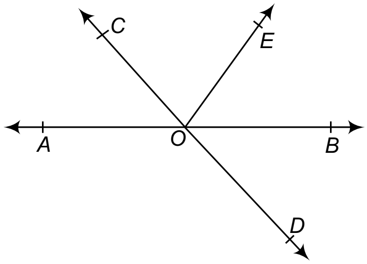
4 Lines and Angles

Exercise 4.1

Question 1. In figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.



Solution Here, $\angle AOC$ and $\angle BOD$ are vertically opposite angles.

$\therefore$ $\angle AOC = \angle BOD$

$\Rightarrow$ $\angle AOC = 40°$ [$\because \angle BOD = 40°$ (Given)] ...(i)

We have, $\angle AOC + \angle BOE = 70°$ (Given)

$40° + \angle BOE = 70°$ [From Eq. (i)]

$\Rightarrow$ $\angle BOE = 30°$

Also, $\angle AOC + \angle COE + \angle BOE = 180°$ (Linear pair axiom)

$\Rightarrow$ $40° + \angle COE + 30° = 180°$

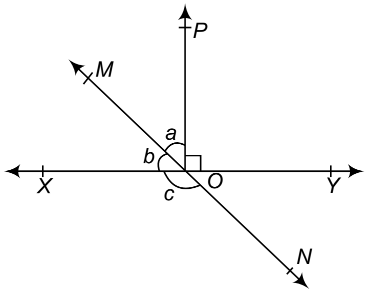
$\Rightarrow$ $\angle COE = 110°$

Now, $\angle COE + \text{reflex } \angle COE = 360°$ (Angles at a point)

$110° + \text{reflex } \angle COE = 360°$

$\Rightarrow$ $\text{Reflex } \angle COE = 250°$

Question 2. In figure, lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$. find c.



Solution We have, $\angle POY = 90°$

$\Rightarrow \quad \angle POY + \angle POX = 180^\circ$ (Linear pair axiom)

$\Rightarrow \quad \angle POX = 90^\circ$

$\Rightarrow \quad a + b = 90^\circ$ ...(i)

Also, $a : b = 2 : 3$ (Given)

$\Rightarrow$ Let $a = 2k, b = 3k$

Now, from Eq. (i), we get

$$2k + 3k = 90^\circ$$

$\Rightarrow \quad 5k = 90^\circ$

$\Rightarrow \quad k = 18^\circ$

$\therefore \quad a = 2 \times 18^\circ = 36^\circ$

and $b = 3 \times 18^\circ = 54^\circ$

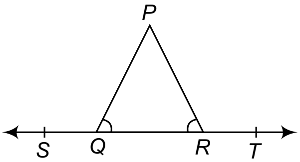
Now, $\angle MOX + \angle XON = 180^\circ$ (Linear pair axiom)

$$b + c = 180^\circ$$

$\Rightarrow \quad 54^\circ + c = 180^\circ$

$\Rightarrow \quad c = 126^\circ$

Question 3. In figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Solution $\because \quad \angle PQS + \angle PQR = 180^\circ$ (Linear pair axiom) ...(i)

and $\angle PRT + \angle PRQ = 180^\circ$ (Linear pair axiom)...(ii)

From Eqs. (i) and (ii), we get

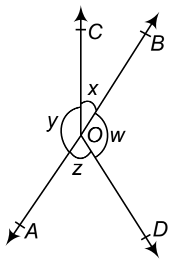
$$\angle PQS + \angle PQR = \angle PRT + \angle PRQ$$

$$\angle PQS + \angle PRQ = \angle PRT + \angle PRQ$$

[Given, $\angle PQR = \angle PRQ$]

$\Rightarrow \quad \angle PQS = \angle PRT$

Question 4. In figure, if $x + y = w + z$, then prove that AOB is a line.



Solution $\because \quad x + y + w + z = 360°$ (Angle at a point)

$x + y = w + z$ (Given)...(i)

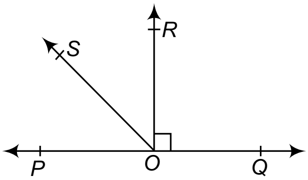
$\therefore \quad x + y + x + y = 360°$ [From Eq. (i)]

$2(x + y) = 360° \Rightarrow x + y = 180°$ (Linear pair axiom)

Hence, AOB is a straight line.

Question 5. In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



Solution We have,

$$\angle POR = \angle ROQ = 90°$$

($\because$ Given that, OR is perpendicular to PQ)

$\therefore \quad \angle POS + \angle ROS = 90°$

$\Rightarrow \quad \angle ROS = 90° - \angle POS$

On adding $\angle ROS$ both sides, we get

$2\angle ROS = 90° - \angle POS + \angle ROS$

$\Rightarrow \quad 2\angle ROS = (90° + \angle ROS) - \angle POS$

$\Rightarrow \quad 2\angle ROS = \angle QOS - \angle POS$

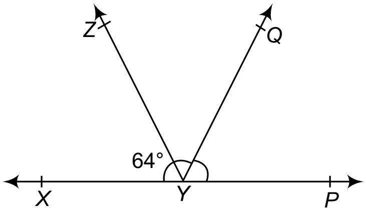
($\because \angle QOS = \angle ROQ + \angle ROS = 90° + \angle ROS$)

$\Rightarrow \quad \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Hence proved.

Question 6. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Solution Here, YQ bisects $\angle ZYP$.



Hence, $\angle ZYQ = \angle QYP = \frac{1}{2} \angle ZYP$ ...(i)

Given, $\angle XYZ = 64°$ ...(ii)

$\because \quad \angle XYZ + \angle ZYQ + \angle QYP = 180°$ (Linear pair axiom)

$\Rightarrow \quad 64° + \angle ZYQ + \angle ZYQ = 180°$ [From Eqs. (i) and (ii)]

$\Rightarrow \quad 2 \angle ZYQ = 180° - 64°$

$\Rightarrow \quad \angle ZYQ = \frac{1}{2} \times 116°$

$\Rightarrow \quad \angle ZYQ = 58°$

$\therefore \quad \angle XYQ = \angle XYZ + \angle ZYQ$

$= 64° + 58°$

$= 122°$

Now, $\angle QYP + \text{reflex } \angle QYP = 360°$

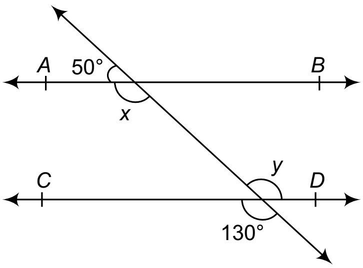
$58° + \text{reflex } \angle QYP = 360°$

$\Rightarrow \quad \text{reflex } \angle QYP = 302°$

4 Lines and Angles

Exercise 4.2

Question 1. In figure, find the values of x and y and then show that $AB \parallel CD$.



Solution $\because$ $x + 50° = 180°$ (Linear pair)

$\Rightarrow$ $x = 130°$

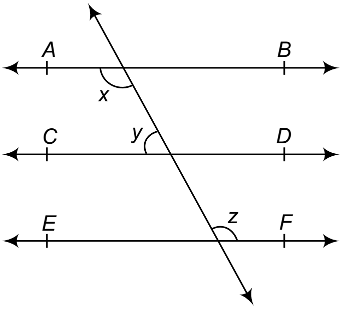
$\therefore$ $y = 130°$ (Vertically opposite angle)

Here, $\angle x = \angle COD = 130°$

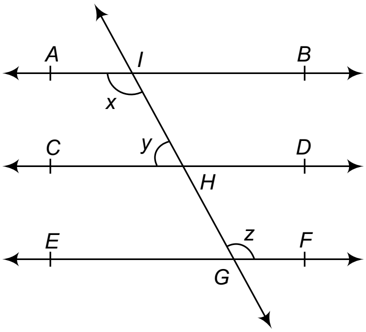
These are corresponding angles for lines AB and CD.

Hence, $AB \parallel CD$

Question 2. In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x.



Solution $y : z = 3 : 7$ (Given)



$\Rightarrow$ Let $y = 3k, z = 7k$

$x = \angle CHG$ (Corresponding angles)...(i)

$\angle CHG = z$ (Alternate angles)...(ii)

From Eqs. (i) and (ii), we get

$x = z$ ...(iii)

Now, $x + y = 180°$

(Internal angles on the same side of the transversal)

$\Rightarrow$ $z + y = 180°$ [From Eq. (iii)]

$7k + 3k = 180°$

$\Rightarrow$ $10k = 180°$

$\Rightarrow$ $k = 18$

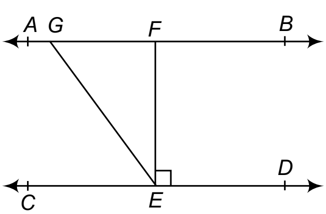
$\therefore$ $y = 3 \times 18° = 54°$

and $z = 7 \times 18° = 126°$

$\therefore$ $x = z$

$\Rightarrow$ $x = 126°$

Question 3. In figure, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Solution $\because$ $\angle AGE = \angle GED$ (Alternate interior angles)

But $\angle GED = 126°$

$\Rightarrow$ $\angle AGE = 126°$ ...(i)

$\therefore$ $\angle GEF + \angle FED = 126°$

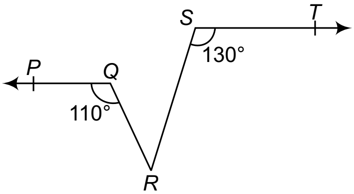
$\Rightarrow$ $\angle GEF + 90° = 126°$ $(\because EF \perp CD)$

$\Rightarrow \quad \angle GEF = 36°$

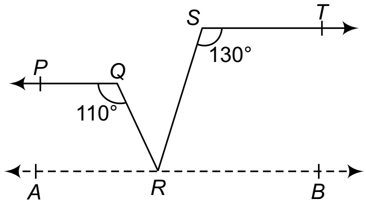
Also, $\quad \angle AGE + \angle FGE = 180°$ (Linear pair axiom)

$\Rightarrow \quad 126° + \angle FGE = 180° \Rightarrow \angle FGE = 54°$

Question 4. In figure, if $PQ \parallel ST$, $\angle PQR = 110°$ and $\angle RST = 130°$, find $\angle QRS$.



Solution Drawing a line parallel to ST through R.



$\Rightarrow \quad PQ \parallel ST$

Also, $\quad AB \parallel PQ \parallel ST$

$\because \quad \angle PQR + \angle QRA = 180°$

($\because$ Interior angles on the same side of transversal)

$\Rightarrow \quad 110° + \angle QRA = 180°$

$\Rightarrow \quad \angle QRA = 70°$

$\because \quad \angle TSR = 130°$ and $ST \parallel AB$

$\therefore \quad \angle ARS = 130°$ (Alternate interior angle)

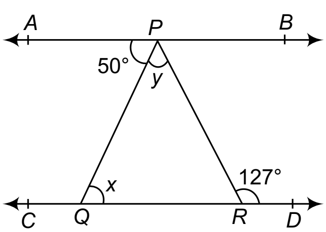
$\because \quad \angle ARS = \angle ARQ + \angle QRS$

$\Rightarrow \quad 130° = 70° + \angle QRS$

$\Rightarrow \quad \angle QRS = 130° - 70° = 60°$

$\Rightarrow \quad \angle QRS = 60°$

Question 5. In figure, if $AB \parallel CD$, $\angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.



Solution We have, $AB \parallel CD$

$\Rightarrow \quad \angle APQ = \angle PQR$ (Alternate interior angles)

$\Rightarrow \quad 50^\circ = x$

$\Rightarrow \quad x = 50^\circ$

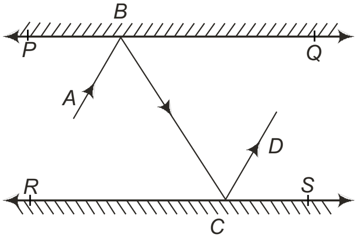
Now, $\quad \angle PQR + \angle QPR = 127^\circ$

(Exterior angle is equal to sum of interior opposite angles of a triangle)

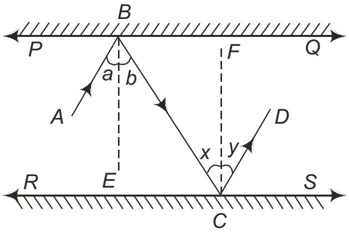
$\Rightarrow \quad 50^\circ + \angle QPR = 127^\circ$

$\Rightarrow \quad y = 77^\circ$.

Question 6. In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.



Solution Draw $BE \perp PQ$ and $CF \perp RS$.



$\Rightarrow \quad BE \parallel CF$

Also, $\quad \angle a = \angle b$ ...(i)

$\angle x = \angle y$ ...(ii)

($\because$ Angle of incidence = angle of reflection)

Now, $\quad \angle b = \angle x$ (Alternate interior angles)

$\Rightarrow \quad 2\angle b = 2\angle x$

$\Rightarrow \quad \angle b + \angle b = \angle x + \angle x$

$\Rightarrow \quad \angle a + \angle b = \angle x + \angle y$ [From Eq. (i) and (ii)]

$\Rightarrow \quad \angle ABC = \angle DCB$

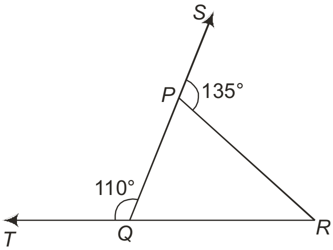
$\Rightarrow \quad AB \parallel CD$ ($\because$ Alternate interior angles axiom)

Hence proved.

4 Lines and Angles

Exercise 4.3

Question 1. In figure, sides QP and RQ of ΔPQR are produced to points S and T, respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.



Solution $\because$ $\angle RPS + \angle RPQ = 180°$ (Linear pair axiom)

$\therefore$ $135° + \angle RPQ = 180°$ [$\because \angle RPS = 135°$ (Given)]

$\Rightarrow$ $\angle RPQ = 45°$

Now, $\angle RPQ + \angle PRQ = \angle PQT$

($\because$ Sum of interior opposite angles = exterior angle)

$\Rightarrow$ $45° + \angle PRQ = 110°$ [$\because \angle PQT = 110°$ (Given)]

$\Rightarrow$ $\angle PRQ = 65°$

Alternate Method

$\because$ $\angle TQP + \angle PQR = 180°$ (Linear pair axiom)

$\therefore$ $110° + \angle PQR = 180°$ [$\because \angle TQP = 110°$ (Given)]

$\Rightarrow$ $\angle PQR = 70°$

and $\angle RPS + \angle RPQ = 180°$ (Linear pair axiom)

$\therefore$ $135° + \angle RPQ = 180°$ [$\because \angle RPS = 135°$ (Given)]

$\Rightarrow$ $\angle RPQ = 45°$

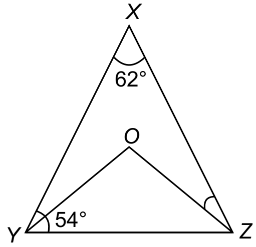
$\because$ In ΔPQR,

$\Rightarrow$ $\angle PQR + \angle QRP + \angle RPQ = 180°$

$\Rightarrow$ $70° + \angle PRQ + 45° = 180°$

$\Rightarrow$ $\angle PRQ = 180° - 115° = 65°$

Question 2. In figure, $\angle X = 62°$, $\angle XYZ = 54°$, if YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



Solution In ΔXYZ,

$\because$ $\angle X + \angle Y + \angle Z = 180°$

(Sum of all angles of triangle is equal to 180°).

$\therefore$ $62° + \angle Y + \angle Z = 180°$ [$\because \angle x = 62°$ (Given)]

$\Rightarrow$ $\angle Y + \angle Z = 118°$

$\Rightarrow$ $\frac{1}{2}\angle Y + \frac{1}{2}\angle Z = \frac{1}{2} \times 118°$

$\Rightarrow$ $\angle OYZ + \angle OZY = 59°$

($\because YO$ and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$)

$\Rightarrow$ $\angle OZY + \frac{1}{2} \times 54° = 59°$ ($\because \angle OYZ = \frac{1}{2} \angle XYZ$)

$\Rightarrow$ $\angle OZY = 59° - 27°$

$\Rightarrow$ $\angle OZY = 32°$

$\therefore$ In ΔYOZ,

$\because$ $\angle YOZ + \angle OYZ + \angle OZY = 180°$

(Sum of all angles of triangle is equal to 180°)

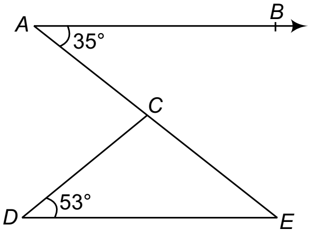
$\therefore$ $\angle YOZ = 180° - (\angle OYZ + \angle OZY)$

$= 180° - (27° + 32°)$

$= 180° - 59°$

$\Rightarrow$ $\angle YOZ = 121°$ ($\because \angle OYZ = \frac{1}{2} \angle XYZ = \frac{1}{2} \times 54 = 27°$)

Question 3. In figure, if $AB \parallel DE$, $\angle BAC = 35°$ and $\angle CDE = 53°$, find $\angle DCE$.



Solution We have, $AB \parallel DE$

$\Rightarrow \quad \angle AED = \angle BAE$ (Alternate interior angles)

Now, $\angle BAE = \angle BAC$

$\Rightarrow \quad \angle BAE = 35°$ $[\because \angle BAC = 35° \text{ (Given)}]$

$\therefore \quad \angle AED = 35°$

In ΔDCE,

$\because \quad \angle DCE + \angle CED + \angle EDC = 180°$

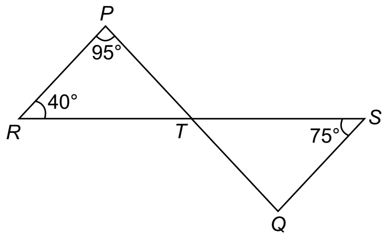
($\because$ Sum of all angles of triangle is equal to 180°)

$\Rightarrow \quad \angle DCE + 35° + 53° = 180°$ $(\because \angle AED = \angle CED = 35°)$

$\Rightarrow \quad \angle DCE = 180° - (35° + 53°)$

$\Rightarrow \quad \angle DCE = 92°$

Question 4. In figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.



Solution $\because \quad \angle PTS = \angle RPT + \angle PRT$

(Exterior angle = Sum of interior opposite angles)

$\angle PTS = 95° + 40°$ $\left[\because \angle RPT = 95° \text{ (Given)} \text{ and } \angle PRT = 40°\right]$

$\Rightarrow \quad \angle PTS = 135°$

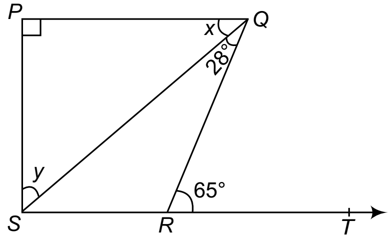
Also, $\angle TSQ + \angle SQT = \angle PTS$

(Exterior angle = Sum of interior opposite angles)

$\Rightarrow \quad 75° + \angle SQT = 135°$

$\Rightarrow \quad \angle SQT = 60°$ $[\because \angle TSQ = 75° \text{ (Given)}]$

Question 5. In figure, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of x and y.



Solution Here, $PQ \parallel SR$

$\Rightarrow \quad \angle PQR = \angle QRT$ (Alternate interior angles)

$\Rightarrow \quad x + 28° = 65° \Rightarrow x = 37°$

Now, in right angled ΔSPQ, we have $\angle P = 90°$.

$\therefore \quad \angle P + x + y = 180°$

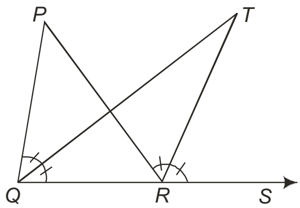
($\because$ Sum of all angles of a triangle is equal to 180°)

$\Rightarrow \quad 90° + 37° + y = 180°$

$\Rightarrow \quad 127° + y = 180° \Rightarrow y = 53°$

Question 6. In figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that

$$\angle QTR = \frac{1}{2} \angle QPR$$



Solution In ΔPQR,

$\because \quad \angle QPR + \angle PQR = \angle PRS \quad \ldots(i)$

($\because$ Sum of interior opposite angles = Exterior angle)

Now, in ΔTQR,

$\because \quad \angle QTR + \angle TQR = \angle TRS \quad \ldots(ii)$

($\because$ Sum of interior opposite angles = Exterior angle)

Now, $\quad \angle TQR = \frac{1}{2} \angle PQR \quad \ldots(iii)$

($\because QT$ and RT are the bisectors of $\angle PQR$ and $\angle PRS$, respectively)

$\angle TRS = \frac{1}{2} \angle PRS \quad \ldots(iv)$

From Eq. (i), we get

$$\frac{1}{2} \angle QPR + \frac{1}{2} \angle PQR = \frac{1}{2} \angle PRS$$

From Eqs. (ii) and (iii), we get

$$\angle QTR + \frac{1}{2} \angle PQR = \angle TRS \quad \ldots(v)$$

From Eqs. (ii) and (v), we get

$$\frac{1}{2} \angle QPR + \angle TQR = \angle QTR + \angle TQR$$

$\Rightarrow \quad \frac{1}{2} \angle QPR = \angle QTR \Rightarrow \angle QTR = \frac{1}{2} \angle QPR$

Hence proved.