# 2 Polynomials

## Exercise 2.1

**Question 1.** Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

**Solution**

(i) $4x^2 - 3x + 7$ is an expression having only non-negative integral powers of $x$. So, it is a polynomial.

(ii) $y^2 + \sqrt{2}$ is an expression having only non-negative integral power of $y$. So, it is a polynomial.

(iii) $3\sqrt{t} + t\sqrt{2}$ is an expression in which one term namely $3\sqrt{t}$ has rational power of $t$. So, it is not a polynomial.

(iv) $y + \frac{2}{y}$ is an expression in which one term namely $\frac{2}{y} \Rightarrow$ *i.e.*, $2y^{-1}$ has negative power of $y$. So, it is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$ is an expression which has 3 variables.

**Question 2.** Write the coefficients of $x^2$ in each of the following

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

**Solution** (i) The coefficient of $x^2$ in $2 + x^2 + x$ is 1.

(ii) The coefficient of $x^2$ in $2 - x^2 + x^3$ is $-1$.

(iii) The coefficient of $x^2$ in $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$.

(iv) The coefficient of $x^2$ in $\sqrt{2}x - 1$ is 0.

**Question 3.** Give one example each of a binomial of degree 35, and of a monomial of degree 100.

**Solution** (i) $y^{35} + 2$ is a binomial of degree 35.

(ii) $y^{100}$ is a monomial of degree 100.

**Question 4.** Write the degree of each of the following polynomials.

(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$

(iii) $5t - \sqrt{7}$ (iv) 3

**Solution** (i) In a polynomial $5x^3 + 4x^2 + 7x$, the highest power of variable $x$ is 3, hence degree of polynomial is 3.

(ii) In a polynomial $4 - y^2$, the highest power of variable $y = 2$, hence degree of polynomial is 2.

(iii) In a polynomial $5t - \sqrt{7}$, the highest power of variable $t = 1$, hence the degree of polynomial is 1.

(iv) In a polynomial 3, the highest power of variable $y = 0$, hence the degree of polynomial is 0.

**Question 5.** Classify the following as linear, quadratic and cubic polynomials.

(i) $x^2 + x$ (ii) $x - x^3$ (iii) $y + y^2 + 4$ (iv) $1 + x$

(v) $3t$ (vi) $r^2$ (vii) $7x^3$

**Solution** (i) The degree of polynomial $x^2 + 2$ is 2, hence it is a quadratic polynomial.

(ii) The degree of polynomial $x - x^3$ is 3, hence it is a cubic polynomial.

(iii) The degree of polynomial $y + y^2 + 4$ is 2, hence it is a quadratic polynomial.

(iv) The degree of polynomial $1 + x$ is 1, hence it is a linear polynomial.

(v) The degree of polynomial $3t$ is 1, hence it a linear polynomial.

(vi) The degree of polynomial $r^2$ is 2, hence it is a quadratic polynomial.

(vii) The degree of polynomial $7x^3$ is 3, hence it is a cubic polynomial.

# 2 Polynomials

## Exercise 2.2

**Question 1.** Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

**Solution** Let $p(x) = 5x - 4x^2 + 3$

(i) The value of $p(x) = 5x - 4x^2 + 3$ at $x = 0$ is

$p(0) = 5 \times 0 - 4 \times 0^2 + 3$

$\Rightarrow$ $p(0) = 3$

(ii) The value of $p(x) = 5x - 4x^2 + 3$ at $x = -1$ is

$p(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3$

$\Rightarrow$ $p(-1) = -6$

(iii) The value of $p(x) = 5x - 4x^2 + 3$ at $x = 2$ is

$p(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3$

$\Rightarrow$ $p(2) = -3$

**Question 2.** Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials.

(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$

(iii) $p(x) = x^3$ (iv) $p(x) = (x-1)(x+1)$

**Solution** (i) $p(y) = y^2 - y + 1$

$\therefore$ $p(0) = 0^2 - 0 + 1$

$\Rightarrow$ $p(0) = 1$

$p(1) = 1^2 - 1 + 1$

$\Rightarrow$ $p(1) = 1$

and $p(2) = 2^2 - 2 + 1 = 4 - 2 + 1$

$\Rightarrow$ $p(2) = 3$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$\therefore$ $p(0) = 2 + 0 + 2 \times 0^2 - 0^3$

$\Rightarrow$ $p(0) = 2$

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3$

$\Rightarrow$ $p(1) = 3 + 2 - 1$

$\Rightarrow$ $p(1) = 4$

and $p(2) = 2 + 2 + 2 \times 2^2 - 2^3$

$= 4 + 8 - 8$

$\Rightarrow$ $p(2) = 4$

(iii) $p(x) = x^3$

$\Rightarrow$ $p(0) = 0^3 \Rightarrow p(0) = 0 \Rightarrow p(1) = 1^3$

$\Rightarrow$ $p(1) = 1$

and $p(2) = 2^3 \Rightarrow p(2) = 8$

(iv) $p(x) = (x-1)(x+1)$

$p(0) = (0-1)(0+1)$

$\Rightarrow$ $p(0) = -1$

$p(1) = (1-1)(1+1)$

$\Rightarrow$ $p(1) = 0$

and $p(2) = (2-1)(2+1)$

$\Rightarrow$ $p(2) = 3$

**Question 3.** Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi, \ x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, x = 1, -1$ (iv) $p(x) = (x+1)(x-2), x = -1, 2$

(v) $p(x) = x^2, x = 0$ (vi) $p(x) = lx + m, x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

**Solution** (i) $p(x) = 3x + 1$

If $x = -\frac{1}{3}$ is zero of $p(x)$, then $p\left(-\frac{1}{3}\right) = 0$

So, $p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right) + 1 = -1 + 1$

$\Rightarrow$ $p\left(-\frac{1}{3}\right) = 0$

Hence, $x = -\frac{1}{3}$ is zero of $p(x)$.

(ii) $p(x) = 5x - \pi$

If $x = \frac{4}{5}$ is zero of $p(x)$, then $p\left(\frac{4}{5}\right) = 0$

So, $p\left(\frac{4}{5}\right) = 5 \times \frac{4}{5} - \pi$

$\Rightarrow$ $p\left(\frac{4}{5}\right) = 4 - \pi$

Hence, $x = \frac{4}{5}$ is not zero of $p(x)$.

(iii) $p(x) = x^2 - 1$

If $x = -1$ and $1$ are zeroes of $p(x)$, then $p(-1), p(1) = 0$

So, $p(-1) = (-1)^2 - 1 = 1 - 1$

$\Rightarrow$ $p(-1) = 0$

and $p(1) = (1)^2 - 1 = 1 - 1 \Rightarrow p(1) = 0$

Hence, $x = -1$ and $1$ are zeroes of $p(x)$.

(iv) $p(x) = (x + 1)(x - 2)$

If $x = -1$ and $2$ are zeroes of $p(x)$, then $p(-1), p(2) = 0$

So, $p(-1) = (-1 + 1)(-1 - 2)$

$\Rightarrow$ $p(-1) = 0$

and $p(2) = (2 + 1)(2 - 2) = 3(0)$

$\Rightarrow$ $p(2) = 0$

Hence, $x = -1$ and $2$ are zeroes of $p(x)$.

(v) $p(x) = x^2$

If $x = 0$ is zero of $p(x)$, then $p(0) = 0$

So, $p(0) = 0^2 \Rightarrow p(0) = 0$

Hence, $x = 0$ is zero of $p(x)$.

(vi) $p(x) = lx + m$

If $x = -\frac{m}{l}$ is zero of $p(x)$, then $p\left(-\frac{m}{l}\right) = 0$

So, $p\left(-\frac{m}{l}\right) = l \times \left(\frac{-m}{l}\right) + m$

$\Rightarrow$ $p\left(-\frac{m}{l}\right) = -m + m \quad \Rightarrow \quad p\left(-\frac{m}{l}\right) = 0$

Hence, $x = -\frac{m}{l}$ is zero of $p(x)$.

(vii) $p(x) = 3x^2 - 1$

If $x = -\frac{1}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$ are zeroes of $p(x)$, then

$$p\left(-\frac{1}{\sqrt{3}}\right), p\left(\frac{2}{\sqrt{3}}\right) = 0$$

So, $p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1$

$\Rightarrow$ $p\left(-\frac{1}{\sqrt{3}}\right) = 1 - 1 = 0$

and $p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$

$\Rightarrow$ $p\left(\frac{2}{\sqrt{3}}\right) = 4 - 1 = 3$

Hence, $x = -\frac{1}{\sqrt{3}}$ is zero of $p(x)$ and $x = \frac{2}{\sqrt{3}}$ is not zero of $p(x)$.

(viii) $p(x) = 2x + 1$

If $x = \frac{1}{2}$ is zero of $p(x)$, then $p\left(\frac{1}{2}\right) = 0$

So, $$p\left(\frac{1}{2}\right) = 2 \times \frac{1}{2} + 1$$

$$\Rightarrow \quad p\left(\frac{1}{2}\right) = 1 + 1 = 2$$

Hence, $x = \frac{1}{2}$ is not zero of $p(x)$.

**Question 4.** Find the zero of the polynomial in each of the following cases

(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$ (iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$ (vi) $p(x) = ax, a \neq 0$

(vii) $p(x) = cx + d, c \neq 0$ where $c$ and $d$ are real numbers.

**Solution** (i) We have, $p(x) = x + 5$

Now, $p(x) = 0$

$\Rightarrow \quad x + 5 = 0$

$\Rightarrow \quad x = -5$

$\therefore -5$ is a zero of the polynomial $p(x)$.

(ii) We have, $p(x) = x - 5$

Now, $p(x) = 0$

$\Rightarrow \quad x - 5 = 0$

$\Rightarrow \quad x = 5$

$\therefore$ 5 is a zero of the polynomial $p(x)$.

(iii) We have, $p(x) = 2x + 5$

Now, $p(x) = 0$

$\Rightarrow \quad 2x + 5 = 0$

$\Rightarrow \quad x = -\frac{5}{2}$

$\therefore -\frac{5}{2}$ is a zero of the polynomial $p(x)$.

(iv) We have, $p(x) = 3x - 2$

Now, $p(x) = 0$

$\Rightarrow \quad 3x - 2 = 0$

$\Rightarrow \quad x = \frac{2}{3}$

$\therefore \frac{2}{3}$ is a zero of the polynomial $p(x)$.

(v) We have, $p(x) = 3x$

Now, $p(x) = 0$

$\Rightarrow$ $3x = 0$

$\Rightarrow$ $x = 0$

$\therefore$ 0 is a zero of the polynomial $p(x)$.

(vi) We have, $p(x) = ax, a \neq 0$

Now, $p(x) = 0 \Rightarrow ax = 0$

$\Rightarrow$ $x = 0$ $(\because a \neq 0)$

$\therefore$ 0 is a zero of the polynomial $p(x)$.

(vii) We have, $p(x) = cx + d, c \neq 0$

Now, $p(x) = 0$

$\Rightarrow$ $cx + d = 0$

$$x = -\frac{d}{c}$$

$\therefore -\frac{d}{c}$ is a zero of the polynomial $p(x)$.

## Exercise 2.3

**Question 1.** Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) $x$ (iv) $x + \pi$

(v) $5 + 2x$

**Solution** Let $p(x) = x^3 + 3x^2 + 3x + 1$

(i) The zero of $x + 1$ is $-1$. $(\because x + 1 = 0 \Rightarrow x = -1)$

So, $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$\Rightarrow p(-1) = -1 + 3 - 3 + 1$

$\Rightarrow p(-1) = 0$

$\therefore$ Required remainder = 0 (By Remainder theorem)

(ii) The zero of $x - \frac{1}{2}$ is $\frac{1}{2}$.

So, $p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$

$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1 + 6 + 12 + 8}{8}$

$\Rightarrow p\left(\frac{1}{2}\right) = \frac{27}{8}$

$\therefore$ Required remainder $= \frac{27}{8}$ (By Remainder theorem)

(iii) The zero of $x$ is 0.

So, $p(0) = 0^3 + 3(0)^2 + 3(0) + 1$

$= 0 + 0 + 0 + 1$

$\Rightarrow p(0) = 1$

$\therefore$ Required remainder = 1 (By Remainder theorem)

(iv) The zero of $x + \pi$ is $-\pi$.

So, $p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$\Rightarrow p(-\pi) = -\pi^3 + 3\pi^2 - 3\pi + 1$

$\therefore$ Required remainder $= -\pi^3 + 3\pi^2 - 3\pi + 1$ (By Remainder theorem)

(v) The zero of $5 + 2x$ is $x = -\frac{5}{2}$.

So, $p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$

$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

$= \frac{-125 + 150 - 60 + 8}{8} = -\frac{27}{8}$

$\therefore$ Required remainder $= -\frac{27}{8}$ (By Remainder theorem)

**Question 2.** Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

**Solution** The zero of $x - a$ is $a$. $(\because x - a = 0 \Rightarrow x = a)$

Let $p(x) = x^3 - ax^2 + 6x - a$

So, $p(a) = a^3 - a(a)^2 + 6a - a = a^3 - a^3 + 5a$

$\Rightarrow$ $p(a) = 5a$

$\therefore$ Required remainder $= 5a$ (By Remainder theorem)

**Question 3.** Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

**Solution** Let $f(x) = 3x^3 + 7x$

The zero of $7 + 3x$ is $x = -\frac{7}{3}$.

To check $(7 + 3x)$ is a factor of $3x^3 + 7x$, we show

$$f\left(-\frac{7}{3}\right) = 0 \quad \text{(By Factor theorem)}$$

Now,
$$f\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$$
$$= -3 \times \frac{343}{27} - \frac{49}{3}$$
$$= \frac{-1029 - 441}{27} = \frac{-1470}{27} = \frac{-490}{9}$$

Hence, $(7 + 3x)$ is not a factor of $3x^3 + 7x$.

## Exercise 2.4

**Question 1.** Determine which of the following polynomials has $(x + 1)$ a factor.

(i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - (2 + \sqrt{2})\, x + \sqrt{2}$

**Solution** The zero of $x + 1$ is $-1$.

(i) Let $p(x) = x^3 + x^2 + x + 1$

Then, $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$\Rightarrow p(-1) = 0$

So, by the Factor theorem $(x + 1)$ is a factor of $x^3 + x^2 + x + 1$.

(ii) Let $p(x) = x^4 + x^3 + x^2 + x + 1$

Then, $p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1$

$\Rightarrow p(-1) = 1$

So, by the Factor theorem $(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) Let $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Then, $p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1$

$\Rightarrow p(-1) = 1$

So, by the Factor theorem $(x + 1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) Let $p(x) = x^3 - x^2 - (2 + \sqrt{2})\, x + \sqrt{2}$

Then, $p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2}$

So, by the Factor theorem $(x + 1)$ is not a factor of

$x^3 - x^2 - (2 + \sqrt{2})\, x + \sqrt{2}$.

**Question 2.** Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases

(i) $p(x) = 2x^3 + x^2 - 2x - 1,\ g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1,\ g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6,\ g(x) = x - 3$

**Solution** (i) The zero of $g(x) = x + 1$ is $x = -1$.

Then, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$ $[\because p(x) = 2x^3 + x^2 - 2x - 1]$

$= -2 + 1 + 2 - 1$

$\Rightarrow p(-1) = 0$

Hence, $g(x)$ is a factor of $p(x)$.

(ii) The zero of $g(x) = x + 2$ is $-2$.

Then, $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$ $[\because p(x) = x^3 + 3x^2 + 3x + 1]$

$= -8 + 12 - 6 + 1$

$\Rightarrow p(-2) = -1$

Hence, $g(x)$ is not a factor of $p(x)$.

(iii) The zero of $g(x) = x - 3$ is 3.

Then, $p(3) = 3^3 - 4(3)^2 + 3 + 6$ $[\because p(x) = x^3 - 4x^2 + x + 6]$

$= 27 - 36 + 3 + 6$

$\Rightarrow p(3) = 0$

Hence, $g(x)$ is a factor of $p(x)$.

**Question 3.** Find the value of $k$, if $x - 1$ is a factor of $p(x)$ in each of the following cases

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ (iv) $p(x) = kx^2 - 3x + k$

**Solution** The zero of $x - 1$ is 1.

(i) $(x - 1)$ is a factor of $p(x)$, then $p(1) = 0$. (By Factor theorem)

$\Rightarrow 1^2 + 1 + k = 0$ $[\because p(x) = x^2 + x + k]$

$\Rightarrow 2 + k = 0$

$\Rightarrow k = -2$

(ii) $\because (x - 1)$ is a factor of $p(x)$, then $p(1) = 0$ (By Factor theorem)

$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$ $[\because p(x) = 2x^2 + kx + \sqrt{2}]$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$\Rightarrow k = -(2 + \sqrt{2})$

(iii) $\because (x - 1)$ is a factor of $p(x)$, then $p(1) = 0$ (By Factor theorem)

$\Rightarrow k(1)^2 - \sqrt{2} + 1 = 0$ $[\because p(x) = kx^2 - \sqrt{2}x + 1]$

$\Rightarrow k = (\sqrt{2} - 1)$

(iv) $\because (x - 1)$ is a factor of $p(x)$, then $p(1) = 0$ (By Factor theorem)

$\Rightarrow k(1)^2 - 3 + k = 0$ $[\because p(x) = kx^2 - 3x + k]$

$\Rightarrow 2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$

**Question 4.** Factorise

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

**Solution** (i) $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$ (Splitting middle term)

$= 4x(3x - 1) - 1(3x - 1)$

$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$ (Splitting middle term)

$= 2x(x + 3) + 1(x + 3) = (x + 3)(2x + 1)$

(iii) $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$ (Splitting middle term)

$= 3x(2x + 3) - 2(2x + 3) = (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$ (Splitting middle term)

$= x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$

**Question 5.** Factorise

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

**Solution** (i) Let $p(x) = x^3 - 2x^2 - x + 2$, constant term of $p(x)$ is 2.

$\therefore$ Factors of 2 are $\pm 1$ and $\pm 2$.

Now, $p(1) = 1^3 - 2(1)^2 - 1 + 2$

$= 1 - 2 - 1 + 2$

$p(1) = 0$

By trial, we find that $p(1) = 0$, so $(x - 1)$ is a factor of $p(x)$.

So, $x^3 - 2x^2 - x + 2$

$= x^3 - x^2 - x^2 + x - 2x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x^2 - 2x + x - 2)$

$= (x - 1)[x(x - 2) + 1(x - 2)]$

$= (x - 1)(x - 2)(x + 1)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

By trial, we find that $p(5) = (5)^3 - 3(5)^2 - 9(5) - 5$

$= 125 - 75 - 45 - 5 = 0$

So, $(x - 5)$ is a factor of $p(x)$.

So, $x^3 - 3x^2 - 9x - 5$

$= x^3 - 5x^2 + 2x^2 - 10x + x - 5$

$= x^2(x - 5) + 2x(x - 5) + 1(x - 5)$

$= (x - 5)(x^2 + 2x + 1)$

$= (x - 5)(x^2 + x + x + 1)$

$= (x - 5)[x(x + 1) + 1(x + 1)]$

$= (x - 5)(x + 1)(x + 1)$

$= (x - 5)(x + 1)^2$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

By trial, we find that $p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20 = -33 + 33 = 0$

So $(x + 1)$ is a factor of $p(x)$.

So, $x^3 + 13x^2 + 32x + 20$

$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$

$= x^2(x + 1) + 12x(x + 1) + 20(x + 1)$

$= (x + 1)(x^2 + 12x + 20)$

$= (x + 1)(x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 10)(x + 2)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

By trial we find that $p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1 = 0$

So $(y - 1)$ is a factor of $p(y)$.

So, $2y^3 + y^2 - 2y - 1$

$= 2y^3 - 2y^2 + 3y^2 - 3y + y - 1$

$= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1)$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)]$

$= (y - 1)(y + 1)(2y + 1)$

## Exercise 2.5

**Question 1.** Use suitable identities to find the following products

(i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$ (iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

**Solution** (i) $(x+4)(x+10)$

Using identity (iv), *i.e.*, $(x+a)(x+b)=x^2+(a+b)x+ab$.

We have, $(x+4)(x+10)=x^2+(4+10)x+(4\times 10)$ $(\because a=4, b=10)$

$=x^2+14x+40$

(ii) $(x+8)(x-10)$

Using identity (iv), *i.e.*, $(x+a)(x+b)=x^2+(a+b)x+ab$

We have, $(x+8)(x-10)=x^2+[8+(-10)]x+(8)(-10)$ $(\because a=8, b=-10)$

$=x^2-2x-80$

(iii) $(3x+4)(3x-5)$

Using identity Eq. (iv), *i.e.*,

$(x+a)(x+b)=x^2+(a+b)x+ab$

We have, $(3x+4)(3x-5)=(3x)^2+(4-5)x+(4)(-5)$ $(\because a=4, b=-5)$

$=9x^2-x-20$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

Using identity (iii), *i.e.*, $(x+y)(x-y)=x^2-y^2$

We have, $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=(y^2)^2-\left(\frac{3}{2}\right)^2=y^4-\frac{9}{4}$

(v) $(3-2x)(3+2x)$

Using identity (iii), *i.e.*, $(x+y)(x-y)=x^2-y^2$

We have, $(3-2x)(3+2x)=3^2-(2x)^2=9-4x^2$

**Question 2.** Evaluate the following products without multiplying directly

(i) $103\times 107$ (ii) $95\times 96$

(iii) $104\times 96$

**Solution** (i) $103\times 107=(100+3)(100+7)$

$=100\times 100+(3+7)(100)+(3\times 7)$ [Using identity (iv)]

$=10000+1000+21=11021$

(ii) $95\times 96=(100-5)(100-4)$

$=100\times 100+[(-5)+(-4)]\,100+(-5\times -4)$ [Using identity (iv)]

$=10000-900+20=9120$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - 4^2$ [Using identity (iii)]

$= 10000 - 16 = 9984$

**Question 3.** Factorise the following using appropriate identities

(i) $9x^2 + 6xy + y^2$ (ii) $4y^2 - 4y + 1$

(iii) $x^2 - \dfrac{y^2}{100}$

**Solution** (i) $9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2$

$= (3x + y)^2$ [Using identity (i)]

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2$ [Using identity (ii)]

(iii) $x^2 - \dfrac{y^2}{100} = x^2 - \left(\dfrac{y}{10}\right)^2 = \left(x + \dfrac{y}{10}\right)\left(x - \dfrac{y}{10}\right)$ [Using identity (iii)]

**Question 4.** Expand each of the following, using suitable identity

(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ (iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$ (vi) $\left(\dfrac{1}{4}a - \dfrac{1}{2}b + 1\right)^2$

**Solution** (i) $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y)$

$+ 2(2y)(4z) + 2(4z)(x)$

[Using identity (v)]

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y)$

$+ 2(-y)(z) + 2(z)(2x)$ [Using identity (v)]

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(iii) $(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y)$

$+ 2(3y)(2z) + 2(2z)(-2x)$ [Using identity (v)]

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b)$

$+ 2(-7b)(-c) + 2(-c)(3a)$ [Using identity (v)]

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y)$

$+ 2(5y)(-3z) + 2(-3z)(-2x)$ [Using identity (v)]

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + 1^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)$

$+ 2\left(-\frac{1}{2}b\right)(1) + 2\,(1)\left(\frac{1}{4}a\right)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{ab}{4} - b + \frac{1}{2}a$ [Using identity (v)]

**Question 5.** Factorise

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

**Solution** (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2\,(2x)\,(3y) + 2\,(3y)\,(-4z) + 2\,(-4z)\,(2x)$

$= (2x + 3y - 4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)\,(y) + 2\,(y)\,(2\sqrt{2}z) + 2\,(2\sqrt{2}z)\,(-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

**Question 6.** Write the following cubes in expanded form

(i) $(2x + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x + 1\right)^3$

(iv) $\left(x - \frac{2}{3}y\right)^3$

**Solution** (i) $(2x + 1)^3 = (2x)^3 + 1^3 + 3\,(2x)\,(1)\,(2x + 1)$

[Using identity $(x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$]

$= 8x^3 + 1 + 6x\,(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x = 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3\,(2a)(3b)(2a - 3b)$

[Using identity $(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$]

$= 8a^3 - 27b^3 - 18ab\,(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $\left(\frac{3}{2}x + 1\right)^3 = \left(\frac{3}{2}x\right)^3 + 1^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x + 1\right)$

[Using identity $(x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$]

$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x + 1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x = \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $\left(x-\frac{2}{3}y\right)^3 = x^3-\left(\frac{2}{3}y\right)^3-3x\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

[Using identity $(x-y)^3 = x^3-y^3-3xy\ (x-y)$]

$= x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right) = x^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$

$= x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3$

**Question 7.** Evaluate the following using suitable identities

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

**Solution** (i) $(99)^3 = (100-1)^3 = 100^3-(1)^3-3\times100\times1(100-1)$

[Using identity $(x-y)^3 = x^3-y^3-3xy\ (x-y)$]

$= 1000000-1-300(100-1)$

$= 1000000-1-30000+300$

$= 970299$

(ii) $(102)^3 = (100+2)^3 = 100^3+2^3+3\times100\times2\,(100+2)$

[Using identity $(x+y)^3 = x^3+y^3+3xy\ (x+y)$]

$= 1000000+8+600(100+2)$

$= 1000000+8+60000+1200 = 1061208$

(iii) $(998)^3 = (1000-2)^3 = 1000^3-2^3-3\times1000\times2\,(1000-2)$

[Using identity $(x-y)^3 = x^3-y^3-3xy\ (x-y)$]

$= 1000000000-8-6000(1000-2)$

$= 1000000000-8-6000000+12000$

$= 994011992$

**Question 8.** Factorise each of the following

(i) $8a^3+b^3+12a^2b+6ab^2$ (ii) $8a^3-b^3-12a^2b+6ab^2$

(iii) $27-125a^3-135a+225a^2$ (iv) $64a^3-27b^3-144a^2b+108ab^2$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

**Solution** (i) $8a^3+b^3+12a^2b+6ab^2 = (2a)^3+b^3+3\times2a\times b\,(2a+b)$

$= (2a+b)^3$ [Using identity (vi)]

(ii) $8a^3-b^3-12a^2b+6ab^2 = (2a)^3+(-b)^3+3\times2a\times(-b)[(2a)+(-b)]$

$= (2a)^3-(b)^3-3\times2a\times b\,(2a-b)$ [Using identity (vii)]

$= (2a-b)^3$

(iii) $27-125a^3-135a+225a^2 = (3)^3+(-5a)^3+3\times3\times(-5a)[(3)+(-5a)]$

$= (3)^3-(5a)^3-3\times3\times5a\,(3-5a)$ [Using identity (vii)]

$= (3-5a)^3$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$$= (4a)^3 + (-3b)^3 + 3 \times 4a \times (-3b)[4a + (-3b)]$$

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b\,(4a - 3b)$$

$$= (4a - 3b)^3 \qquad \text{[Using identity (vii)]}$$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \frac{1}{6^3} - 3 \times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$

$$= \left(3p - \frac{1}{6}\right)^3 \qquad \text{[Using identity (vii)]}$$

**Question 9.** Verify

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

**Solution** (i) We know that,

$$(x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$$

$$\Rightarrow \quad x^3 + y^3 = (x + y)^3 - 3xy\,(x + y)$$

$$= (x + y)[(x + y)^2 - 3xy]$$

$$= (x + y)[x^2 + y^2 + 2xy - 3xy]$$

$$= (x + y)[x^2 + y^2 - xy]$$

$$= \text{RHS}$$

Hence proved.

(ii) We know that,

$$(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$$

$$\Rightarrow \quad x^3 - y^3 = (x - y)^3 + 3xy\,(x - y)$$

$$= (x - y)[(x - y)^2 + 3xy]$$

$$= (x - y)[x^2 + y^2 + 2xy + 3xy]$$

$$= (x - y)[x^2 + y^2 + xy]$$

$$= \text{RHS}$$

Hence proved.

**Question 10.** Factorise each of the following

(i) $27y^3 + 125z^3$ (ii) $64m^3 - 343n^3$

[**Hint** See question 9]

**Solution** (i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$$= (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$$

$$= (3y + 5z)(9y^2 - 15yz + 25z^2)$$

(ii) $64\,m^3 - 343\,n^3 = (4m)^3 - (7n)^3$

$$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$$

$$= (4m - 7n)[16m^2 + 28mn + 49n^2]$$

**Question 11.** Factorise $27x^3 + y^3 + z^3 - 9xyz$.

**Solution** $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3 \times 3x \times y \times z$

$$= (3x + y + z)[(3x)^2 + y^2 + z^2 - 3xy - yz - z(3x)] \text{[Using identity (viii)]}$$
$$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$$

**Question 12.** Verify that

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

**Solution** We have,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$$
$$= \frac{1}{2}(x + y + z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$$
$$= \frac{1}{2}(x + y + z)[x^2 + x^2 + y^2 + y^2 + z^2 + z^2 - 2xy - 2yz - 2zx]$$
$$= \frac{1}{2}(x + y + z)[x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx]$$
$$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

**Question 13.** If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3\,xyz$.

**Solution** We know that,

$$x^3 + y^3 + z^3 - 3\,xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

[Using identity (viii)]

$$= 0\,(x^2 + y^2 + z^2 - xy - yz - zx)$$

($\because x + y + z = 0$ given)

$$= 0$$

$\Rightarrow \quad x^3 + y^3 + z^3 = 3\,xyz$

Hence proved.

**Question 14.** Without actually calculating the cubes, find the value of each of the following

(i) $(-12)^3 + (7)^3 + (5)^3$ (ii) $(28)^3 + (-15)^3 + (-13)^3$

**Solution** We know that, $x^3 + y^3 + z^3 - 3xyz$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

If $x + y + z = 0$, then $x^3 + y^3 + z^3 - 3xyz = 0$

or $\quad x^3 + y^3 + z^3 = 3xyz$

(i) We have to find the value of $(-12)^3 + (7)^3 + (5)^3$.

Here, $-12+7+5=0$

So, $(-12)^3 + (7)^3 + (5)^3 = 3 \times (-12)(7)(5)$

$= -1260$

(ii) We have to find the value of $(28)^3 + (-15)^3 + (-13)^3$.

Here, $28 + (-15) + (-13) = 28 - 15 - 13 = 0$

So, $(28)^3 + (-15)^3 + (-13)^3 = 3 \times (28)(-15)(-13)$

$= 16380$

**Question 15.** Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given

(i) Area $25a^2 - 35a + 12$

(ii) Area $35y^2 + 13y - 12$

**Solution** (i) We have, area of rectangle

$= 25a^2 - 35a + 12$

$= 25a^2 - 20a - 15a + 12$

$= 5a(5a-4) - 3(5a-4) = (5a-4)(5a-3)$

Possible expression for length $= 5a - 3$

and breadth $= 5a - 4$

(ii) We have, Area of rectangle $= 35y^2 + 13y - 12 = 35y^2 - 15y + 28y - 12$

$= 5y(7y-3) + 4(7y-3) = (7y-3)(5y+4)$

Possible expression on for length $= 7y - 3$

and breadth $= 5y + 4$

**Question 16.** What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume $3x^2 - 12x$

(ii) Volume $12ky^2 + 8ky - 20k$

**Solution** (i) We have, volume of cuboid $= 3x^2 - 12x = 3x(x-4)$

One possible expressions for the dimensions of the cuboid is 3, $x$ and $x-4$.

(ii) We have, volume of cuboid $= 12ky^2 + 8ky - 20k$

$= 12ky^2 + 20ky - 12ky - 20k$

$= 4ky\,(3y+5) - 4k(3y+5)$

$= (3y+5)(4ky - 4k)$

$= (3y+5)\,4k(y-1)$

One possible expressions for the dimensions of the cuboid is $4k$, $3y+5$ and $y-1$.