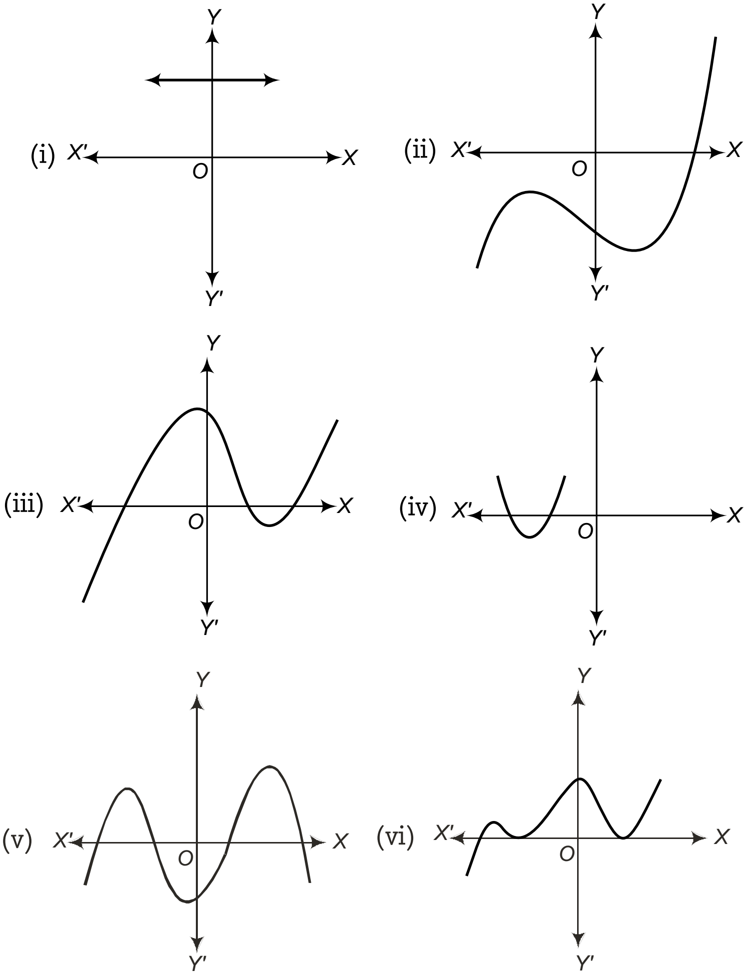
2 Polynomials

Exercise 2.1

Question 1. The graphs of $y = p(x)$ are given in figures below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



Solution

(i) There are no zeroes as the graph does not intersect the *X*-axis.

(ii) The number of zeroes is one as the graph intersects the *X*-axis at one point only.

(iii) The number of zeroes is three as the graph intersects the *X*-axis at three points.

(iv) The number of zeroes is two as the graph intersects the *X*-axis at two points.

(v) The number of zeroes is four as the graph intersects the *X*-axis at four points.

(vi) The number of zeroes is three as the graph intersects the *X*-axis at three points.

2 Polynomials

Exercise 2.2

Question 1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$ (iv) $4u^2 + 8u$

(v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Solution

(i) We have, $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

So, the value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, *i.e.*, when $x = 4$ or $x = -2$. So, the zeroes of $x^2 - 2x = -8$ are 4 and -2.

$\therefore$ Sum of the zeroes $= 4 - 2 = 2 = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\dfrac{(-2)}{1} = 2$

And product of the zeroes $= 4(-2) = -8 = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = -\dfrac{8}{1} = -8$

(ii) We have, $4s^2 - 4s + 1 = 4s^2 - 2s - 2s + 1 = 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1) = (2s - 1)^2$

So, the value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$ or $s = \dfrac{1}{2}$.

$\therefore$ Zeroes of the polynomial are $\dfrac{1}{2}$ and $\dfrac{1}{2}$.

$\therefore$ Sum of the zeroes $= \dfrac{1}{2} + \dfrac{1}{2} = 1 = -\dfrac{\text{Coefficient of } s}{\text{Coefficient of } s^2} = -\left(\dfrac{-4}{4}\right) = 1$

And product of the zeroes $= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{1}{4} = \dfrac{\text{Constant term}}{\text{Coefficient of } s^2} = \dfrac{1}{4}$

(iii) We have, $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (3x + 1)(2x - 3)$

The value of $6x^2 - 3 - 7x$ is 0, when the value of $(3x + 1)(2x - 3)$ is 0, *i.e.*, when $3x + 1 = 0$, $2x - 3 = 0$, *i.e.*, when $x = -\dfrac{1}{3}$ or $x = \dfrac{3}{2}$

$\therefore$ The zeroes of $6x^2 - 3 - 7x$ are $-\dfrac{1}{3}$ and $\dfrac{3}{2}$.

Therefore, sum of the zeroes $= -\dfrac{1}{3} + \dfrac{3}{2} = \dfrac{7}{6}$

$= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\dfrac{(-7)}{6} = \dfrac{7}{6}$

And product of zeroes $= \left(-\dfrac{1}{3}\right)\left(\dfrac{3}{2}\right) = -\dfrac{1}{2} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = -\dfrac{3}{6} = -\dfrac{1}{2}$

(iv) We have, $4u^2 + 8u = 4u(u+2)$

The value of $4u^2 + 8u$ is 0 when the value of $4u(u+2) = 0$, *i.e.,* when $u = 0$ or $u + 2 = 0$, *i.e.,* when $u = 0$ or $u = -2$

$\therefore$ The zeroes of $4u^2 + 8u$ are 0 and –2.

Therefore, sum of the zeroes

$$= 0 + (-2) = -2 = -\frac{\text{Coefficient of } u}{\text{Coefficient of } u^2} = -\frac{8}{4} = -2$$

And product of zeroes $= (0)(-2) = 0 = \dfrac{\text{Constant term}}{\text{Coefficient of } u^2} = \dfrac{0}{4} = 0$

(v) We have, $t^2 - 15 = (t - \sqrt{15})(t + \sqrt{15})$

The value of $t^2 - 15$ is 0 when the value of $(t - \sqrt{15})(t + \sqrt{15})$ is 0, *i.e.,* when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, *i.e.,* when $t = \sqrt{15}$ or $t = -\sqrt{15}$.

$\therefore$ The zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Therefore, sum of the zeroes $= \sqrt{15} + (-\sqrt{15}) = 0$

$$= -\frac{\text{Coefficient of } t}{\text{Coefficient of } t^2} = -\frac{0}{1} = 0$$

And product of the zeroes $= (\sqrt{15}) \times (-\sqrt{15}) = -15$

$$= \frac{\text{Constant term}}{\text{Coefficient of } t^2} = -\frac{15}{1} = -15$$

(vi) We have, $3x^2 - x - 4 = 3x^2 + 3x - 4x - 4$

$$= 3x(x+1) - 4(x+1) = (x+1)(3x-4)$$

The value of $3x^2 - x - 4$ is 0 when the value of $(x+1)(3x-4)$ is 0, *i.e.,* when $x + 1 = 0$ or $3x - 4 = 0$, *i.e.,* when $x = -1$ or $x = \dfrac{4}{3}$

$\therefore$ The zeroes of $3x^2 - x - 4$ are -1 and $\dfrac{4}{3}$.

Therefore, sum of the zeroes $= -1 + \dfrac{4}{3} = \dfrac{-3+4}{3} = \dfrac{1}{3}$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{(-1)}{3} = \frac{1}{3}$$

And product of the zeroes $= (-1)\left(\dfrac{4}{3}\right) = -\dfrac{4}{3}\,\dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = -\dfrac{4}{3}$

Question 2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) $0, \sqrt{5}$ (iv) 1, 1

(v) $-\frac{1}{4}, \frac{1}{4}$ (vi) 4, 1

Solution Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

(i) Here, $\alpha + \beta = \dfrac{1}{4}$ and $\alpha\beta = -1$

Thus, the polynomial formed

$$= x^2 - (\text{Sum of the zeroes})\,x + \text{Product of the zeroes}$$

$$= x^2 - \left(\frac{1}{4}\right)x - 1 = x^2 - \frac{x}{4} - 1$$

The other polynomials are $k\left(x^2 - \frac{x}{4} - 1\right)$.

If $k = 4$, then the polynomial is $4x^2 - x - 4$.

(ii) Here, $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

Thus, the polynomial formed

$$= x^2 - (\text{Sum of the zeroes})\, x + \text{Product of the zeroes}$$

$$= x^2 - (\sqrt{2})\, x + \frac{1}{3} \text{ or } x^2 - \sqrt{2}x + \frac{1}{3}$$

Other polynomial are $k\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$.

If $k = 3$, then the polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) Here, $\alpha + \beta = 0$ and $\alpha\beta = \sqrt{5}$

Thus, the polynomial formed

$$= x^2 - (\text{Sum of the zeroes})\, x + \text{Product of the zeroes}$$

$$= x^2 - (0)x + \sqrt{5}$$

$$= x^2 + \sqrt{5}$$

(iv) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then,

$$\alpha + \beta = 1 = -\frac{(-1)}{1} = -\frac{b}{a}$$

and $$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -1$ and $c = 1$.

$\therefore$ One quadratic polynomial which satisfy the given conditions is $x^2 - x + 1$.

(v) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then,

$$\alpha + \beta = -\frac{1}{4} = -\frac{1}{4} = -\frac{b}{a}$$

and $$\alpha\beta = \frac{1}{4} = \frac{c}{a}$$

If $a = 4$, then $b = 1$ and $c = 1$.

$\therefore$ One quadratic polynomial which satisfy the given conditions is $4x^2 + x + 1$.

(vi) Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β. Then,

$$\alpha + \beta = 4 = -\frac{(-4)}{1} = -\frac{b}{a}$$

and $$\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -4$ and $c = 1$.

$\therefore$ One quadratic polynomial which satisfy the given conditions is $x^2 - 4x + 1$.

2 Polynomials

Exercise 2.3

Question 1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Solution (i) Here, dividend and divisor are both in standard forms. So, we have

$$
\begin{array}{r|l}
 & x - 3 \\
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\qquad + \\ \hline
 & -3x^2 + 7x - 3 \\
 & -3x^2 \qquad + 6 \\
 & + \qquad\qquad - \\ \hline
 & 7x - 9 \\ \hline
\end{array}
$$

$\therefore$ The quotient is $x - 3$ and the remainder is $7x - 9$.

(ii) Here, the dividend is already in the standard form and the divisor is not in the standard form. It can be written as $x^2 - x + 1$.

We have,

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
x^2 - x + 1 & x^4 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\ \hline
 & x^3 - 4x^2 + 4x + 5 \\
 & x^3 - x^2 + x \\
 & - \quad + \quad - \\ \hline
 & -3x^2 + 3x + 5 \\
 & -3x^2 + 3x - 3 \\
 & + \quad - \quad + \\ \hline
 & 8 \\ \hline
\end{array}
$$

$\therefore$ The quotient is $x^2 + x - 3$ and the remainder is 8.

(iii) We have, divisor $(-x^2 + 2)$ and dividend $x^4 - 5x + 6$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 - 5x + 6 \\
 & x^4 - 2x^2 \\
 & \underline{-\quad +} \\
 & 2x^2 - 5x + 6 \\
 & 2x^2 \qquad - 4 \\
 & \underline{- \qquad\quad +} \\
 & -5x + 10
\end{array}
$$

$\therefore$ The quotient is $-x^2 - 2$ and the remainder is $-5x + 10$.

Question 2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Solution (i) Let us divide $2t^4 + 3t^3 - 2t^2 - 9t - 12$ by $t^2 - 3$, we get

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 \qquad\quad - 6t^2 \\
 & \underline{- \qquad\qquad +} \\
 & 3t^3 + 4t^2 - 9t - 12 \\
 & 3t^3 \qquad\quad - 9t \\
 & \underline{- \qquad\qquad +} \\
 & 4t^2 \qquad - 12 \\
 & 4t^2 \qquad - 12 \\
 & \underline{- \qquad\quad +} \\
 & 0
\end{array}
$$

Since, the remainder is 0. Therefore, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) Let us divide $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $x^2 + 3x + 1$, we get

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & \underline{- \qquad - \qquad -} \\
 & -4x^3 - 10x^2 + 2x + 2 \\
 & -4x^3 - 12x^2 - 4x \\
 & \underline{+ \qquad + \qquad +} \\
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
 & \underline{- \qquad - \qquad -} \\
 & 0
\end{array}
$$

Since, the remainder is 0. Therefore, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) Let us divide $x^5 - 4x^3 + x^2 + 3x + 1$ by $x^3 - 3x + 1$, we get

$$\begin{array}{r|l} & x^2 - 1 \\ x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\ & x^5 - 3x^3 + x^2 \\ & - \quad + \quad - \\ \hline & -x^3 + 3x + 1 \\ & -x^3 + 3x - 1 \\ & + \quad - \quad + \\ \hline & 2 \\ \hline \end{array}$$

Here, remainder is 2 ($\neq 0$). Therefore, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Question 3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solution Since, two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore$ $$x = \sqrt{\frac{5}{3}} \text{ and } x = -\sqrt{\frac{5}{3}}$$

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ or $3x^2 - 5$ is a factor of the given polynomial.

Now, we apply the division algorithm to the given polynomial and $3x^2 - 5$.

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ 3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & 3x^4 \quad - 5x^2 \\ & - \quad + \\ \hline & 6x^3 + 3x^2 - 10x - 5 \\ & 6x^3 \quad - 10x \\ & - \quad + \\ \hline & 3x^2 - 5 \\ & 3x^2 - 5 \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

First term of quotient is $\frac{3x^4}{3x^2} = x^2$

Second term of quotient is $\frac{6x^3}{3x^2} = 2x$

Third term of quotient is $\frac{3x^2}{3x^2} = 1$.

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1) + 0$

$= (3x^2 - 5)(x + 1)^2$

Quotient $= x^2 + 2x + 1 = (x + 1)^2$

$\therefore$ Zeroes of $(x + 1)^2$ are -1 and -1.

Hence, all its zeroes are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.

Question 4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Solution Given, $p(x) = x^3 - 3x^2 + x + 2, q(x) = x - 2$ and $r(x) = -2x + 4$. By division algorithm, we know that Divident = Divisor × Quotient + Remainder
$p(x) = q(x) \times g(x) + r(x)$

Therefore, $x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = (x - 2) \times g(x)$

$$\Rightarrow \qquad g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$$

On dividing $x^3 - 3x^2 + 3x - 2$ by $x - 2$, we get

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

First term of $g(x) = \frac{x^3}{x} = x^2$

Second term of $g(x) = \frac{-x^2}{x} = -x$

Third term of $g(x) = \frac{x}{x} = 1$

Hence, $\qquad g(x) = x^2 - x + 1$

Question 5. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$ (ii) deg $q(x)$ = deg $r(x)$

(iii) deg $q(x) = 0$

Solution Let $\quad q(x) = 3x^2 + 2x + 6$, degree of $q(x) = 2$

$p(x) = 12x^2 + 8x + 24$, degree of $p(x) = 2$

Here, $\quad$ deg $p(x)$ = deg $q(x)$

(i) Using division algorithm,

We have, $\qquad p(x) = q(x) \times g(x) + r(x)$

On dividing $12x^2 + 8x + 24$ by $3x^2 + 2x + 6$, we get

$$\begin{array}{r|l} & 4 \\ \hline 3x^2 + 2x + 6 & 12x^2 + 8x + 24 \\ & 12x^2 + 8x + 24 \\ & - \quad - \quad - \\ \hline & 0 \\ \hline \end{array}$$

Since, the remainder is zero, therefore $3x^2 + 2x + 6$ is a factor of $12x^2 + 8x + 24$.

$\therefore \quad g(x) = 4$ and $r(x) = 0$

(ii) $p(x) = x^5 + 2x^4 + 3x^3 + 5x^2 + 2$

$q(x) = x^2 + x + 1$, degree of $q(x) = 2$

$g(x) = x^3 + x^2 + x + 1$

$r(x) = 2x^2 - 2x + 1$, degree of $r(x) = 2$

Here, deg $q(x)$ = deg $r(x)$

On dividing $x^5 + 2x^4 + 3x^3 + 5x^2 + 2$ by $x^2 + x + 1$, we get

$$\begin{array}{r|l} & x^3 + x^2 + x + 1 \\ \hline x^2 + x + 1 & x^5 + 2x^4 + 3x^3 + 5x^2 + 2 \\ & x^5 + x^4 + x^3 \\ & - \quad - \quad - \\ \hline & x^4 + 2x^3 + 5x^2 + 2 \\ & x^4 + x^3 + x^2 \\ & - \quad - \quad - \\ \hline & x^3 + 4x^2 + 2 \\ & x^3 + x^2 + x \\ & - \quad - \quad - \\ \hline & 3x^2 - x + 2 \\ & x^2 + x + 1 \\ & - \quad - \quad - \\ \hline & 2x^2 - 2x + 1 \\ \hline \end{array}$$

Here, $g(x) = x^3 + x^2 + x + 1$ and $r(x) = 2x^2 - 2x + 1$

(iii) Let $p(x) = 2x^4 + 8x^3 + 6x^2 + 4x + 12$, $r(x) = 2$, degree of $r(x) = 0$

$g(x) = x^4 + 4x^3 + 3x^2 + 2x + 1 \Rightarrow q(x) = 10$

Here, deg $r(x) = 0$

On dividing $2x^4 + 8x^3 + 6x^2 + 4x + 12$ by 2, we get

$$\begin{array}{r|l} & x^4 + 4x^3 + 3x^2 + 2x + 1 \\ \hline 2 & 2x^4 + 8x^3 + 6x^2 + 4x + 12 \\ & 2x^4 \\ & - \\ \hline & 8x^3 + 6x^2 + 4x + 12 \\ & 8x^3 \\ & - \\ \hline & 6x^2 + 4x + 12 \\ & 6x^2 \\ & - \\ \hline & 4x + 12 \\ & 4x \\ & - \\ \hline & 12 \\ & 2 \\ & - \\ \hline & 10 \\ \hline \end{array}$$

Here, $g(x) = x^4 + 4x^3 + 3x^2 + 2x + 1$ and $q(x) = 10$

2 Polynomials

Exercise 2.4 (Optional)*

Question 1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case

(i) $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2;\ \ 2, 1, 1$

Solution (i) Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get $a = 2, b = 1, c = -5$ and $d = 2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= \frac{1 + 1 - 10 + 8}{4} = \frac{0}{4} = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$= 2 + 1 - 5 + 2 = 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= 2(-8) + 4 + 10 + 2 = -16 + 16 = 0$$

$\therefore \frac{1}{2}$, 1 and -2 are the zeroes of $2x^3 + x^2 - 5x + 2$.

So, $\alpha = \frac{1}{2}, \beta = 1$

and $\gamma = -2$

Therefore, $\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = \frac{1 + 2 - 4}{2} = -\frac{1}{2} = -\frac{b}{a}$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \left(\frac{1}{2}\right)(1) + (1)(-2) + (-2)\left(\frac{1}{2}\right)$$

$$= \frac{1}{2} - 2 - 1 = \frac{1 - 4 - 2}{2} = -\frac{5}{2} = \frac{c}{a}$$

and $\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = -1 = -\frac{2}{2} = -\frac{d}{a}$

(ii) Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

$a = 1, b = -4, c = 5$ and $d = -2$

$$p(2) = (2)^3 - 4(2)^2 + 5(2) - 2 = 8 - 16 + 10 - 2 = 0$$

$$p(1) = (1)^3 - 4(1)^2 + 5(1) - 2 = 1 - 4 + 5 - 2 = 0$$

$\therefore$ 2, 1 and 1 are the zeroes of $x^3 - 4x^2 + 5x - 2$.

So, $\alpha = 2,\ \beta = 1$ and $\gamma = 1$

Therefore, $\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -\frac{(-4)}{1} = -\frac{b}{a}$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (2)(1) + (1)(1) + (1)(2)$$
$$= 2 + 1 + 2 = 5 = \frac{5}{1} = \frac{c}{a}$$

and $\alpha\beta\gamma = (2)(1)(1) = 2 = -\frac{(-2)}{1} = -\frac{d}{a}$

Question 2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time and the product of its zeroes as 2, – 7 and – 14 respectively.

Solution Let the cubic polynomial be $ax^3 + bx^2 + cx + d$ and its zeroes be α, β and γ.

Then, $\alpha + \beta + \gamma = 2 = -\frac{(-2)}{1} = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = -7 = -\frac{7}{1} = \frac{c}{a}$

and $\alpha\beta\gamma = -14 = -\frac{14}{1} = -\frac{d}{a}$

$\therefore a = 1$, then $b = -2$, $c = -7$ and $d = 14$

So, one cubic polynomial which satisfy the given conditions will be $x^3 - 2x^2 - 7x + 14$.

Question 3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a, $a + b$, find a and b.

Solution Since, $(a - b)$, a, $(a + b)$ are the zeroes of the polynomial $x^3 - 3x^2 + x + 1$.

Therefore, sum of the zeroes $= (a - b) + a + (a + b) = \frac{-(-3)}{1} = 3$

So, $3a = 3 \Rightarrow a = 1$

$\therefore$ Sum of the products of ts zeroes taken two at a time

$$= a(a - b) + a(a + b) + (a + b)(a - b) = \frac{1}{1} = 1$$

$\Rightarrow$ $a^2 - ab + a^2 + ab + a^2 - b^2 = 1$

$\Rightarrow$ $3a^2 - b^2 = 1$

So, $3(1)^2 - b^2 = 1$

$\Rightarrow$ $3 - b^2 = 1$

$\Rightarrow$ $b^2 = 2$ or $b = \pm\sqrt{2}$

Hence, $a = 1$ and $b = \pm\sqrt{2}$

Question 4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Solution We have, $2 \pm \sqrt{3}$ are two zeroes of the polynomial

$$p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35.$$

Let $x = 2 \pm \sqrt{3}$

So, $x - 2 = \pm\sqrt{3}$

On squaring, we get $x^2 - 4x + 4 = 3$, *i.e.*, $x^2 - 4x + 1 = 0$

Let us divide $p(x)$ by $x^2 - 4x + 1$ to obtain other zeroes.

$$\begin{array}{r|l} & x^2 - 2x - 35 \\ \hline x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\ & \underset{-}{x^4} \underset{+}{- 4x^3} \underset{-}{+ x^2} \\ \hline & - 2x^3 - 27x^2 + 138x - 35 \\ & \underset{+}{- 2x^3} \underset{-}{+ 8x^2} \underset{+}{- 2x} \\ \hline & - 35x^2 + 140x - 35 \\ & \underset{+}{- 35x^2} \underset{-}{+ 140x} \underset{+}{- 35} \\ \hline & 0 \\ \hline \end{array}$$

$\therefore$ $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

$= (x^2 - 4x + 1)(x^2 - 7x + 5x - 35)$

$= (x^2 - 4x + 1)[x(x - 7) + 5(x - 7)]$

$= (x^2 - 4x + 1)(x + 5)(x - 7)$

So, $(x + 5)$ and $(x - 7)$ are other factors of $p(x)$.

So, -5 and 7 are other zeroes of the given polynomial.

Question 5. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Solution Let us divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$.

$$\begin{array}{r|l} & x^2 - 4x + (8 - k) \\ \hline x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\ & \underset{-}{x^4} \underset{+}{- 2x^3} \underset{-}{+ kx^2} \\ \hline & - 4x^3 + (16 - k)x^2 - 25x + 10 \\ & \underset{+}{- 4x^3} \underset{-}{+ 8x^2} \quad \underset{+}{- 4kx} \\ \hline & (8 - k)x^2 + (4k - 25)x + 10 \\ & \underset{-}{(8 - k)x^2} \underset{+}{- 2(8 - k)x} \underset{-}{+ (8 - k)k} \\ \hline & (2k - 9)x - (8 - k)k + 10 \\ \hline \end{array}$$

$\therefore$ Remainder $= (2k - 9)x - (8 - k)k + 10$

But the remainder is given as $x + a$.

On comparing their coefficients, we have

$2k - 9 = 1 \Rightarrow 2k = 10 \Rightarrow k = 5$

and $-(8 - k)k + 10 = a$

So, $a = -(8 - 5)5 + 10$

$= -3 \times 5 + 10 = -15 + 10 = -5$

Hence, $k = 5$ and $a = -5$