3 Pair of Linear Equations in Two Variables

Exercise 3.1

Question 1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Solution Let the present age of father be x years and the age of daughter be y years.

Seven years ago father's age = $(x - 7)$ yr

Seven years ago daughter's age = $(y - 7)$ yr

According to the problem, $(x - 7) = 7(y - 7)$

or $x - 7y = -42$ ...(i)

After 3 yr father's age = $(x + 3)$ yr

After 3 yr daughter's age = $(y + 3)$ yr

According to the condition given in the questions,

$$x + 3 = 3(y + 3)$$

or $x - 3y = 6$ ...(ii)

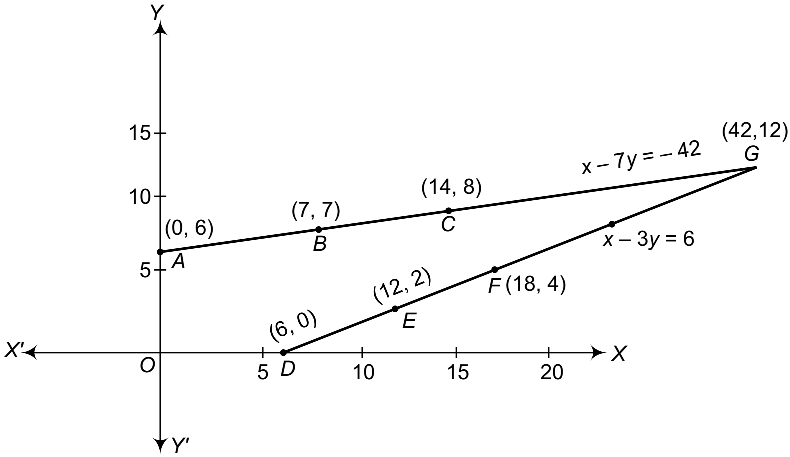
To find the equivalent geometric representation, we find some points on the line representing each equation. These solutions are given below in the table.

From Eq. (i), $x - 7y = -42$

x	0	7	14	42
$y = \frac{x + 42}{7}$	6	7	8	12
Points	A	B	C	G

From Eq. (ii), $x - 3y = 6$

x	6	12	18	42
$y = \frac{x - 6}{3}$	0	2	4	12
Points	D	E	F	G



Plot the points $A(0, 6)$, $B(7, 7)$, $C(14, 8)$ and join them to get a straight line ABC. Similarly, plot the points $D(6, 0)$, $E(12, 2)$ and $F(18, 4)$ and join them to get a straight line DEF. And these two lines intersect at point $G(42, 12)$.

Question 2. The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹ 1300. Represent this situation algebraically and geometrically.

Solution Let us denote the cost of one bat be ₹ x and one ball be ₹ y.

Then, the algebraic representation is given by the following equations

$$3x + 6y = 3900 \quad \ldots\text{(i)}$$

and

$$x + 2y = 1300 \quad \ldots\text{(ii)}$$

To obtain the equivalent geometric representation, we find two points on the line representing each equation. That is, we find two solutions of each equations.

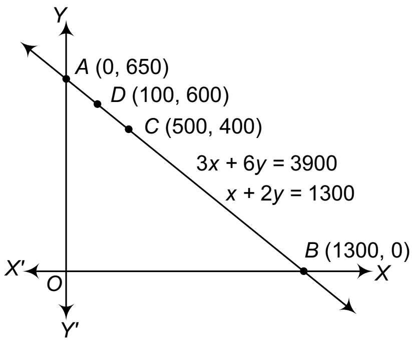
These solutions are given below in the table

For, $3x + 6y = 3900$

x	0	1300
y	650	0

For, $x + 2y = 1300$

x	500	100
y	400	600



We plot the points $A(0, 650)$ and $B(1300, 0)$ to obtain the geometric representation of $3x + 6y = 3900$ and $C(500, 400)$ and $D(100, 600)$ to obtain the geometric representation of $x + 2y = 1300$. We observe that these line are coincident.

Question 3. The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically and geometrically.

Solution Let us denote the cost of 1 kg of apples be ₹ x and cost of 1 kg of grapes be ₹ y.

Then, the algebraic representation is given by the following equations

$$2x + y = 160 \quad \ldots(i)$$

and

$$4x + 2y = 300 \quad \ldots(ii)$$

Then, the algebraic representation is given by the following equations

$$3x + 6y = 3900 \quad \ldots(i)$$

and

$$x + 2y = 1300 \quad \ldots(ii)$$

To obtain the equivalent geometric representation, we find two points on the line representing each equation. That is, we find two solutions of each equations.

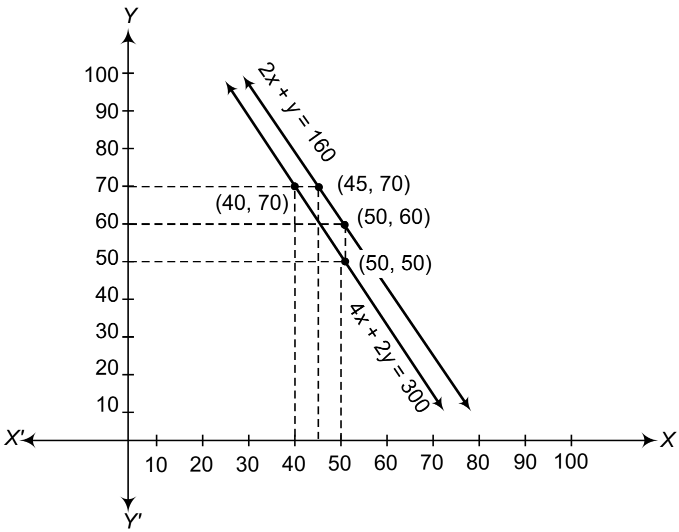
These solutions are given below in the table

For, $3x + 6y = 3900$

x	0	1300
y	650	0

For, $x + 2y = 1300$

x	500	100
y	400	600



Geometric representation is shown in the above figure which is a pair of parallel lines.

3 Pair of Linear Equations in Two Variables

Exercise 3.2

Question 1. From the pair of linear equations in the following problems and find their solution graphically.

(i) 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Solution (i) Let the number of boys be x and the number of girls be y.

Then, the equations formed are

$$x + y = 10 \quad \dots(i)$$

$$y = x + 4 \quad \dots(ii)$$

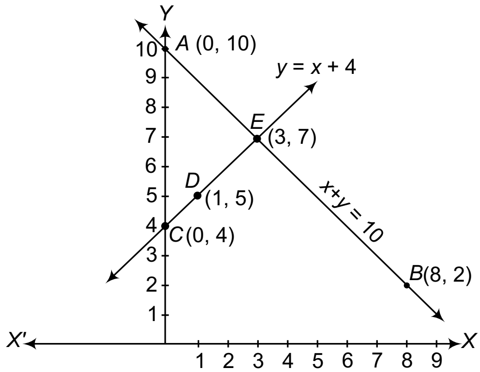
Let us draw the graphs of Eqs. (i) and (ii) by finding two solutions for each of the equations.

∴ Table for line, $x + y = 10$

x	0	8	3
$y = 10 - x$	10	2	7
Points	A	B	E

Table for line, $y = x + 4$

x	0	1	3
$y = x + 4$	4	5	7
Points	C	D	E



Plotting these points, we draw the lines AB and CE passing through them to represent the equations. The two lines AB and CE intersect at the point $E(3, 7)$. So, $x = 3$ and $y = 7$ is the required solution of the pair of linear equations.

i.e., Number of boys = 3,
Number of girls = 7

(ii) Let us denote the cost of one pencil be x and one pen be ₹ y.
Then, the equations formed are

$$5x + 7y = 50 \quad \ldots(i)$$

and

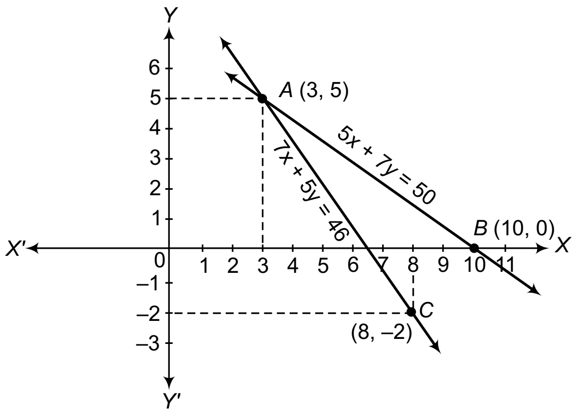
$$7x + 5y = 46 \quad \ldots(ii)$$

Table for line, $5x + 7y = 50$

x	10	3
y	0	5

Table for line, $7x + 5y = 46$

x	8	3
y	– 2	5



Plotting these points, we draw the lines AB and AC passing through them to represent the equations. The two lines AB and AC intersect at the point $A(3, 5)$. So, $x = 3$ and $y = 5$ is the required solution of the pair of linear equation, *i.e.*, cost of one pencil, x = ₹ 3 and cost of one pen, y = ₹ 5.

Question 2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$
(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$
(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Solution (i) The given pair of linear equations is

$$5x - 4y + 8 = 0 \quad \ldots(i)$$

and

$$7x + 6y - 9 = 0 \quad \ldots(ii)$$

On comparing with $ax + by + c = 0$
We get, $a_1 = 5$, $b_1 = -4$, $c_1 = 8$; $a_2 = 7$, $b_2 = 6$, $c_2 = -9$

Here, $\frac{5}{7} \neq \frac{-4}{6}$ $\left(\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right)$

So, lines (i) and (ii) are intersecting lines.

(ii) The given pair of linear equations is

$$9x + 3y + 12 = 0 \quad \ldots(i)$$

and $$18x + 6y + 24 = 0 \quad \ldots(ii)$$

On comparing with $ax + by + c = 0$

We get, $a_1 = 9, b_1 = 3, c_1 = 12; a_2 = 18, b_2 = 6, c_2 = 24$

Here, $\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$ $\left(\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$

$\Rightarrow$ $\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

So, lines (i) and (ii) are coincident lines.

(iii) The given pair of linear equations is

$$6x - 3y + 10 = 0 \quad \ldots(i)$$

and $$2x - y + 9 = 0 \quad \ldots(ii)$$

On comparing with $ax + by + c = 0$

We get, $a_1 = 6, b_1 = -3, c_1 = 10; \quad a_2 = 2, b_2 = -1, c_2 = 9$

Here, $\frac{6}{2} = \frac{-3}{-1} \neq \frac{10}{9}$ $\left(\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\right)$

$\Rightarrow$ $\frac{3}{1} = \frac{3}{1} \neq \frac{10}{9}$

So, lines (i) and (ii) are parallel lines.

Question 3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent or inconsistent.

(i) $3x + 2y = 5; 2x - 3y = 7$

(ii) $2x - 3y = 8; 4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$

(iv) $5x - 3y = 11; -10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

Solution (i) The given equation can be rewritten as

$$3x + 2y - 5 = 0, \quad 2x - 3y - 7 = 0$$

On comparing with $ax + by + c = 0$

We get, $a_1 = 3, \; b_1 = 2, \; c_1 = -5; \quad a_2 = 2, \; b_2 = -3, \; c_2 = -7$

Here, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3}$

Thus, $\frac{3}{2} \neq \frac{2}{-3}$, *i.e.*, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the pair of linear equations is consistent.

(ii) The given equation can be rewritten as

$$2x - 3y - 8 = 0; \quad 4x - 6y - 9 = 0$$

On comparing with $ax + by + c = 0$

We get, $a_1 = 2, b_1 = -3, c_1 = -8; \quad a_2 = 4, b_2 = -6, c_2 = -9$

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$

Thus, $\frac{1}{2} = \frac{1}{2} \neq \frac{8}{9}$, *i.e.,* $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the pair of linear equations is inconsistent.

(iii) The given equation can be rewritten as

$$\frac{3}{2}x + \frac{5}{3}y - 7 = 0; \quad 9x - 10y - 14 = 0$$

On comparing with $ax + by + c = 0$

We get, $a_1 = \frac{3}{2}, b_1 = \frac{5}{3}, c_1 = -7; a_2 = 9, b_2 = -10, c_2 = -14$

Here, $\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}, \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = -\frac{1}{6},$

Thus, $\frac{1}{6} \neq -\frac{1}{6}$, *i.e.,* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the pair of linear equations is consistent.

(iv) The given equation can be rewritten as

$$5x - 3y - 11 = 0; \quad -10x + 6y + 22 = 0$$

On comparing with $ax + by + c = 0$

We get, $a_1 = 5, b_1 = -3, c_1 = -11; \quad a_2 = -10, b_2 = 6, c_2 = 22$

Here, $\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2},$

$\frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$

Thus, $-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$

i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the pair of linear equations is consistent (or dependent).

(v) The given equation can be rewritten as

$$\frac{4}{3}x + 2y - 8 = 0; \quad 2x + 3y - 12 = 0$$

Here, $a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8; \quad a_2 = 2, b_2 = 3, c_2 = -12$

$\Rightarrow$ $\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3}, \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$

Thus, $\frac{2}{3} = \frac{2}{3} = \frac{2}{3}$, *i.e.,* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the pair of linear equations is consistent (or dependent).

Question 4. Which of the following pairs of linear equations are consistent or inconsistent? If consistent obtain the solution graphically?

(i) $x + y = 5, 2x + 2y = 10$ (ii) $x - y = 8, 3x - 3y = 16$

(iii) $2x + y - 6 = 0,$ (iv) $2x - 2y - 2 = 0,$

$4x - 2y - 4 = 0$ $4x - 4y - 5 = 0$

Solution (i) Given pair of lines are $x + y = 5$ and $2x + 2y = 10$

Here, $a_1 = 1, b_1 = 1, c_1 = -5; a_2 = 2, b_2 = 2, c_2 = -10$

Here, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2} \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given pair of linear equations are coincident.

Therefore, these lines have infinitely many common solutions.

Hence, the given pair of linear equations is consistent.

Now, $x + y = 5$ ...(i)

$\Rightarrow y = 5 - x$

If $x = 0$, $y = 5$

If $x = 5$, $y = 0$

x	0	5
$y = 5 - x$	5	0
Points	*A*	*B*

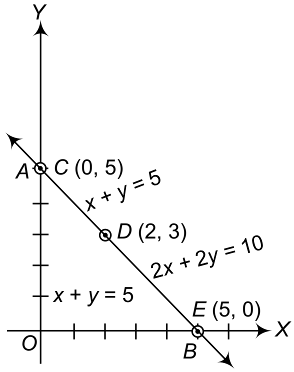
and $2x + 2y = 10$ ...(ii)

$\Rightarrow y = \frac{10 - 2x}{2}$

If $x = 0$, $y = 5$

If $x = 2$, $y = 3$

x	0	2	5
$y = (10 - 2x)/2$	5	3	0
Points	*C*	*D*	*E*



Plotting the points $A(0, 5)$ and $B(5, 0)$, we get the line AB. Again, plotting the points $C(0, 5)$, $D(2, 3)$ and $E(5, 0)$, we get the line CDE.

We observe that the lines represented by Eqs. (i) and (ii) are coincident.

(ii) Given pair of lines are $x - y = 8$, $3x - 3y = 16$

Here, $a_1 = 1, b_1 = -1, c_1 = -8;\ a_2 = 3, b_2 = -3, c_2 = -16$

Here, $$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$$

and $$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the lines represented by the given equations are parallel. Therefore, it has no solution. Hence, the given pair of lines is inconsistent.

(iii) Given pair of lines are $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$

Here, $a_1 = 2, b_1 = 1, c_1 = -6;\ a_2 = 4, b_2 = -2, c_2 = -4$

Here, $$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{-2}$$

and $$\frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

$\therefore$ $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the given pair of linear equations are intersecting, therefore these lines have unique solution.

Hence, given pair of linear equation is consistent.

We have, $2x + y - 6 = 0$

$\Rightarrow$ $y = 6 - 2x$

When $x = 0, y = 6$

When $x = 3, y = 0$

x	0	3	2
y	6	0	2
Point	*A*	*B*	*E*

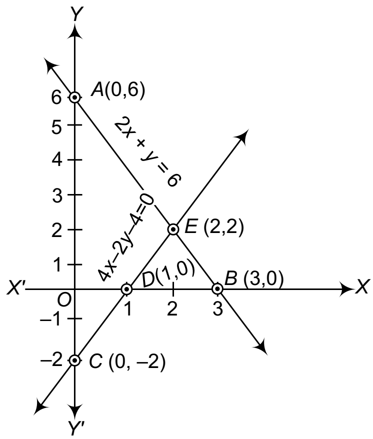
and $4x - 2y - 4 = 0$

$\Rightarrow$ $y = 2x - 2$

When $x = 0, y = -2$

When $x = 1, y = 0$

x	0	1	2
y	–2	0	2
Point	*C*	*D*	*E*



Plotting the points $A(0, 6)$ and $B(3, 0)$, we get the straight line AB. Plotting the points $C(0, -2)$ and $D(1, 0)$, we get the straight line CD. The lines AB and CE intersect at $E(2, 2)$.

(iv) Given, $2y - 2x - 2 = 0$ and $4x - 4y - 5 = 0$

Here, $a_1 = 2, b_1 = -2, c_1 = -2; a_2 = 4, b_2 = -4, c_2 = -5$

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

$\therefore \qquad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pairs of linear equations are parallel.

Therefore, these lines have no solution.

Hence, the given pair has inconsistent.

Question 5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solution Let the length of the garden be x metre and its width be y metre.

Then, perimeter of rectangular garden = 2 (length + width) = $2(x + y)$

Therefore, half perimeter = $(x + y)$

But it is given as 36.

$\therefore \qquad (x + y) = 36 \qquad$ ...(i)

Also, $x = y + 4$, *i.e.*, $x - y = 4 \qquad$ ...(ii)

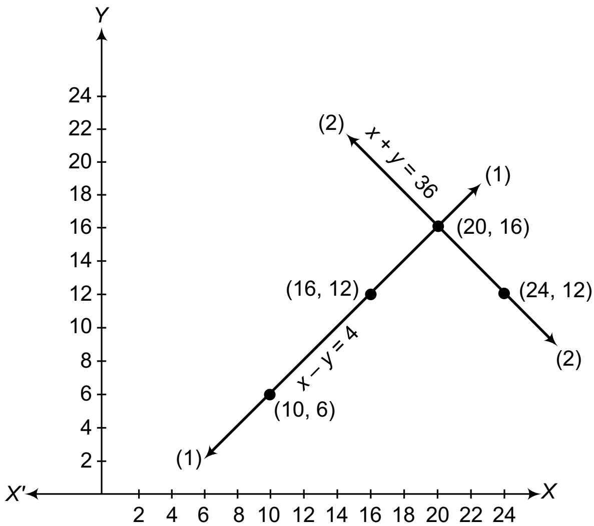
For finding the solution of Eqs. (i) and (ii) graphically, we form the following table

For $x + y = 36$

x	20	24
y	16	12

For $x - y = 4$

x	10	16	20
y	6	12	16



Draw the graphs by joining points (20, 16) and (24, 12) and points (10, 6) and (16, 12). The two lines intersect at point (20, 16) as shown in the graph.

Hence, length = 20 m and width = 16 m

Question 6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is

(i) intersecting lines

(ii) parallel lines

(iii) coincident lines

Solution Given, linear equation is

$$2x + 3y - 8 = 0 \quad \ldots(i)$$

(i) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ Any intersecting line may be taken as $5x + 2y - 9 = 0$.

(ii) For parallel lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore$ Any line parallel to Eq. (i) may be taken as $6x + 9y + 7 = 0$.

(iii) For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore$ Any line coincident to Eq. (i) may be taken as $4x + 6y - 16 = 0$.

Question 7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and x-axis and shade the triangular region.

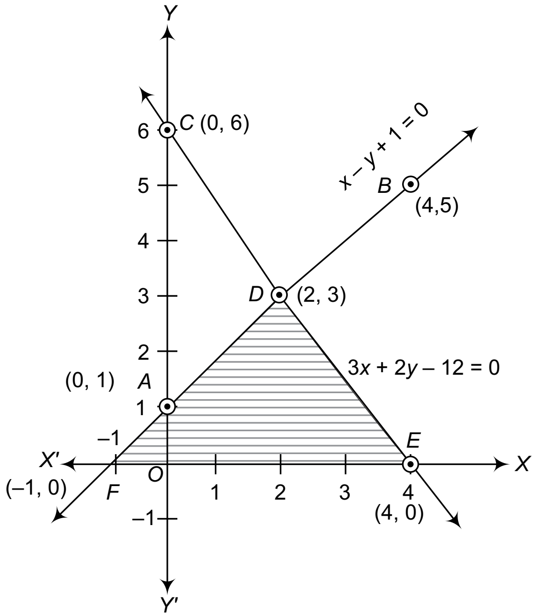
Solution Pair of linear equations are

$$x - y + 1 = 0 \quad \ldots(i)$$

x	0	4	2
$y = x + 1$	1	5	3
Points	A	B	D

$$3x + 2y - 12 = 0 \quad \ldots(ii)$$

x	0	2	4
$y = \dfrac{12 - 3x}{2}$	6	3	0
Points	C	D	E



Plot the points $A(0, 1)$, $B(4, 5)$ and join them to get a line AB. Similarly, plot the points $C(0, 6)$, $D(2, 3)$ and join them to form a line CD.

Clearly, the two lines intersect each other at the point $D(2, 3)$. Hence, $x = 2$ and $y = 3$ is the solution of the given pair of equations. The line CD cuts the x-axis at the point $E(4, 0)$ and the line AB cuts the x-axis at the point $F(-1, 0)$. Hence, the coordinates of the vertices of the triangle are $D(2, 3)$, $E(4, 0)$ and $F(-1, 0)$.

3 Pair of Linear Equations in Two Variables

Exercise 3.3

Question 1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14, x - y = 4$

(ii) $s - t = 3, \frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3, 9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3, 0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0, \sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2, \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Solution (i) We have, $x + y = 14$ ...(i)

and $x - y = 4$ ...(ii)

From Eq. (ii), $y = x - 4$ ...(iii)

Substituting y from Eq. (iii) in Eq. (i), we get

$$x + x - 4 = 14 \quad \Rightarrow \quad 2x = 18 \quad \Rightarrow \quad x = 9$$

On substituting $x = 9$ in Eq. (iii), we get

$$y = 9 - 4 = 5 \quad \Rightarrow \quad y = 5$$

$\therefore$ $x = 9, \ y = 5$

(ii) We have, $s - t = 3$ ...(i)

and $\frac{s}{3} + \frac{t}{2} = 6$ ...(ii)

From Eq. (i), $s = t + 3$ ...(iii)

On substituting s from Eq. (iii) in Eq. (ii), we get

$$\frac{t + 3}{3} + \frac{t}{2} = 6$$

$$\frac{2(t + 3) + 3t}{6} = 6$$

$\Rightarrow$ $2(t + 3) + 3t = 36$

$\Rightarrow$ $5t + 6 = 36 \Rightarrow 5t = 30$

$\Rightarrow$ $t = 6$

From Eq. (iii), $s = 6 + 3 = 9$

Hence, $s = 9$ and $t = 6$

(iii) We have, $3x - y = 3$ ...(i)

and $9x - 3y = 9$ ...(ii)

From Eq. (i), $y = 3x - 3$ ...(iii)

On substituting y from Eq. (iii) in Eq. (ii), we get

$$9x - 3(3x - 3) = 9$$

$\Rightarrow$ $9 = 9$

It is a true statement. Hence, every solution of Eq. (i) is a solution of Eq. (ii) and *vice-versa*.

On putting $x = k$ in Eq. (i), we get

$$3k - y = 3 \Rightarrow y = 3k - 3$$

$\therefore x = k,\ y = 3k - 3$ is a solution for every real k.

Hence, infinitely many solutions exist.

(iv) We have, $0.2x + 0.3y = 1.3$ ...(i)

and $0.4x + 0.5y = 2.3$ ...(ii)

$\Rightarrow$ $\frac{4}{10}x + \frac{5}{10}y = \frac{23}{10}$

From Eq. (i), $\frac{2x}{10} + \frac{3y}{10} = \frac{13}{10}$

$\Rightarrow$ $2x + 3y = 13 \Rightarrow 3y = 13 - 2x$

$\Rightarrow$ $y = \frac{13 - 2x}{3}$ ...(iii)

On substituting y from Eq. (iii) in Eq. (ii), we get

$$\frac{4}{10}x + \frac{5}{10} \times \frac{(13 - 2x)}{3} = \frac{23}{10}$$

$\Rightarrow$ $4x + \frac{5}{3}(13 - 2x) = 23$

$\Rightarrow$ $12x + 5(13 - 2x) = 3 \times 23$

$\Rightarrow$ $12x + 65 - 10x = 69$

$\Rightarrow$ $2x = 69 - 65 = 4 \Rightarrow x = 2$

On substituting $x = 2$ in Eq. (iii), we get

$$y = \frac{13 - 2 \times 2}{3} = \frac{9}{3} \Rightarrow y = 3$$

Hence, $x = 2$ and $y = 3$

(v) We have, $\sqrt{2}x + \sqrt{3}y = 0$ ...(i)

and $\sqrt{3}x - \sqrt{8}y = 0$ ...(ii)

From Eq. (ii), $\sqrt{8}y = \sqrt{3}x \Rightarrow y = \frac{\sqrt{3}x}{\sqrt{8}}$ ...(iii)

On substituting y from Eq. (iii) in Eq. (i), we get

$$\sqrt{2}x + \sqrt{3} \times \left(\frac{\sqrt{3}x}{\sqrt{8}}\right) = 0$$

$\Rightarrow$ $\sqrt{2}x + \frac{3x}{\sqrt{8}} = 0$

$\Rightarrow$ $\sqrt{2} \times \sqrt{8}x + 3x = 0$

$\Rightarrow$ $\sqrt{16}x + 3x = 0$

$\Rightarrow$ $4x + 3x = 0$

$\Rightarrow$ $7x = 0$

$\Rightarrow$ $x = 0$

Putting $x = 0$ in Eq. (iii), $y = 0$

(vi) We have, $$\frac{3x}{2} - \frac{5y}{3} = -2 \quad \text{...(i)}$$

and $$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \quad \text{...(ii)}$$

From Eq. (ii),

$$\frac{y}{2} = \frac{13}{6} - \frac{x}{3} = \frac{13 - 2x}{6},$$

$$\Rightarrow \quad y = 2 \times \frac{(13 - 2x)}{6}$$

$$\Rightarrow \quad y = \frac{(13 - 2x)}{3} \quad \text{...(iii)}$$

On substituting y from Eq. (iii) in Eq. (i), we get

$$\frac{3x}{2} - \frac{5}{3} \times \frac{(13 - 2x)}{3} = -2$$

$$\Rightarrow \quad \frac{3x}{2} - \frac{5}{9}(13 - 2x) = -2$$

$$\Rightarrow \quad 18 \times \frac{3x}{2} - 18 \times \frac{5}{9}(13 - 2x) = -2 \times 18 \quad \text{(Multiply by 18)}$$

$$\Rightarrow \quad 27x - 10(13 - 2x) = -36$$

$$\Rightarrow \quad 27x - 130 + 20x = -36$$

$$\Rightarrow \quad 27x + 20x = 130 - 36$$

$$\Rightarrow \quad 47x = 94$$

$$\Rightarrow \quad x = \frac{94}{47}$$

$$\Rightarrow \quad x = 2$$

On substituting $x = 2$ in Eq. (iii), we get

$$y = \frac{13 - 2 \times 2}{3} = \frac{9}{3} = 3$$

i.e., $$y = 3$$

Question 2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of m for which $y = mx + 3$.

Solution We have, $$2x + 3y = 11 \quad \text{...(i)}$$

and $$2x - 4y = -24 \quad \text{...(ii)}$$

From Eq. (ii), $$4y = 2x + 24$$

or $$y = \frac{x + 12}{2} \quad \text{...(iii)}$$

On substituting y from Eq. (iii) in Eq. (i)

$$2x + 3\left(\frac{x + 12}{2}\right) = 11$$

$$\Rightarrow \quad 4x + 3(x + 12) = 11 \times 2$$

$$\Rightarrow \quad 4x + 3x + 36 = 22$$

$$\Rightarrow \quad 7x = 22 - 36$$

$$\Rightarrow \quad 7x = -14$$

$$\Rightarrow \quad x = -2$$

On substituting $x = -2$ in Eq. (iii), $y = \frac{-2+12}{2} \Rightarrow y = \frac{10}{2} = 5$

On substituting $x = -2$ and $y = 5$ in the equation $y = mx + 3$, we get

$$5 = m \times (-2) + 3 \Rightarrow 5 = -2m + 3$$

$\Rightarrow$ $$2m = 3 - 5 = -2 \Rightarrow m = -1$$

Question 3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Solution (i) Let the two numbers be x and y $(x > y)$.

We are given that, $x - y = 26$ and $x = 3y$

The linear equations are $x - y = 26$ ...(i)

and $x = 3y$ ...(ii)

On substituting x from Eq. (ii) in Eq. (i), we get

$$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13$$

From Eq. (ii), $x = 3 \times 13 = 39 \Rightarrow x = 39$

Hence, the two numbers are 39 and 13.

(ii) Let the supplementary angles be $x°$ and $y°$ $(x° > y°)$

Now, we are given that

$$x° + y° = 180° \quad \text{...(i)}$$

and $$x° - y° = 18° \quad \text{...(ii)}$$

From Eq. (ii), $$y° = x° - 18° \quad \text{...(iii)}$$

On substituting $y°$ from Eq. (iii) in Eq. (i),

$$x° + x° - 18° = 180°$$

$\Rightarrow$ $$2x° = 198° \Rightarrow x° = 99°$$

Then, from Eq. (iii), $y° = 99° - 18° = 81° \Rightarrow y° = 81°$

Hence, the angles are 99° and 81°.

(iii) Let cost of one bat = ₹ x

Cost of one ball = ₹ y

We are given that,

Cost of 7 bats and 6 balls = ₹ 3800, *i.e.*, $7x + 6y = 3800$ ...(i)

Cost of 3 bats and 5 balls = ₹ 1750, *i.e.*, $3x + 5y = 1750$ ...(ii)

From Eq. (ii), $5y = 1750 - 3x \Rightarrow y = \frac{1750 - 3x}{5}$ ...(iii)

On substituting y from Eq. (iii) in Eq. (i), we get

$$7x + 6 \times \frac{(1750 - 3x)}{5} = 3800$$

On multiplying by 5, we get

$$35x + 6 \times (1750 - 3x) = 5 \times 3800$$

$$\Rightarrow \quad 35x + 10500 - 18x = 19000$$

$$\Rightarrow \quad 35x - 18x = 19000 - 10500$$

$$\Rightarrow \quad 17x = 8500$$

$$x = 500$$

From Eq. (iii),

$$y = \frac{1750 - 3 \times 500}{5}$$

$$= \frac{1750 - 1500}{5}$$

$$= \frac{250}{5}$$

$$\Rightarrow \quad y = 50$$

Hence, cost of one bat = ₹ 500

Cost of one ball = ₹ 50

(iv) Let fixed charge be ₹ x and charge per km be ₹ y.

Then, by given conditions,

$$x + 10y = 105 \quad ...(i)$$

and $$x + 15y = 155 \quad ...(ii)$$

From Eq. (i), $x = (105 - 10y)$ ...(iii)

On substituting x from Eq. (iii) in Eq. (ii), we get

$$105 - 10y + 15y = 155$$

$$\Rightarrow \quad 5y = 155 - 105 = 50$$

$$\Rightarrow \quad y = 10$$

From Eq. (iii),

$$x = 105 - 10 \times 10 = 105 - 100$$

$$\Rightarrow \quad x = 5$$

Hence, fixed charges = ₹ 5

Rate per kilometre = ₹ 10

Amount to be paid for travelling 25 km = $x + 25y = 5 + 10 \times 25$

$= 5 + 250 =$ ₹ 255

(v) Let $\frac{x}{y}$ be the fraction where x and y are positive integers.

Then, by given conditions,

$$\frac{x+2}{y+2} = \frac{9}{11} \text{ and } \frac{x+3}{y+3} = \frac{5}{6}$$

$\Rightarrow \quad 11 \times (x+2) = 9 \times (y+2)$ and $6 \times (x+3) = 5 \times (y+3)$

$\Rightarrow \quad 11x + 22 = 9y + 18 \Rightarrow 6x + 18 = 5y + 15$

$\Rightarrow \quad 11x - 9y + 4 = 0$ ...(i)

and $\quad 6x - 5y + 3 = 0$ ...(ii)

From Eq. (ii), $\quad 5y = 6x + 3$

or $\quad y = \frac{6x+3}{5}$ ...(iii)

On substituting y from Eq. (iii) in Eq. (i),

$$11x - 9 \times \left(\frac{6x+3}{5}\right) + 4 = 0$$

$\Rightarrow \quad 55x - 9 \times (6x+3) + 20 = 0$

$\Rightarrow \quad 55x - 54x - 27 + 20 = 0 \quad \Rightarrow x = 7$

From Eq. (iii), $y = \frac{6 \times 7 + 3}{5} \Rightarrow y = \frac{42+3}{5} = \frac{45}{5} \Rightarrow y = 9$

Hence, the fraction is $\frac{7}{9}$.

(vi) Let x (in year) be the present age of Jacob's son and y (in year) be the present age of Jacob. Five years hence, it has relation by given condition,

$$(y+5) = 3(x+5) \Rightarrow y + 5 = 3x + 15$$

$\Rightarrow \quad 3x - y + 10 = 0$ ...(i)

Five years ago, it has relation

$$(y-5) = 7(x-5)$$

$\Rightarrow \quad 7x - y - 30 = 0$ ...(ii)

From Eq. (i), $\quad y = 3x + 10$ ...(iii)

On substituting y from Eq. (iii) in Eq. (ii),

$$7x - (3x+10) - 30 = 0$$

$\Rightarrow \quad 4x - 40 = 0$

$\Rightarrow \quad 4x = 40$

$\Rightarrow \quad x = 10$

On substituting $x = 10$ in Eq. (iii), we get

$$y = 3 \times 10 + 10$$

$\Rightarrow \quad y = 40$

Hence, the present age of Jacob is 40 yr and present age of his son is 10 yr.

3 Pair of Linear Equations in Two Variables

Exercise 3.4

Question 1. Solve the following pair of linear equations by the elimination method and the substitution method.

(i) $x + y = 5$ and $2x - 3y = 4$

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Solution (i) **Solution by elimination method**

We have, $x + y = 5$ ...(i)

and $2x - 3y = 4$ ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 1 and adding, we get

$$3(x + y) + 1(2x - 3y) = 3 \times 5 + 1 \times 4$$

$\Rightarrow$ $3x + 3y + 2x - 3y = 15 + 4$

$\Rightarrow$ $5x = 19 \Rightarrow x = \frac{19}{5}$

From Eq. (i), substituting $x = \frac{19}{5}$, we get

$$\frac{19}{5} + y = 5$$

$\Rightarrow$ $y = 5 - \frac{19}{5} \Rightarrow y = \frac{25 - 19}{5} = \frac{6}{5}$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

Solution by substitution method

We have, $x + y = 5$ ...(i)

and $2x - 3y = 4$ ...(ii)

From Eq. (i), $y = 5 - x$ ...(iii)

On substituting y from Eq. (iii) in Eq. (ii),

$$2x - 3(5 - x) = 4$$

$\Rightarrow$ $2x - 15 + 3x = 4$

$\Rightarrow$ $5x = 19$

$\Rightarrow$ $x = \frac{19}{5}$

Then, from Eq. (iii), substituting $x = \frac{19}{5}$,

$$y = 5 - \frac{19}{5} \Rightarrow y = \frac{25 - 19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

(ii) **Solution by elimination method**

We have, $3x + 4y = 10$ ...(i)

and $2x - 2y = 2$ ...(ii)

On multiplying Eq. (i) by 1 and Eq. (ii) by 2 and adding, we get

$$1(3x + 4y) + 2(2x - 2y) = 1 \times 10 + 2 \times 2$$

$\Rightarrow$ $3x + 4y + 4x - 4y = 10 + 4$

$\Rightarrow$ $7x = 14 \Rightarrow x = 2$

From Eq. (i), substituting $x = 2$ in Eq. (i), we get

$$3 \times 2 + 4y = 10 \Rightarrow 4y = 4$$

$\Rightarrow$ $y = 1$

Hence, $x = 2$ and $y = 1$

Solution by substitution method

We have, $3x + 4y = 10$ ...(i)

and $2x - 2y = 2$ ...(ii)

From Eq. (ii), $2y = 2x - 2$

$\Rightarrow$ $y = x - 1$ (Divide by 2)...(iii)

On substituting y from Eq. (iii) in Eq. (i),

$$3x + 4(x - 1) = 10$$

$\Rightarrow$ $3x + 4x - 4 = 10$

$\Rightarrow$ $7x = 14 \Rightarrow x = 2$

From Eq. (iii), $y = 2 - 1 = 1$

$\Rightarrow$ $y = 1$

Hence, $x = 2$ and $y = 1$

(iii) **Solution by elimination method**

We have, $3x - 5y = 4$ ...(i)

and $9x = 2y + 7$ ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 1, we get

$$9x - 15y = 12 \quad \text{...(iii)}$$

and $9x - 2y = 7$ ...(iv)

On subtracting Eq. (iv) from Eq. (iii),

$$(9x - 15y) - (9x - 2y) = 12 - 7$$

$\Rightarrow$ $-15y + 2y = 5$

$\Rightarrow$ $-13y = 5 \Rightarrow y = -\frac{5}{13}$

From Eq. (i), substituting $y = -\frac{5}{13}$, we get

$$3x - 5\left(-\frac{5}{13}\right) = 4$$

$\Rightarrow$ $39x + 25 = 52$

$\Rightarrow$ $39x = 27 \Rightarrow x = \frac{9}{13}$

Hence, $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

Solution by substitution method

We have, $$3x - 5y = 4 \quad \ldots(i)$$

$$9x = 2y + 7 \quad \ldots(ii)$$

From Eq. (ii) $2y = 9x - 7$ or $y = \dfrac{9x - 7}{2}$ ...(iii)

Substituting y from Eq. (iii) in Eq. (i), we get

$$3x - 5 \times \left(\frac{9x - 7}{2}\right) = 4 \Rightarrow 6x - 45x + 35 = 8$$

$$\Rightarrow \quad -39x = -27 \Rightarrow x = \frac{9}{13}$$

From Eq. (iii), substituting $x = \dfrac{9}{13}$, we get

$$y = \frac{9 \times \frac{9}{13} - 7}{2} = \frac{81 - 91}{2 \times 13} = -\frac{10}{26}$$

$$\Rightarrow \quad y = -\frac{5}{13}$$

Hence, $$x = \frac{9}{13} \text{ and } y = -\frac{5}{13}$$

(iv) **Solution by elimination method**

We have, $$\frac{x}{2} + \frac{2y}{3} = -1 \quad \ldots(i)$$

and $$x - \frac{y}{3} = 3 \quad \ldots(ii)$$

On multiplying Eq. (i) by 1 and Eq. (ii) by 2, we get

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad \ldots(iii)$$

and $$2x - \frac{2y}{3} = 6 \quad \ldots(iv)$$

On adding Eq. (iii) and Eq. (iv), we get

$$\frac{x}{2} + 2x = -1 + 6 \Rightarrow \frac{5}{2}x = 5 \Rightarrow x = 2$$

Now, from Eq. (ii) substituting $x = 2$, we get

$$2 - \frac{y}{3} = 3 \Rightarrow -\frac{y}{3} = 1 \Rightarrow y = -3$$

Hence, $$x = 2 \text{ and } y = -3$$

Solution by substitution method

We have, $$\frac{x}{2} + \frac{2y}{3} = -1 \quad \ldots(i)$$

and $$x - \frac{y}{3} = 3 \quad \ldots(ii)$$

From Eq. (ii), $$\frac{y}{3} = x - 3$$

$$\Rightarrow \quad y = 3(x - 3) \quad \ldots(iii)$$

On substituting $y = 3(x - 3)$ in Eq. (i), we get

$$\frac{x}{2} + \frac{2}{3} \times 3\,(x - 3) = -1$$

$$\Rightarrow \quad \frac{x}{2} + 2(x - 3) = -1$$

$$\Rightarrow \quad x + 4(x - 3) = -2$$

$$\Rightarrow \quad 5x - 12 = -2 \Rightarrow 5x = 10 \Rightarrow x = 2$$

Now, from Eq. (iii) substituting $x = 2$, we get

$$y = 3(2 - 3) = -3$$

Hence, $x = 2$ and $y = -3$

Question 2. Form the pair of linear equations in the following problems and find their solutions (if they exist) by the elimination method

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$, if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution (i) Let the fraction be $\frac{x}{y}$.

According to the given conditions,

$$\frac{x+1}{y-1} = 1;\ \frac{x}{y+1} = \frac{1}{2} \Rightarrow x + 1 = y - 1;\ 2x = y + 1$$

$$\Rightarrow \quad x - y = -2 \quad \text{...(i)}$$

$$\text{and} \quad 2x - y = 1 \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii),

$$(2x - y) - (x - y) = 1 + 2 \Rightarrow x = 3$$

On substituting $x = 3$ in Eq. (i), $3 - y = -2 \Rightarrow y = 5$

Hence, the fraction is $\frac{3}{5}$.

(ii) Let present age of Nuri = x (in years). Present age of Sonu = y (in years)

According to the given conditions,

Five years ago, $x - 5 = 3(y - 5) \Rightarrow x - 3y = -10$ ...(i)

Ten years later, $x + 10 = 2(y + 10) \Rightarrow x - 2y = 10$ ...(ii)

On subtracting Eq. (i) from Eq. (ii),

$$(x - 2y) - (x - 3y) = 10 + 10$$

$$\Rightarrow \quad -2y + 3y = 20$$

$$\Rightarrow \quad y = 20$$

From Eq. (ii), substituting $y = 20$, we get

$$x = 2y + 10 = 2 \times 20 + 10$$

$$\Rightarrow \quad x = 50$$

Therefore, present age of Nuri = 50 yr

and present age of Sonu = 20 yr

(iii) Let x be the digit at unit's place and y be the digit at ten's place of the value of the number

$$x + y = 9 \quad \text{...(i)}$$

Value of the number = $x + 10y$

When we reverse the digits, the value of the new number = $y + 10x$

We are given that $9 \times (x + 10y) = 2 \times (y + 10x)$

$$\Rightarrow \quad 9x + 90y = 2y + 20x$$

$$\Rightarrow \quad 88y = 11x$$

$$\Rightarrow \quad x = 8y \quad \text{...(ii)}$$

On substituting x from Eq. (ii) in Eq. (i),

$$8y + y = 9$$

$$\Rightarrow \quad 9y = 9$$

$$\Rightarrow \quad y = 1$$

Then, from Eq. (ii), $x = 8 \times 1 \Rightarrow x = 8$

Hence, the number is

$$x + 10y = 8 + 10 \times 1 = 8 + 10 = 18$$

(iv) Let number of ₹ 50 notes = x

Number of ₹ 100 notes = y

By given condition, $x + y = 25$ ...(i)

Also, $50 \times x + 100 \times y = 2000$

or $x + 2y = 40$ (Divide by 50)...(ii)

Subtracting Eq. (i) from Eq. (ii)

$$(x + 2y) - (x + y) = 40 - 25$$

$$\Rightarrow \quad y = 15$$

On substituting $y = 15$ in Eq. (i), we get

$$x + 15 = 25 \Rightarrow x = 10$$

$\therefore$ Number of ₹ 50 notes $x = 10$

Number of ₹ 100 notes $y = 15$

(v) Let the fixed charges for the first three days be ₹ x.

Let the additional charge per day be ₹ y.

Saritha paid for 7 days = ₹ 27

By given condition,

$$x + 4 \times y = 27$$

(∵ ₹ $4y$ are to be paid for 4 extra days)

$$\Rightarrow \quad x + 4y = 27 \quad \ldots(i)$$

and in the case of Susy,

$$x + 2y = 21 \quad \ldots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$2y = 27 - 21 \Rightarrow y = 3$$

From Eq. (i), substituting $y = 3$, we get

$$x + 4 \times 3 = 27 \Rightarrow x = 15$$

Fixed charge for first three days = ₹ 15

Addition charge per extra day = ₹ 3

3 Pair of Linear Equations in Two Variables

Exercise 3.5

Question 1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$ and $3x - 9y - 2 = 0$

(ii) $2x + y = 5$ and $3x + 2y = 8$

(iii) $3x - 5y = 20$ and $6x - 10y = 40$

(iv) $x - 3y - 7 = 0$ and $3x - 3y - 15 = 0$

Solution (i) We have, $x - 3y - 3 = 0$ ...(i)

and $3x - 9y - 2 = 0$ ...(ii)

Here, $a_1 = 1, b_1 = -3, c_1 = -3; a_2 = 3, b_2 = -9, c_2 = -2$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, no solution exists.

(ii) We have, $2x + y = 5$ ...(i)

and $3x + 2y = 8$ ...(ii)

Here, $a_1 = 2, b_1 = 1, c_1 = -5; a_2 = 3, b_2 = 2, c_2 = 8$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, we have a unique solution.

$$2x + y - 5 = 0$$

$$3x + 2y - 8 = 0$$

By cross multiplication, we get

$$\frac{x}{\begin{matrix} 1 & -5 \\ 2 & -8 \end{matrix}} = \frac{y}{\begin{matrix} -5 & 2 \\ -8 & 3 \end{matrix}} = \frac{1}{\begin{matrix} 2 & 1 \\ 3 & 2 \end{matrix}}$$

$$\Rightarrow \quad \frac{x}{\{(1)(-8) - (2)(-5)\}} = \frac{y}{\{(-5)(3) - (-8)(2)\}} = \frac{1}{\{(2)(2) - (3)(1)\}}$$

$$\Rightarrow \quad \frac{x}{(-8 + 10)} = \frac{y}{(-15 + 16)} = \frac{1}{(4 - 3)}$$

$$\Rightarrow \quad \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$$\Rightarrow \quad \frac{x}{2} = \frac{1}{1} \text{ and } \frac{y}{1} = \frac{1}{1}$$

$$\Rightarrow \quad x = 2 \text{ and } y = 1$$

(iii) We have, $3x - 5y = 20$ ...(i)

and $6x - 10y = 40$ ...(ii)

Here, $a_1 = 3, b_1 = -5, c_1 = -20$; $a_2 = 6, b_2 = -10, c_2 = -40$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad \left(\text{Each} = \frac{1}{2}\right)$$

There are infinitely many solutions.

(iv) We have, $x - 3y - 7 = 0$ ...(i)

and $3x - 3y - 15 = 0$ ...(ii)

Here, $a_1 = 1, b_1 = -3, c_1 = -7$; $a_2 = 3, b_2 = -3, c_2 = -15$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-3} = 1$$

Here, $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

Hence, unique solution exists.

$$x - 3y - 7 = 0$$
$$3x - 3y - 15 = 0$$

By cross multiplication, we have

$$\frac{x}{\begin{matrix} -3 & -7 \\ -3 & -15 \end{matrix}} = \frac{y}{\begin{matrix} -7 & 1 \\ -15 & 3 \end{matrix}} = \frac{1}{\begin{matrix} 1 & -3 \\ 3 & -3 \end{matrix}}$$

$$\Rightarrow \quad \frac{x}{\{(-3)(-15) - (-3)(-7)\}} = \frac{y}{\{(-7)(3) - (-15)(1)\}} = \frac{1}{\{(1)(-3) - (3)(-3)\}}$$

$$\Rightarrow \quad \frac{x}{(45 - 21)} = \frac{y}{(-21 + 15)} = \frac{1}{(-3 + 9)}$$

$$\Rightarrow \quad \frac{x}{24} = -\frac{y}{6} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{x}{24} = \frac{1}{6} \text{ and } -\frac{y}{6} = \frac{1}{6}$$

$$\Rightarrow \quad x = \frac{24}{6} \text{ and } y = -\frac{6}{6}$$

$$\Rightarrow \quad x = 4 \text{ and } y = -1$$

Question 2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$
$$(a - b)x + (a + b)\, y = 3a + b - 2$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$
$$(2k - 1)\,x + (k - 1)\,y = 2k + 1$$

Solution (i) We have, $2x + 3y - 7 = 0$

and $\quad (a - b)\,x + (a + b)\,y - (3a + b - 2) = 0$

Here, $\quad a_1 = 2, b_1 = 3, c_1 = -7;$

$$a_2 = a - b, b_2 = a + b, c_2 = 3a + b - 2$$

For infinite number of solutions, we have

$$\frac{2}{a - b} = \frac{3}{a + b} = \frac{-7}{-(3a + b - 2)} \qquad \left(\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\right)$$

$$\Rightarrow \quad \frac{a - b}{2} = \frac{a + b}{3} = \frac{3a + b - 2}{7}$$

From first and second, we have

$$\frac{a - b}{2} = \frac{a + b}{3}$$

$$\Rightarrow \quad 3a - 3b = 2a + 2b$$

$$\Rightarrow \quad a = 5b \qquad \ldots\text{(i)}$$

From second and third, we have

$$\frac{a + b}{3} = \frac{3a + b - 2}{7}$$

$$\Rightarrow \quad 7a + 7b = 9a + 3b - 6$$

$$\Rightarrow \quad 4b = 2a - 6$$

$$\Rightarrow \quad 2b = a - 3 \qquad \text{(Divide by 2)}\ldots\text{(ii)}$$

From Eqs. (i) and (ii), eliminating a,

$$2b = 5b - 3 \Rightarrow b = 1$$

Substituting $b = 1$ in Eq. (i), we get

$$a = 5 \times 1 \Rightarrow a = 5$$

(ii) We have, $\quad 3x + y - 1 = 0 \qquad \ldots$(i)

and $\quad (2k - 1)\,x + (k - 1)y - (2k + 1) = 0 \qquad \ldots$(ii)

Here, $a_1 = 3, b_1 = 1, c_1 = -1; a_2 = 2k - 1, b_2 = k - 1, c_2 = -(2k + 1)$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{2k - 1}, \frac{b_1}{b_2} = \frac{1}{k - 1}, \frac{c_1}{c_2} = \frac{1}{(2k + 1)}$$

For no solution $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$ *i.e.*, $\dfrac{3}{2k - 1} = \dfrac{1}{k - 1} \Rightarrow 3\,(k - 1) = 2k - 1$

$$\Rightarrow \quad 3k - 2k = -1 + 3 \Rightarrow k = 2$$

Now, if $k = 2$, then we have

$$\frac{a_1}{a_2} = \frac{3}{4-1} = 1, \frac{b_1}{b_2} = \frac{1}{2-1} = 1, \frac{c_1}{c_2} = \frac{1}{(2k+1)} = \frac{1}{4+1} = \frac{1}{5}$$

Thus, for $k = 2$, we have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, it has no solution when $k = 2$.

Question 3. Solve the following pair of linear equations by the substitution and cross multiplication methods

$$8x + 5y = 9$$
$$3x + 2y = 4$$

Solution We have, $8x + 5y = 9$ and $3x + 2y = 4$

or $$8x + 5y - 9 = 0 \quad \ldots(i)$$

and $$3x + 2y - 4 = 0 \quad \ldots(ii)$$

By substitution method

From Eq. (ii),

$$x = \frac{4-2y}{3} \quad \ldots(iii)$$

Substitute the value of x, from Eq. (iii) in Eq. (i), we get

$$8\left(\frac{4-2y}{3}\right) + 5y - 9 = 0$$

$\Rightarrow$ $$32 - 16y + 15y - 27 = 0$$

$\Rightarrow$ $$-y + 5 = 0 \Rightarrow y = 5$$

From Eq. (iii) substitute $y = 5$, we get

$$x = \frac{4 - 2 \times 5}{3} = \frac{4-10}{3} = \frac{-6}{3} = -2$$

By cross multiplication method

We have, $$8x + 5y - 9 = 0$$

and $$3x + 2y - 4 = 0$$

$$\frac{x}{\begin{matrix} 5 & -9 \\ 2 & -4 \end{matrix}} = \frac{y}{\begin{matrix} -9 & 8 \\ -4 & 3 \end{matrix}} = \frac{1}{\begin{matrix} 8 & 5 \\ 3 & 2 \end{matrix}}$$

Then, we have, $$\frac{x}{\{(5)(-4) - (2)(-9)\}} = \frac{y}{\{(-9)(3) - (-4)(8)\}}$$

$$= \frac{1}{\{(8) \times (2) - (3) \times (5)\}}$$

$\Rightarrow$ $$\frac{x}{(-20+18)} = \frac{y}{(-27+32)} = \frac{1}{(16-15)}$$

$\Rightarrow$ $$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$$

$\Rightarrow$ $$x = -2 \text{ and } y = 5$$

Question 4. Form the pair of linear equations in the following problems and find their solution (if they exist) by any algebraic method

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay ₹ 1000 as hostel charges whereas a student B, who takes food for 26 days, pays ₹ 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 sq units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 sq units. Find the dimensions of the rectangle.

Solution. (i) Let fixed hostel charge (monthly) = ₹ c

and cost of food for one day = ₹ x

In the case of A student,

$$20x + c = 1000 \quad \text{...(i)}$$

In the case of B student,

$$26x + c = 1180 \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$26x - 20x = 1180 - 1000$$

$\Rightarrow$ $6x = 180 \Rightarrow x = 30$

From Eq. (i), $c = 1000 - 20x$

$= 1000 - 20 \times 30 = 1000 - 600 = 400$

Hence, monthly fixed charges = ₹ 400

Cost of food per day = ₹ 30

(ii) Let the fraction be $\frac{x}{y}$.

According to the given condition,

$$\frac{x-1}{y}=\frac{1}{3} \text{ and } \frac{x}{y+8}=\frac{1}{4}$$

$\Rightarrow$ $3x - y = 3$ ...(i)

and $4x - y = 8$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$(4x - y) - (3x - y) = 8 - 3 \Rightarrow x = 5$$

On putting $x = 5$ in Eq. (i), we get

$$3 \times 5 - y = 3 \Rightarrow y = 12$$

Hence, the fraction is $\frac{5}{12}$.

(iii) Let number of right answers is x and number of wrong answers is y.

Total number of questions $= x + y$

In first case, marks awarded for x right answer $= 3x$

Marks lost for y wrong answer $= y \times 1 = y$

Then, $3x - y = 40$ (By given condition) ...(i)

($\because$ Losing marks we take '–' sign)

In second case, marks awarded for x right answer $= 4x$

Marks lost for y wrong answer $= 2y$

Then, $4x - 2y = 50$ (By given condition)...(ii)

From Eq. (i), $y = 3x - 40$ ...(iii)

From Eq. (ii) and Eq. (iii) eliminating y,

$$4x - 2(3x - 40) = 50 \Rightarrow 4x - 6x + 80 = 50$$

$\Rightarrow$ $2x = 30 \Rightarrow x = 15$

On putting $x = 15$ in Eq. (iii),

$$y = 3 \times 15 - 40 \Rightarrow y = 45 - 40 \Rightarrow y = 5$$

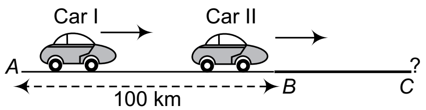
Total number of questions $= x + y = 15 + 5 = 20$

(iv) Let speed of car I $= x$ kmh^{-1}

and speed of car II $= y$ kmh^{-1}

Car I starts from point A and car II starts from point B.

First case



Two cars meet at point C after 5 h.

AC = distance travelled by car I in 5 h $= 5x$ km

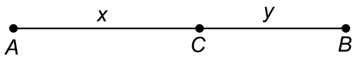
BC = distance travelled by car II in 5 h = $5y$ km

Here, $AC - BC = AB$

$\Rightarrow \quad 5x - 5y = 100 \qquad (\because AB = 100\,\text{km})$

$\Rightarrow \quad x - y = 20 \qquad$ (Divide by 5)...(i)

Second case



Two cars meet at C after one hour.

i.e., $x + y = 100 \qquad$ ...(ii)

On adding Eq. (i) and Eq. (ii), we get

$$2x = 120 \Rightarrow x = 60\,\text{kmh}^{-1}$$

On putting $x = 60$ in Eq. (ii), we get

$$60 + y = 100 \Rightarrow y = 40\,\text{km h}^{-1}$$

Hence, the speed of the two cars are respectively 60 kmh^{-1} and 40 kmh^{-1}.

(v) Let x and y be the length and breadth of a rectangle.

In first case, area is reduced by 9 sq units. When length = $x - 5$ units

and breadth = $y + 3$ units

$\because$ Area of rectangle = Length $\times$ Breadth = xy

According to the given condition,

$$xy - (x - 5) \times (y + 3) = 9$$

$\Rightarrow \quad xy - (xy + 3x - 5y - 15) = 9$

$\Rightarrow \quad -3x + 5y + 15 = 9$

$\Rightarrow \quad 3x - 5y = 6 \qquad$ ...(i)

In second case, area increase by 67 sq units. When length = $x + 3$ and breadth = $y + 2$.

$\therefore \quad (x + 3) \times (y + 2) - xy = 67$

$\Rightarrow \quad xy + 2x + 3y + 6 - xy = 67$

$\Rightarrow \quad 2x + 3y = 61 \qquad$ ...(ii)

On multiplying Eq. (i) by 3 and Eq. (ii) by 5, we get

$9x - 15y = 18 \qquad$ ...(iii)

$10x + 15y = 305 \qquad$ ...(iv)

On adding Eq. (iii) and Eq. (iv),

$$19x = 323 \Rightarrow x = 17$$

On substituting $x = 17$ in Eq. (ii), we get

$$34 + 3y = 61 \Rightarrow 3y = 27 \Rightarrow y = 9$$

Hence, length = 17 units and breadth = 9 units

3 Pair of Linear Equations in Two Variables

Exercise 3.6

Question 1. Solve the following pairs of equations by reducing them to a pair of linear equations

(i) $\frac{1}{2x}+\frac{1}{3y}=2$ and $\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ and $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14$ and $\frac{3}{x}-4y=23$

(iv) $\frac{5}{(x-1)}+\frac{1}{(y-2)}=2$ and $\frac{6}{(x-1)}-\frac{3}{(y-2)}=1$

(v) $\frac{7x-2y}{xy}=5$ and $\frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy$ and $2x+4y=5xy$

(vii) $\frac{10}{(x+y)}+\frac{2}{(x-y)}=4$ and $\frac{15}{(x+y)}-\frac{5}{(x-y)}=-2$

(viii) $\frac{1}{(3x+y)}+\frac{1}{(3x-y)}=\frac{3}{4}$ and $\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}$

Solution (i) We have, $\frac{1}{2x}+\frac{1}{3y}=2$ and $\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

On putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, we get

$$\frac{1}{2}u+\frac{1}{3}v=2 \text{ and } \frac{1}{3}u+\frac{1}{2}v=\frac{13}{6}$$

On multiplying by 6 on both sides, in both equations, we get

$$\Rightarrow \quad 3u+2v=12 \quad \ldots(i)$$

$$2u+3v=13 \quad \ldots(ii)$$

On multiplying Eq. (i) by 3 and Eq. (ii) by 2, then subtracting later from first, we get

$$3(3u+2v)-2(2u+3v)=3\times 12-2\times 13$$

$$\Rightarrow \quad 9u-4u=36-26 \Rightarrow 5u=10 \Rightarrow u=2$$

Then, substituting $u=2$ in Eq. (i), we get

$$6+2v=12 \Rightarrow 2v=6 \Rightarrow v=3$$

Now, $\quad u=2$ and $v=3$

$$\Rightarrow \quad \frac{1}{x}=2 \text{ and } \frac{1}{y}=3$$

$\Rightarrow \qquad x = \frac{1}{2}$ and $y = \frac{1}{3}$

(ii) We have, $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ and $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

On putting $\frac{1}{\sqrt{x}} = u$ and $\frac{1}{\sqrt{y}} = v$, we get

$$2u + 3v = 2 \qquad \ldots(i)$$

and $$4u - 9v = -1 \qquad \ldots(ii)$$

On multiplying Eq. (i) by 3 and Eq. (ii) by 1 and then adding, we get

$$3 \times (2u + 3v) + 1 \times (4u - 9v) = 3 \times 2 - |x|$$

$$\Rightarrow \qquad 10u = 5 \quad \Rightarrow \quad u = \frac{1}{2}$$

On putting $u = \frac{1}{2}$ in Eq. (i),

$$2 \times \frac{1}{2} + 3v = 2 \quad \Rightarrow 3v = 1 \quad \Rightarrow v = \frac{1}{3}$$

Now, $$u = \frac{1}{2} \text{ and } v = \frac{1}{3}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{x}} = \frac{1}{2} \text{ and } \frac{1}{\sqrt{y}} = \frac{1}{3}$$

$$\Rightarrow \qquad \sqrt{x} = 2 \text{ and } \sqrt{y} = 3 \Rightarrow x = 4 \text{ and } y = 9$$

(iii) We have, $\frac{4}{x} + 3y = 14$ and $\frac{3}{x} - 4y = 23$

On putting $\frac{1}{x} = X$, we get

$$4X + 3y = 14 \qquad \ldots(i)$$

and $$3X - 4y = 23 \qquad \ldots(ii)$$

On multiplying Eq. (i) by 4 by Eq. (ii) and 3 and then adding, we get

$$16X + 9X = 4 \times 14 + 3 \times 23$$

$$\Rightarrow \qquad 25x = 56 + 69$$

$$\Rightarrow \qquad 25X = 125 \quad \Rightarrow X = 5$$

Then, $$\frac{1}{x} = 5 \Rightarrow x = \frac{1}{5}$$

From Eq. (i), substituting $X = 5$, we get

$$4 \times 5 + 3y = 14 \quad \Rightarrow \quad 3y = 14 - 20$$

$$\Rightarrow \qquad 3y = -6 \quad \Rightarrow \quad y = -2$$

Hence, $$x = \frac{1}{5} \text{ and } y = -2$$

(iv) We have, $\frac{5}{(x-1)} + \frac{1}{(y-2)} = 2$ and $\frac{6}{(x-1)} - \frac{3}{(y-2)} = 1$

On putting $\frac{1}{x-1} = u$ and $\frac{1}{y-2} = v$, we get

$$5u + v = 2 \quad \ldots\text{(i)}$$

and $$6u - 3v = 1 \quad \ldots\text{(ii)}$$

On multiplying Eq. (i) by 3 and Eq. (ii) by 1, then adding, we get

$$15u + 6u = 6 + 1 \Rightarrow 21u = 7 \Rightarrow u = \frac{1}{3}$$

From Eq. (i), $5 \times \frac{1}{3} + v = 2 \Rightarrow v = 2 - \frac{5}{3} = \frac{1}{3}$

$\Rightarrow$ $$v = \frac{1}{3}$$

Now, $$u = \frac{1}{3} \text{ and } v = \frac{1}{3}$$

$\Rightarrow$ $$\frac{1}{x-1} = \frac{1}{3} \text{ and } \frac{1}{y-2} = \frac{1}{3}$$

$\Rightarrow$ $$x - 1 = 3 \text{ and } y - 2 = 3$$

$\Rightarrow$ $$x = 3 + 1$$

and $$y = 3 + 2$$

$\Rightarrow$ $$x = 4 \text{ and } y = 5$$

(v) We have, $\frac{7x - 2y}{xy} = 5$ and $\frac{8x + 7y}{xy} = 15$

$\Rightarrow$ $$\frac{7x}{xy} - \frac{2y}{xy} = 5 \text{ and } \frac{8x}{xy} + \frac{7y}{xy} = 15$$

$\Rightarrow$ $$\frac{7}{y} - \frac{2}{x} = 5 \text{ and } \frac{8}{y} + \frac{7}{x} = 15$$

On putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get

$$7v - 2u = 5 \quad \ldots\text{(i)}$$

and $$8v + 7u = 15 \quad \ldots\text{(ii)}$$

On multiplying Eq. (i) by 7 and Eq. (ii) by 2, then adding, we get

$$49v + 16v = 35 + 30$$

$\Rightarrow$ $$65v = 65 \Rightarrow v = 1$$

On substituting $v = 1$ in Eq. (ii), we get

$$8 \times 1 + 7u = 15$$

$\Rightarrow$ $$7u = 15 - 8$$

$\Rightarrow$ $$7u = 7$$

$\Rightarrow$ $$u = 1$$

Now, $u = 1$ and $v = 1 \Rightarrow \frac{1}{x} = 1$ and $\frac{1}{y} = 1 \Rightarrow x = 1$ and $y = 1$

(vi) We have, $6x + 3y = 6xy$ and $2x + 4y = 5xy$

Dividing by xy, we get

$$\frac{6}{y} + \frac{3}{x} = 6 \text{ and } \frac{2}{y} + \frac{4}{x} = 5$$

or $$3\left(\frac{1}{x}\right) + 6\left(\frac{1}{y}\right) = 6 \text{ and } 4\left(\frac{1}{x}\right) + 2\left(\frac{1}{y}\right) = 5$$

On putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get

$$3u + 6v = 6 \quad \text{...(i)}$$

and $$4u + 2v = 5 \quad \text{...(ii)}$$

On multiplying Eq. (i) by 4 and Eq. (ii) by 3, subtracting later from first, we get

$$4(3u + 6v) - 3(4u + 2v) = 4 \times 6 - 3 \times 5$$

$$\Rightarrow \quad 12u + 24v - 12u - 6v = 24 - 15$$

$$\Rightarrow \quad 18v = 9 \Rightarrow v = \frac{1}{2}$$

On substituting $v = \frac{1}{2}$ in Eq. (ii),

$$4u + 2 \times \frac{1}{2} = 5 \Rightarrow 4u = 4 \Rightarrow u = 1$$

Now, $u = 1$ and $v = \frac{1}{2} \Rightarrow \frac{1}{x} = 1$ and $\frac{1}{y} = \frac{1}{2} \Rightarrow x = 1$ and $y = 2$

(vii) We have, $\frac{10}{(x + y)} + \frac{2}{(x - y)} = 4$ and $\frac{15}{(x + y)} - \frac{5}{(x - y)} = -2$

On putting $\frac{1}{x + y} = u$ and $\frac{1}{x - y} = v$

$$\therefore \quad 10u + 2v = 4 \quad \text{...(i)}$$

and $$15u - 5v = -2 \quad \text{...(ii)}$$

On multiplying Eq. (i) by 5 and Eq. (ii) by 2 and then adding, we get

$$5(10u + 2v) + 2(15u - 5v) = 5 \times 4 + 2 \times (-2)$$

$$\Rightarrow \quad 50u + 10v + 30u - 10v = 20 - 4$$

$$\Rightarrow \quad 80u = 16 \Rightarrow u = \frac{1}{5}$$

From Eq. (i), substituting $u = \frac{1}{5}$, we get

$$10 \times \frac{1}{5} + 2v = 4 \Rightarrow 2v = 2 \Rightarrow v = 1$$

Now, $$u = \frac{1}{5} \text{ and } v = 1$$

$$\Rightarrow \quad \frac{1}{x + y} = \frac{1}{5} \text{ and } \frac{1}{x - y} = 1$$

$$\Rightarrow \quad x + y = 5 \quad \text{...(iii)}$$

and $$x - y = 1 \quad \text{...(iv)}$$

On adding Eqs. (iii) and (iv), we get

$$2x = 6 \Rightarrow x = 3$$

On putting $x = 3$ in Eq. (iii),

$$3 + y = 5 \Rightarrow y = 2$$

Hence, $x = 3$ and $y = 2$

(viii) We have, $\frac{1}{(3x + y)} + \frac{1}{(3x - y)} = \frac{3}{4}$ and $\frac{1}{2(3x + y)} - \frac{1}{2(3x - y)} = -\frac{1}{8}$

On putting $\frac{1}{(3x + y)} = u$ and $\frac{1}{(3x - y)} = v$, we get

$$u + v = \frac{3}{4} \quad \text{...(i)}$$

and $\frac{1}{2}u - \frac{1}{2}v = -\frac{1}{8} \Rightarrow u - v = -\frac{1}{4}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2u = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow \quad u = \frac{1}{4}$$

From Eq. (i), $\frac{1}{4} + v = \frac{3}{4}$

$$\Rightarrow \quad v = \frac{2}{4} = \frac{1}{2}$$

Now, $u = \frac{1}{4}$ and $v = \frac{1}{2}$

$$\Rightarrow \quad \frac{1}{3x + y} = \frac{1}{4} \text{ and } \frac{1}{3x - y} = \frac{1}{2}$$

$$\Rightarrow \quad 3x + y = 4 \quad \text{...(iii)}$$

and $3x - y = 2$ ...(iv)

On adding Eqs. (iii) and (iv), we get

$$6x = 6 \Rightarrow x = 1$$

Substituting $x = 1$ in Eq. (iii), we get

$$3 + y = 4 \Rightarrow y = 1$$

Hence, $x = 1$ and $y = 1$

Question 2. Formulate the following problems as a pair of equations and hence find their solutions.

(i) Ritu can row downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours, if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 min longer. Find the speed of the train and the bus separately.

Solution (i) Let the speed of Ritu in still water = x kmh^{-1}

and speed of current = y kmh^{-1}

Then, speed downstream = $(x + y)$ kmh^{-1}

Speed upstream = $(x - y)$ kmh^{-1}

$$\text{Time} = \frac{\text{Speed}}{\text{Distance}}$$

$$\frac{20}{x + y} = 2 \text{ and } \frac{4}{x - y} = 2 \qquad \text{(By condition)}$$

$\Rightarrow$ $x + y = 10$ ...(i)

and $x - y = 2$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2x = 12 \Rightarrow x = 6$$

On putting $x = 6$ in Eq. (i), $6 + y = 10 \Rightarrow y = 4$

$\therefore$ Speed of Ritu in still water = 6 kmh^{-1}

Speed of current = 4 kmh^{-1}

(ii) Let 1 woman finish the work in x days and let 1 man finish the work in y days.

Work of 1 woman in day $= \frac{1}{x}$

Work of 1 man in 1 day $= \frac{1}{y}$

Work of 2 women and 5 men in one day $= \frac{2}{x} + \frac{5}{y} = \frac{5x + 2y}{xy}$

The number of days required for complete work $= \frac{xy}{5x + 2y}$

We are given that, $\frac{xy}{5x + 2y} = 4$

Similarly, in second case

$$\frac{xy}{6x + 3y} = 3 \qquad \text{(Given)}$$

Then, $$\frac{5x + 2y}{xy} = \frac{1}{4} \text{ and } \frac{6x + 3y}{xy} = \frac{1}{3}$$

$\Rightarrow$ $$\frac{20}{y} + \frac{8}{x} = 1 \text{ and } \frac{18}{y} + \frac{9}{x} = 1$$

On putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore \quad 20v + 8u = 1 \quad \ldots(i)$

and $\quad 18v + 9u = 1 \quad \ldots(ii)$

On multiplying Eq. (i) by 9 and Eq. (ii) by 8, then subtracting later from first, we get

$$180v - 144v = 9 - 8$$

$$\Rightarrow \quad 36v = 1$$

$$\Rightarrow \quad v = \frac{1}{36}$$

On substituting $v = \frac{1}{36}$ in Eq. (ii), we get

$$18 \times \frac{1}{36} + 9u = 1$$

$$\Rightarrow \quad 9u = 1 - \frac{1}{2} \Rightarrow u = \frac{1}{18}$$

Now, $\quad u = \frac{1}{18}$ and $v = \frac{1}{36}$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{18} \text{ and } \frac{1}{y} = \frac{1}{36}$$

$$\Rightarrow \quad x = 18 \text{ and } y = 36$$

Hence, single woman finishes the work in 18 days and single man finishes the work in 36 days.

(iii) Let the speed of the train = x kmh^{-1} and the speed of the bus = y kmh^{-1}

In first case, Roohi travels 60 km by train and 240 km by bus in 4 h.

Thus, $\quad \frac{60}{x} + \frac{240}{y} = 4$

$$\Rightarrow \quad \frac{15}{x} + \frac{60}{y} = 1 \quad \text{(By given condition)}\ldots(i)$$

Similarly , in second case, she travels 100 km by train and 200 km by bus in $4\frac{1}{6}$ h.

$$\therefore \quad \frac{100}{x} + \frac{200}{y} = 4 + \frac{1}{6} \quad \left(\because 10 \text{ min} = \frac{1}{6} \text{ h}\right)$$

$$\Rightarrow \quad \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} = 1 \quad \left(\text{Multiply by } \frac{6}{25}\right)\ldots(ii)$$

On putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, we get

$$15u + 60v = 1 \quad \ldots(iii)$$

and $\quad 24u + 48v = 1 \quad \ldots(iv)$

On multiplying Eq. (i) by 24 and Eq. (iv) by 15, then subtracting from Eq. (iv)

$$24\,(15u + 60v) - 15(24u + 48v) = 24 - 15$$

$$\Rightarrow \quad 1440\,v - 720\,v = 9$$

$$\Rightarrow \quad 720v = 9$$

$$\Rightarrow \quad v = \frac{9}{720} = \frac{1}{80}$$

Substituting $v = \frac{1}{80}$ in Eq. (iii), we get

$$15u + 60 \times \frac{1}{80} = 1$$

$$\Rightarrow \quad 15u = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \quad u = \frac{1}{60}$$

Now, $u = \frac{1}{60}$ and $v = \frac{1}{80}$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{60} \text{ and } \frac{1}{y} = \frac{1}{80}$$

$$\Rightarrow \quad x = 60 \text{ and } y = 80$$

Hence, the speed of the train = 60 kmh^{-1}

and the speed of the bus = 80 kmh^{-1}

Exercise 3.7 (Optional)*

Question 1. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Solution Let the ages of Ani and Biju be x year and y year respectively.

According to the given condition,

$$x - y = \pm 3 \qquad \ldots(i)$$

Also, age of Ani's father Dharam = $2x$ year

and age of Biju's sister = $\frac{y}{2}$ year

According to the given condition,

$$2x - \frac{y}{2} = 30$$

$$\Rightarrow \quad 4x - y = 60 \qquad \ldots(ii)$$

Case I When $x - y = 3$ ...(iii)

On subtracting Eq. (iii) from Eq. (i), we get

$$3x = 57 \Rightarrow x = 19 \text{ yr}$$

On putting $x = 19$ in Eq. (iii), we get

$$19 - y = 3 \Rightarrow y = 16 \text{ yr}$$

$$\therefore \quad x = 19 \text{ yr and } y = 16 \text{ yr}$$

Case II When $x - y = -3$ ...(iv)

On subtracting Eq. (iv) from Eq. (ii), we get

$$\therefore \quad 3x = 60 + 3$$

$$\Rightarrow \quad 3x = 63$$

$$\Rightarrow \quad x = 21$$

On putting $x = 21$ in Eq. (iv), we get

$$21 - y = -3 \Rightarrow y = 24 \text{ yr}$$

Hence, age of Ani is 19 yr and age of Biju is 16 yr or age of Ani is 21 yr and age of Biju is 24 yr.

Question 2. One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital?" [From the Bijaganita of Bhaskara II]

[**Hint** $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$].

Solution Let the amount of their respective capital's be ₹x and ₹ y.

∴ According to the given condition,

$$x + 100 = 2(y - 100)$$

$$\Rightarrow \quad x - 2y = -300 \quad \ldots(i)$$

and

$$6(x - 10) = y + 10$$

$$\Rightarrow \quad 6x - y = 70 \quad \ldots(ii)$$

On multiplying Eq. (i) by 2 and subtracting from Eq. (i)

$$x - 12x = -300 - 140$$

$$\Rightarrow \quad -11x = -440 \Rightarrow x = ₹\ 40$$

On putting $x = 40$ in Eq. (i), we get

$$40 - 2y = -300$$

$$\Rightarrow \quad 2y = 340$$

$$\Rightarrow \quad y = ₹\ 170$$

Hence, the amount of their respective capitals are ₹ 40 and ₹ 170.

Question 3. A train covered a certain distance at a uniform speed. If the train would have been 10 kmh^{-1} faster, it would have taken 2 hours less than the scheduled time. And, if the train were slower by 10 kmh^{-1}; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution Let the actual speed of the train be x kmh^{-1} and actual time taken by y yr.

$$\because \quad \text{Distance} = \text{Speed} \times \text{Times} = (xy)\ \text{km}$$

∴ According to the given condition,

$$xy = (x + 10)(y - 2)$$

$$\Rightarrow \quad xy = xy - 2x + 10y - 20$$

$$\Rightarrow \quad 2x - 10y + 20 = 0$$

$$\Rightarrow \quad x - 5y + 10 = 0 \quad \text{(Divide by 2)} \ldots(i)$$

and

$$xy = (x - 10)(y + 3)$$

$$\Rightarrow \quad xy = xy + 3x - 10y - 30$$

$$\Rightarrow \quad 3x - 10y - 30 = 0 \quad \ldots(ii)$$

On multiplying Eq. (i) by 3 and subtracting Eq. (ii) from Eq. (i),

$$3 \times (x - 5y + 10) - (3x - 10y - 30) = 0$$

$$\Rightarrow \quad -5y = 60$$

$$\Rightarrow \quad y = 12$$

On putting $y = 12$ in Eq. (i), we get

$$x - 5 \times 12 + 10 = 0$$

$$\Rightarrow \quad x - 60 + 10 = 0$$

$$\Rightarrow \quad x = 50$$

Hence, the distance covered by the train $= 50 \times 12 = 600$ km

Question 4. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Solution Let the number of students in the class be x and the number of rows be y.

$\therefore$ The number of rows in each row $= \frac{x}{y}$

According to the given condition,

$$x = \left(\frac{x}{y} + 3\right)(y - 1)$$

$$\Rightarrow \quad x = x - \frac{x}{y} + 3y - 3$$

$$\Rightarrow \quad \frac{x}{y} - 3y + 3 = 0 \quad \ldots\text{(i)}$$

and

$$x = \left(\frac{x}{y} - 3\right)(y + 2)$$

$$\Rightarrow \quad x = x + \frac{2x}{y} - 3y - 6$$

$$\Rightarrow \quad \frac{2x}{y} - 3y - 6 = 0 \quad \ldots\text{(ii)}$$

Putting $\frac{x}{y} = u$ in Eqs. (i) and (ii),

$$u - 3y + 3 = 0 \quad \ldots\text{(iii)}$$

and

$$2u - 3y - 6 = 0 \quad \ldots\text{(iv)}$$

On subtracting Eq. (iii) from Eq. (iv),

$$u - 9 = 0 \Rightarrow u = 9$$

On substituting $u = 9$ in Eq. (iii), we get

$$9 - 3y + 3 = 0 \Rightarrow 3y = 12$$

$$\Rightarrow \quad y = 4 \quad \ldots\text{(v)}$$

Now,

$$u = 9 \Rightarrow \frac{x}{y} = 9$$

$$\frac{x}{4} = 9 \quad \text{[From Eq. (v), } y = 4]$$

$$\Rightarrow \quad x = 36$$

$\therefore$ The number of students in the class $= xy = 36 \times 4 = 144$

Question 5. In a ΔABC, $\angle C = 3$, $\angle B = 2(\angle A + \angle B)$. Find the three angles.

Solution We have,

$$\angle C = 3\ \angle B = 2\ (\angle A + \angle B) \quad \ldots\text{(i)}$$

$\because$ The sum of the three angles of a triangle is 180°.

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + \angle B + 2(\angle A + \angle B) = 180°$ [From Eq. (i)]

$\Rightarrow \quad 3\angle A + 3\angle B = 180°$

$\Rightarrow \quad \angle A + \angle B = 60°$ (Divide by 3) ...(ii)

Again $\quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + \angle B + 3\angle B = 180°$ [From Eq. (i)]

$\Rightarrow \quad \angle A + 4\angle B = 180°$ ...(iii)

On subtracting Eq. (ii) from Eq. (iii),

$$3\angle B = 120°$$

$\Rightarrow \quad \angle B = 40°$

On putting $\angle B = 40°$ in Eq. (ii), we get

$$\angle A + 40° = 60°$$

$\Rightarrow \quad \angle A = 20°$

On putting $\angle B = 40°$ in Eq. (i), we get

$$\angle C = 3 \times 40° = 120°$$

Hence, the angles are $\angle A = 20°$, $\angle B = 40°$ and $\angle C = 120°$.

Question 6. Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and the Y-axis.

Solution The given equations are

$5x - y = 5$ ...(i)

and $\quad 3x - y = 3$ ...(ii)

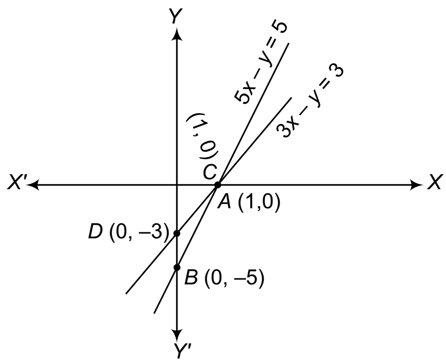
Table for $5x - y = 5$

x	1	0
y	0	– 5
Points	*A*	*B*

Table for $3x - y = 3$

x	1	0
y	0	–3
Points	*C*	*D*

Now, we plot the points *A* (1, 0) and *B* (0, – 5) on a graph paper and join these points to form a line *AB*. Also we plot the points *C* (1, 0) and *D* (0, – 3) on the same graph paper and join these points form the line *CD*.



$\therefore$ Required triangle is of $\Delta\, ABD$ and whose vertices are A (1, 0), B (0, – 5) and D (0, – 3).

Question 7. Solve the following pair of linear equations

(i) $px + qy = p - q$
$qx - py = p + q$

(ii) $ax + by = c$
$bx + ay = 1 + c$

(iii) $\dfrac{x}{a} - \dfrac{y}{b} = 0$
$ax + by = a^2 + b^2$

(iv) $(a - b)\,x + (a + b)\,y = a^2 - 2ab - b^2$
$(a + b)\,(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$
$-378x + 152y = -604$

Solution (i) Given pair of linear equations is

$$px + qy = p - q \qquad \ldots(i)$$

and $$qx - py = p + q \qquad \ldots(ii)$$

On multiplying Eq. (i) by p and Eq. (ii) by q and then adding, we get

$$p^2x + q^2x = (p^2 - pq) + (pq + q^2)$$

$\Rightarrow$ $$x\,(p^2 + q^2) = p^2 + q^2 \Rightarrow x = 1$$

On putting $x = 1$ in Eq. (i), we get

$$p + qy = p - q \Rightarrow qy = -q \Rightarrow y = -1$$

Hence, solution is $x = 1$ and $y = -1$.

(ii) Given pair of linear equations is

$$ax + by = c \qquad \ldots(i)$$

and $$bx + ay = 1 + c \qquad \ldots(ii)$$

On multiplying Eq. (i) by b and Eq. (ii) by a and subtracting Eq. (ii) from Eq. (i),

$$b^2y - a^2y = bc - (a + ac)$$

$\Rightarrow$ $$y\,(b^2 - a^2) = \frac{bc - a - ac}{b^2 - a^2}$$

$\Rightarrow$ $$y = \frac{a + ac - bc}{a^2 - b^2}$$

On putting the value of y in Eq. (i), we get

$$ax + b\left(\frac{a + ac - bc}{a^2 - b^2}\right) = c$$

$$\Rightarrow \quad ax = c - \left(\frac{ab + abc - b^2c}{a^2 - b^2}\right)$$

$$\Rightarrow \quad ax = \frac{a^2c - b^2c - ab - abc + b^2c}{a^2 - b^2}$$

$$\Rightarrow \quad ax = \frac{a\,(ac - b - bc)}{a^2 - b^2}$$

$$\Rightarrow \quad x = \frac{ac - b - bc}{a^2 - b^2}$$

Hence, solution is $x = \dfrac{ac - b - bc}{a^2 - b^2}$ and $y = \dfrac{a + ac - bc}{a^2 - b^2}$.

(iii) Given pair of linear equations is

$$\frac{x}{a} - \frac{y}{b} = 0$$

and $\quad ax + by = a^2 + b^2$

$\Rightarrow \quad bx - ay = 0 \quad \ldots\text{(i)}$

and $\quad ax + by = a^2 + b^2 \quad \ldots\text{(ii)}$

On multiplying Eq. (i) by b and Eq. (ii) by a and adding, we get

$$b^2x + a^2x = a\,(a^2 + b^2)$$

$$\Rightarrow \quad x = \frac{a\,(a^2 + b^2)}{(b^2 + a^2)} = a$$

On putting $x = a$ in Eq. (i), we get

$$ba - ay = 0$$

$$\Rightarrow \quad y = b$$

Hence, solution is $x = a$ and $y = b$.

(iv) Given pair of linear equations is

$$(a - b)\,x + (a + b)\,y = a^2 - 2ab - b^2 \quad \ldots\text{(i)}$$

and $\quad (a + b)\,(x + y) = a^2 + b^2$

$$\Rightarrow \quad x\,(a + b) + (a + b)\,y = a^2 + b^2 \quad \ldots\text{(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$(a - b)\,x - (a + b)\,x = -\,2ab - 2b^2$$

$$\Rightarrow \quad -\,2bx = -\,2b\,(a + b)$$

$$\Rightarrow \quad x = (a + b)$$

On putting $x = (a + b)$ in Eq. (i), we get

$$(a - b)\,(a + b) + (a + b)\,y = a^2 - 2ab - b^2$$

$$\Rightarrow \quad a^2 - b^2 + (a + b)\,y = a^2 - 2ab - b^2$$

$$\Rightarrow \quad y = \frac{-\,2ab}{a + b}$$

Hence, the solution is $x = a + b$ and $y = \dfrac{-2ab}{a + b}$.

(v) Given pair of linear equations is

$$152x - 378y = -74 \quad \ldots(i)$$

and $$-378x + 152y = -604 \quad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$-226x - 226y = -678$$

$$\Rightarrow \quad x + y = 3 \quad \text{(Divide by } -226) \ldots(iii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$530x - 530y = 530$$

$$\Rightarrow \quad x - y = 1 \quad \text{(Divide by 530)} \ldots(iv)$$

On adding Eqs. (iii) and (iv), we get

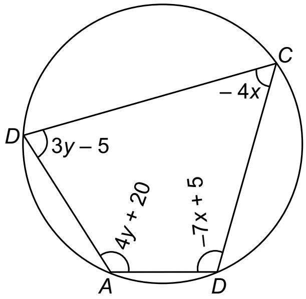
$$2x = 4 \Rightarrow x = 2$$

On putting $x = 2$ in Eq. (iii), we get

$$2 + y = 3 \Rightarrow y = 1$$

Hence, the solution is $x = 2$ and $y = 1$.

Question 8. *ABCD* is a cyclic quadrilateral. Find the angles of the cyclic quadrilateral.



Solution Since, in cyclic quadrilateral, the sum of two opposite angles is 180°.

$$\therefore \quad \angle B + \angle D = 180^\circ$$

and $$\angle A + \angle C = 180^\circ$$

$$\Rightarrow \quad 3y - 5 - 7x + 5 = 180^\circ$$

and $$4y + 20 - 4x = 180^\circ$$

$$\Rightarrow \quad 3y - 7x = 180^\circ$$

and $$4y - 4x = 160^\circ$$

$$\Rightarrow \quad 3y - 7x = 180^\circ \quad \ldots(i)$$

and $$y - x = 40^\circ \quad \text{(Divide by 4)} \ldots(ii)$$

On multiplying Eq. (ii) by 7 and subtracting from Eq. (i), we get

$$-4y = 180^\circ - 280^\circ$$

$$\Rightarrow \quad -4y = -100 \Rightarrow y = 25^\circ$$

On putting $y = 25^\circ$ in Eq. (ii), we get

$$25^\circ - x = 40^\circ \Rightarrow x = -15^\circ$$

Hence, solution is $x = -15^\circ$ and $y = 25^\circ$.