$$

$$\Rightarrow \quad a + a + a = 180$$

$$\Rightarrow \quad 3a = 180$$

$$\Rightarrow \quad a = 60 \text{ cm}$$

$$\therefore \quad s = \frac{a+a+a}{2} = \frac{180}{2} \qquad (\because 2s = a + b + c)$$

$$s = 90 \text{ cm}$$

$$\text{Area of an equilateral triangle} = \sqrt{s\,(s-a)\,(s-a)\,(s-a)}$$

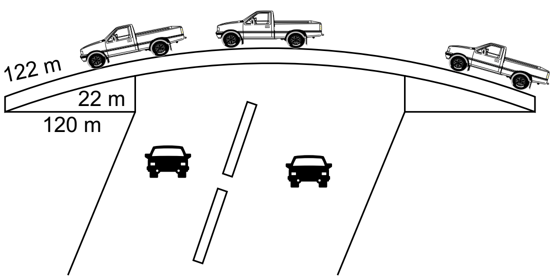
$$[\because \text{Heron's formula, } s = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}]$$

$$= \sqrt{90\,(90-60)\,(90-60)\,(90-60)}$$

$$= \sqrt{30 \times 3 \times 30 \times 30 \times 30} = 30 \times 30\sqrt{3}$$

$$= 900\sqrt{3} \text{ cm}^2$$

Question 2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see figure). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?



Solution Let $a = 122$ m, $b = 22$ m, $c = 120$ m

We have, $\quad b^2 + c^2 = (22)^2 + (120)^2 = 484 + 14400 = 14884 = (122)^2 = a^2$

Hence, the side walls are in right triangled shape.

The area of the triangular side walls $= \left(\frac{1}{2} \times a \times c\right)$

$$= \left(\frac{1}{2} \times 22 \times 120\right) = 11 \times 120$$

$$= 1320 \text{ m}^2$$

Now, yearly rent $= ₹\, 5000$ per m^2

$\therefore$ Monthly rent $= ₹\, 5000 \times \frac{1}{12}$ per m^2

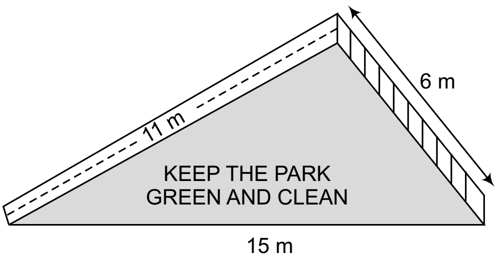
Company hired one of its walls for 3 months.

Thus, rent paid by the company for 3 months $= ₹\, 1320 \times \frac{5000}{12} \times 3$

$$= 110 \times 5000 \times 3$$

$$= ₹\, 1650000$$

Question 3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see figure). If the sides of the wall are 15 m, 11 m and 6m, find the area painted in colour.



Solution The given figure formed a triangle whose sides are

$$a = 15 \text{ m}, b = 11 \text{ m}, c = 6 \text{ m}$$

Now, $s = \frac{(15 + 11 + 6)}{2}$ m

$$= \frac{32}{2} = 16 \text{ m}$$

Therefore, area painted in colour $= \sqrt{16\,(16-15)\,(16-11)\,(16-6)}$

[$\because$ Heron's formula, $s = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$]

$$= \sqrt{16 \times 1 \times 5 \times 10}$$

$$= \sqrt{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 2}$$

$$= 20\sqrt{2} \text{ m}^2$$

Hence, the area painted in colour is $20\sqrt{2}\,\text{m}^2$.

Question 4. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Solution Let the sides of a triangle, $a = 18$ cm, $b = 10$ cm and c

We have, perimeter = 42 cm

$$\Rightarrow \quad a + b + c = 42$$
$$\Rightarrow \quad 18 + 10 + c = 42$$
$$\Rightarrow \quad c = (42 - 28) \text{ cm}$$
$$\Rightarrow \quad c = 14 \text{ cm}$$

Now, $$s = \frac{a+b+c}{2}$$

$$\Rightarrow \quad s = \frac{42}{2} = 21 \text{cm}$$

$\therefore$ Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ (By Heron's formula)

$$= \sqrt{21(21-18)(21-10)(21-14)}$$
$$= \sqrt{21 \times 3 \times 11 \times 7} = \sqrt{7 \times 3 \times 3 \times 11 \times 7} = 21\sqrt{11} \text{ cm}^2$$

Question 5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.

Solution Suppose that the sides in cm, are $12x$, $17x$ and $25x$.

Then, we know that $12x + 17x + 25x = 540$ (Perimeter of triangle)

$$\Rightarrow \quad 54x = 540 \Rightarrow x = 10$$

So, the sides of the triangle are 12×10 cm, 17×10 cm, 25×10 cm *i.e.,* 120 cm, 170 cm, 250 cm.

We have, $$s = \frac{540}{2} \text{ cm} = 270 \text{ cm}$$

$$\text{Area} = \sqrt{270(270-120)(270-170)(270-250)}$$

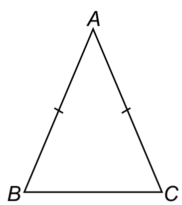
(By Heron's formula)

$$= \sqrt{27 \times 10 \times 150 \times 100 \times 20}$$
$$= 100\sqrt{27 \times 15 \times 10 \times 2}$$
$$= 100\sqrt{9 \times 3 \times 3 \times 5 \times 10 \times 2}$$
$$= 100\sqrt{9 \times 3 \times 3 \times 10 \times 10}$$
$$= 100 \times 3 \times 3 \times 10 = 9000 \text{ cm}^2$$

Question 6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Solution Let in isosceles ΔABC,

$$AB = AC = 12 \text{ cm} \quad \text{(Given)}$$



Perimeter = 30 cm

$$\Rightarrow \quad AB + AC + BC = 30$$

$$\Rightarrow \quad 12 + 12 + BC = 30$$

$$\Rightarrow \quad BC = 30 - 24$$

$$\Rightarrow \quad BC = 6 \text{ cm}$$

We have, $s = \frac{30}{2} \text{ cm} = 15 \text{ cm}$

$$\text{Area of isosceles triangle} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$$

$$= \sqrt{15\,(15-12)\,(15-12)\,(15-6)}$$

$$= \sqrt{15 \times 3 \times 3 \times 9}$$

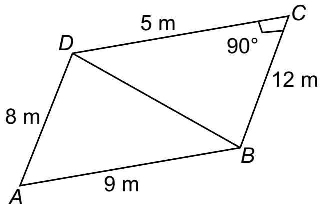
$$= \sqrt{5 \times 3 \times 3 \times 3 \times 3 \times 3} = 9\sqrt{15} \text{ cm}^2$$

7 Heron's Formula

Exercise 7.2

Question 1. A park, in the shape of a quadrilateral $ABCD$, has $\angle C = 90^\circ$, $AB = 9$ m, $BC = 12$ m, $CD = 5$ m and $AD = 8$ m. How much area does it occupy?

Solution In right ΔBCD



We have, $BD^2 = BC^2 + CD^2$ (By Pythagoras theorem)

$$= 12^2 + 5^2 = 144 + 25 = 169 = (13)^2$$

$\Rightarrow$ $BD = 13$ m

Area of quadrilateral $ABCD$ = Area of ΔABD + Area of right ΔBCD

In ΔABD,

We have, $AB = 9$ m, $BD = 13$ m, $DA = 8$ m

$$s = \frac{AB + BD + DA}{2}$$

$$= \frac{9 + 13 + 8}{2} \text{ m} = \frac{30}{2} \text{ m} = 15 \text{ m}$$

Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$ (By Heron's formula)

$$= \sqrt{15\,(15-9)\,(15-13)\,(15-8)}$$

$$= \sqrt{15 \times 6 \times 2 \times 7}$$

$$= \sqrt{3 \times 5 \times 6 \times 2 \times 7}$$

$$= 6\sqrt{35} \text{ m}^2 = 6 \times 5.9 \text{ m}^2 = 35.4 \text{ m}^2 \text{ (Approx.)}$$

Area of right $\Delta BCD = \frac{1}{2} \times BC \times CD$

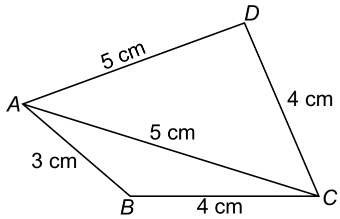
($\because$ Area of triangle $= \frac{1}{2} \times$ Base $\times$ Height)

$$= \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$$

Hence, area of quadrilateral $ABCD = (35.4 + 30) \text{ m}^2 = 65.4 \text{ m}^2$

Question 2. Find the area of a quadrilateral $ABCD$ in which $AB = 3$ cm, $BC = 4$ cm, $CD = 4$ cm, $DA = 5$ cm and $AC = 5$ cm.

Solution Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD



In Δ ABC,

We have, $AB = 3$ cm, $BC = 4$ cm, $CA = 5$ cm

Therefore, $AB^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25 = (5)^2 = CA^2$

Hence, Δ ABC is a right triangle

So, area of Δ $ABC = \frac{1}{2} \times AB \times BC$

($\because$ Area of triangle $= \frac{1}{2} \times$ Base $\times$ Height)

$= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$

In Δ ACD,

We have, $AC = 5$ cm, $CD = 4$ cm, $DA = 5$ cm

Now, $s = \frac{AC + CD + DA}{2}$

$= \frac{5 + 4 + 5}{2}$

$\Rightarrow$ $s = 7$ cm

Area of Δ $ACD = \sqrt{7\,(7-5)\,(7-4)\,(7-5)}$ (By Heron's formula)

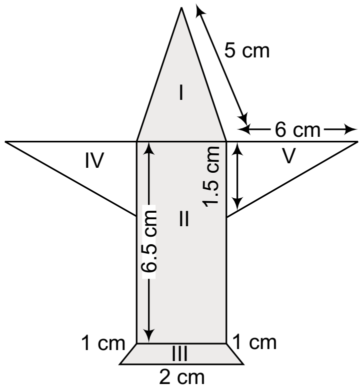
$= \sqrt{7 \times 2 \times 3 \times 2}$

$= 2\sqrt{21} \text{ cm}^2 = 2 \times 4.6 \text{ cm}^2$

$= 9.2 \text{ cm}^2$ (Approx.)

Hence, required area $= (6 + 9.2) \text{ cm}^2 = 15.2 \text{ cm}^2$ (Approx.)

Question 3. Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.



Solution **For part I**

It is a triangle with sides 5 cm, 5 cm and 1 cm.

So, $$s = \frac{5+5+1}{2} = \frac{11}{2} \text{ cm}$$

$\therefore$ Area of part I $= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$ (By Heron's formula)

$$= \sqrt{\frac{11}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{9}{2}}$$

$$= \frac{3}{4}\sqrt{11} = \frac{3}{4} \times 3.316 = 3 \times 0.829 = 2.487 \text{ cm}^2$$

For part II

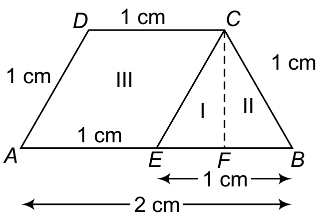
It is a rectangle with sides 6.5 cm and 1 cm

$\therefore$ Area of part II $= 6.5 \times 1$

($\because$ Area of rectangle = Length $\times$ Breadth)

$= 6.5 \text{ cm}^2$

For part III



It is a trapezium *ABCD*.

Δ *EBC* is an equilateral with side 1 cm.

$$\therefore \quad \frac{1}{2} \times EB \times CF = \frac{\sqrt{3}}{4} \times (1)^2$$

$$\Rightarrow \quad \frac{1}{2} \times 1 \times CF = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \quad CF = \frac{\sqrt{3}}{2} \text{ cm}$$

Now, area of trapezium $= \frac{1}{2}$ (Sum of parallel sides $\times$ Height)

$$= \frac{1}{2}(AB + CD) \times CF$$

$$= \frac{1}{2}(2 + 1) \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} \times 3 \times 1.732 = 3 \times 0.433 = 1.299 \text{ cm}^2$$

For part V

It is a right triangle with sides 6 cm and 1.5 cm.

$$\text{Area of part V} = \frac{1}{2} \times 1.5 \times 6 = \frac{1}{2} \times 9 = 4.5 \text{ cm}^2$$

Similarly, area of **part IV** $= 4.5 \text{ cm}^2$

$\therefore$ Total area of paper used $=$ Area of part (I + II + III + IV + V)

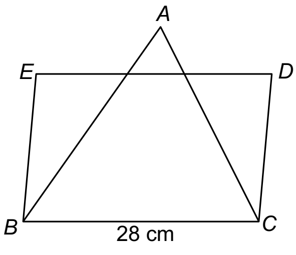
$$= (2.487 + 6.5 + 1.299 + 4.5 + 4.5) \text{ cm}^2$$

$$= 19.286 \text{ cm}^2 = 19.3 \text{ cm}^2 \text{ (Approx.)}$$

Question 4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Solution Let *ABC* be a triangle with sides

$$AB = 26 \text{ cm}, BC = 28 \text{ cm}, CA = 30 \text{ cm}$$



Now,
$$s = \frac{AB + BC + CA}{2}$$

$$= \left(\frac{26 + 28 + 30}{2}\right) \text{ cm}$$

$$\Rightarrow \quad s = \frac{84}{2} \text{ cm}$$

$$\Rightarrow \quad s = 42 \text{ cm}$$

$$\text{Area of } \Delta\, ABC = \sqrt{42\,(42-26)\,(42-28)\,(42-30)}$$

(By Heron's formula)

$$= \sqrt{42 \times 16 \times 14 \times 12}$$

$$= \sqrt{7 \times 2 \times 3 \times 2 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3 \times 2 \times 2}$$

$$= 7 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$= 336 \text{ cm}^2$$

We know that,

$$\text{Area of parallelogram} = \text{Base} \times \text{Height} \quad \ldots(i)$$

We have,

$$\text{Area of parallelogram} = \text{Area of } \Delta\, ABC \quad \text{(Given)}$$

$$= 336 \text{ cm}^2$$

From Eq. (i), we have

$$\text{Base} \times \text{Height} = 336$$

$$\Rightarrow \quad 28 \times \text{Height} = 336$$

$$\Rightarrow \quad \text{Height} = \frac{336}{28}$$

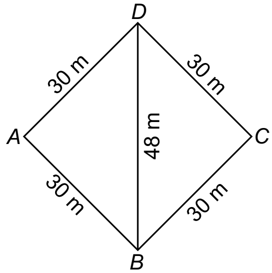
$$\Rightarrow \quad \text{Height} = 12 \text{ cm}$$

Question 5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Solution Let $ABCD$ be a rhombus.

$$\text{Area of the rhombus } ABCD = 2 \times \text{area of } \Delta ABD \quad \ldots(i)$$

(Since, in a rhombus diagonals divides two equal parts)



In $\Delta\, ABD$, we have, $AB = 30\text{ m}$, $BD = 48\text{ m}$, $DA = 30\text{ m}$

$$\therefore \quad s = \frac{30 + 48 + 30}{2} = \frac{108}{2}$$

$\Rightarrow \qquad s = 54 \text{ m}$

$\therefore \qquad \text{Area of } \Delta ABD = \sqrt{54\,(54-30)\,(54-48)\,(54-30)}$

$$= \sqrt{54 \times 24 \times 6 \times 24} \qquad \text{(By Heron's formula)}$$

$$= \sqrt{9 \times 6 \times 24 \times 6 \times 24}$$

$$= 3 \times 6 \times 24 = 432 \text{ m}^2$$

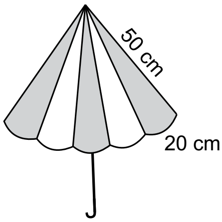
$\therefore$ From Eq. (i),

Area of rhombus $ABCD = 2 \times 432 \text{ m}^2 = 864 \text{ m}^2$

Number of cows $= 18$

$\therefore \quad$ Area of grass field per cow $= \dfrac{864}{18} = 48 \text{ m}^2$

Question 6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?



Solution In an umbrella, each triangular piece is an isosceles triangle with sides 50 cm, 50 cm, 20 cm.

Now, $\qquad s = \dfrac{50 + 50 + 20}{2} \text{ cm}$

$\Rightarrow \qquad s = 60 \text{ cm}$

$\therefore$ Area of each triangular piece

$$= \sqrt{60\,(60-50)\,(60-50)\,(60-20)} \qquad \text{(By Heron's formula)}$$

$$= \sqrt{60 \times 10 \times 10 \times 40}$$

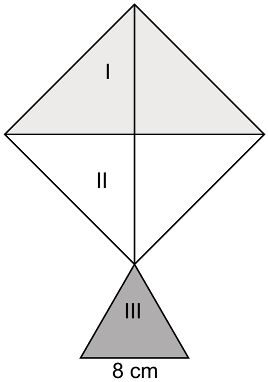
$$= \sqrt{6 \times 10 \times 10 \times 10 \times 4 \times 10}$$

$$= 200\sqrt{6} \text{ cm}^2$$

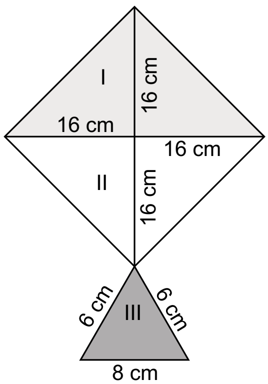
Since, there are 10 triangular piece, in those of them 5-5 are of different colours.

Hence, total area of cloth of each colour $= 5 \times 200\sqrt{6} \text{ cm}^2 = 1000\sqrt{6} \text{ cm}^2$

Question 7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?



Solution Since, the kite is in the shape of a square.



Each diagonal of square =32 cm (Given)

We know that, the diagonals of a square bisect each other at right angle.

$\therefore$ Area of part I $= \frac{1}{2} \times$ Base $\times$ Height

$$= \frac{1}{2} \times 32 \times 16 \text{ cm}^2 = 16 \times 16$$

$$= 256 \text{ cm}^2$$

$$\text{Area of part II} = \frac{1}{2} \times 32 \times 16 \text{ cm}^2$$

$$= 256 \text{ cm}^2$$

For part III

It is a triangle with sides 6 cm, 6 cm and 8 cm.

Now, $s = \frac{6+6+8}{2} = \frac{20}{2}$

$\Rightarrow$ $s = 10\,\text{cm}$

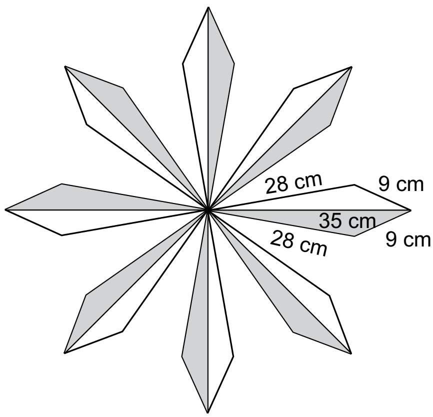
$$\text{Area of triangle} = \sqrt{10\,(10-6)\,(10-6)\,(10-8)} \quad \text{(By Heron's formula)}$$
$$= \sqrt{10 \times 4 \times 4 \times 2}$$
$$= \sqrt{5 \times 2 \times 4 \times 4 \times 2}$$
$$= 8\sqrt{5}\ \text{cm}^2$$
$$= 8 \times 2.24 = 17.92\ \text{cm}^2$$

Hence, paper of I colour has been used = 256 cm^2

Paper of II colour has been used = 256 cm^2

Paper of III colour has been used = 17.92 cm^2

Question 8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 paise per cm^2.



Solution Given, the sides of a triangular tiles are 9 cm, 28 cm and 35 cm.

For each triangular tile, we have

$$s = \frac{9 + 28 + 35}{2} = \frac{72}{2}$$

$\Rightarrow$ $$s = 36\ \text{cm}$$

$\therefore$ $$\text{Area of each triangular tile} = \sqrt{36\,(36-9)\,(36-28)\,(36-35)}$$

(By Heron's formula)

$$= \sqrt{36 \times 27 \times 8 \times 1}$$
$$= \sqrt{(6)^2 \times (3)^2 \times 3 \times (2)^2 \times 2} = 36\sqrt{6}\ \text{cm}^2$$

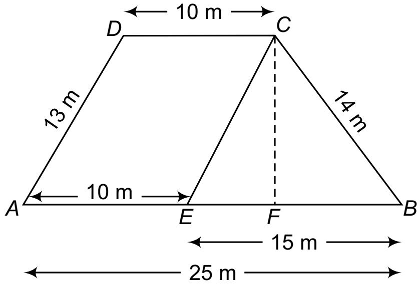
$\therefore$ $$\text{Total area of 16 such tiles} = 16 \times 36 \times \sqrt{6}\ \text{cm}^2$$
$$= 16 \times 36 \times 2.45\ \text{cm}^2 = 1411.20\ \text{cm}^2$$

Total cost of polishing the tiles at the rate of 50 paise per cm^2

$$= ₹\ \frac{50}{100} \times 1411.20 = ₹\ 705.60$$

Question 9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Solution Here, $ABCD$ is a trapezium and $AB \parallel DC$.



Through C, draw $CE \parallel DA$ and $CF \perp AB$

$$AB = 25\text{ m}, BC = 14\text{ m}, CD = 10\text{ m}, DA = 13\text{ m}$$

$$AE = 10\text{ m and } CE = 13\text{ m}$$

$\therefore$ $$EB = 25 - 10 = 15\text{ m}$$

For $\Delta\, EBC$

$$s = \frac{15 + 14 + 13}{2} = \frac{42}{2}$$

$\Rightarrow$ $$s = 21\text{ m}$$

$\therefore$ $$\text{Area of } \Delta\, EBC = \sqrt{21\,(21-15)\,(21-14)\,(21-13)}$$

(By Haron's formula)

$\Rightarrow$ $$\frac{1}{2} \times EB \times CF = \sqrt{21 \times 6 \times 7 \times 8}$$

$$= \sqrt{(21)^2 \times (4)^2}$$

$\Rightarrow$ $$\frac{1}{2} \times 15 \times CF = 84$$

$\Rightarrow$ $$CF = \frac{84 \times 2}{15} = \frac{168}{15} = 11.2\text{ m}$$

Now, area of the trapezium $ABCD$

$$= \frac{1}{2}\text{(Sum of parallel sides)} \times \text{Distance between parallel sides}$$

$$= \frac{1}{2}(AB + CD) \times CF$$

$$= \frac{1}{2} \times (25 + 10) \times 11.2$$

$$= \frac{1}{2} \times 35 \times 11.2 = 35 \times 5.6$$

$$= 196\text{ m}^2$$