7 Quadratic Equations

Exercise 7.1

Question 1. Check whether the following are quadratic equations

(i) $(x+1)^2 = 2(x-3)$

(ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$

(iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$

(vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Solution (i) Given equation is

$$(x+1)^2 = 2(x-3)$$

$$\Rightarrow \quad x^2 + 2x + 1 = 2x - 6 \quad [\because (a+b)^2 = a^2 + 2ab + b^2]$$

$$\Rightarrow \quad x^2 + 1 + 6 = 0$$

$$\Rightarrow \quad x^2 + 7 = 0$$

which is of the form $ax^2 + bx + c = 0$, where $b = 0$

Thus, $(x+1)^2 = 2(x-3)$ is a quadratic equation.

(ii) Given equation is

$$x^2 - 2x = (-2)(3-x)$$

$$\Rightarrow \quad x^2 - 2x = -6 + 2x$$

$$\Rightarrow \quad x^2 - 2x - 2x + 6 = 0 \quad \Rightarrow x^2 - 4x + 6 = 0$$

which is of the form $ax^2 + bx + c = 0$.

Thus, $x^2 - 2x = (-2)(3-x)$ is a quadratic equation.

(iii) Given equation is

$$(x-2)(x+1) = (x-1)(x+3)$$

$$\Rightarrow \quad x^2 - 2x + x - 2 = x^2 - x + 3x - 3$$

$$\Rightarrow \quad x^2 - 2x + x - 2 - x^2 + x - 3x + 3 = 0$$

$$\Rightarrow \quad -3x + 1 = 0$$

which is not of the form $ax^2 + bx + c = 0$, where $a \neq 0$ and b and c are any real numbers.

$\therefore$ It is not a quadratic equation.

(iv) Given equation is

$$(x-3)(2x+1) = x(x+5)$$
$$\Rightarrow \quad 2x^2 - 6x + x - 3 = x^2 + 5x$$
$$\Rightarrow \quad 2x^2 - 6x + x - 3 - x^2 - 5x = 0$$
$$\Rightarrow \quad x^2 - 10x - 3 = 0$$

which is of the form $ax^2 + bx + c = 0$, where $a \neq 0$

Thus, $x^2 - 10x - 3 = 0$ is a quadratic equation.

(v) Given equation is

$$(2x-1)(x-3) = (x+5)(x-1)$$
$$\Rightarrow \quad 2x^2 - x - 6x + 3 = x^2 + 5x - x - 5$$
$$\Rightarrow \quad 2x^2 - x - 6x + 3 - x^2 - 5x + x + 5 = 0$$
$$\Rightarrow \quad x^2 - 11x + 8 = 0$$

which is of the form $ax^2 + bx + c = 0$, where $a \neq 0$

Thus, $x^2 - 11x + 8 = 0$ is a quadratic equation.

(vi) Given equation is

$$x^2 + 3x + 1 = (x-2)^2$$
$$\Rightarrow \quad x^2 + 3x + 1 = x^2 + 4 - 4x \quad [\because (a-b)^2 = a^2 - 2ab + b^2]$$
$$\Rightarrow \quad x^2 + 3x + 1 - x^2 - 4 + 4x = 0$$
$$\Rightarrow \quad 7x - 3 = 0$$

which is not of the form $ax^2 + bx + c = 0$, where $a \neq 0$ and b, c are any real numbers.

$\therefore$ It is not a quadratic equation.

(vii) Given equation is

$$(x+2)^3 = 2x(x^2-1)$$
$$\Rightarrow x^3 + 8 + 3x^2(2) + 3x(2)^2 = 2x^3 - 2x$$
$$[\because (a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b]$$
$$\Rightarrow \quad x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$$
$$\Rightarrow \quad x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$$
$$\Rightarrow \quad -x^3 + 6x^2 + 14x + 8 = 0$$

which is not of the form $ax^2 + bx + c = 0$, $a \neq 0$

Thus, $(x+2)^3 = 2x(x^2-1)$ is not a quadratic equation.

(viii) Given equation is

$$x^3 - 4x^2 - x + 1 = (x-2)^3$$
$$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 - 3(x^2)2 + 3x(2)^2 - (2)^3$$
$$[\because (a-b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b]$$
$$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$$
$$\Rightarrow \quad x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$$
$$\Rightarrow \quad 2x^2 - 13x + 9 = 0$$

which is of the form $ax^2 + bx + c = 0$, $a \neq 0$

Thus, $x^3 - 4x^2 - x + 1 = (x-2)^3$ is a quadratic equation.

Question 2. Represent the following situations in the form of quadratic equations

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metre) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 kmh^{-1} less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Solution (i) Let the breadth of the plot = x m

Then, the length of the plot = $(2x + 1)$ m (By condition)

Area of the rectangular plot = 528 m^2 (Given)

Area of the rectangular plot = Length × Breadth

$\therefore \quad (2x + 1)\,x = 528$

$\Rightarrow \quad 2x^2 + x = 528$

$\Rightarrow \quad 2x^2 + x - 528 = 0$

Since, it is of the form $ax^2 + bx + c = 0$, $a \neq 0$. Thus, it represents the required quadratic equation.

(ii) Let the two consecutive positive integers be x and $x + 1$.

Then, according to the question,

Product of two consecutive integers = 306

$\Rightarrow \quad x\,(x + 1) = 306$

$\Rightarrow \quad x^2 + x = 306$

or $\quad x^2 + x - 306 = 0$

Since, it is of the form $ax^2 + bx + c = 0$, $a \neq 0$. Thus, it represents the required quadratic equation.

(iii) Let the age of Rohan = x yr

Then, his mother's age = $(x + 26)$ yr (By conditions)

After three years,

Rohan's age = $(x + 3)$ yr

Rohan's mother's age = $[(x + 26) + 3]$ yr = $(x + 29)$yr

According to the question,

$(x + 3)\,(x + 29) = 360$

$\Rightarrow \quad x^2 + 29x + 3x + 87 = 360$

$\Rightarrow \quad x^2 + 32x - 273 = 0$

Since, it is of the form $ax^2 + bx + c = 0$, $a \neq 0$. Thus, it represents the required quadratic equation.

(iv) Let the speed of the train = x km/h

Distance travelled by the train = 480 km

Therefore, time taken for travelling 480 km = $\frac{480}{x}$ h $\left(\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$

If the speed had been 8 km/h less *i.e.,* $(x - 8)$ km/h, then

Time taken for travelling 480 km = $\frac{480}{x-8}$ h

According to the question,

$$\Rightarrow \quad \frac{480}{x-8} - \frac{480}{x} = 3$$

$$\Rightarrow \quad \frac{480\,x - 480\,(x-8)}{x\,(x-8)} = 3$$

$$\Rightarrow \quad 480x - 480x + 3840 = 3x\,(x-8)$$

$$3840 = 3x^2 - 24x$$

$$\Rightarrow \quad 3x^2 - 24x - 3840 = 0$$

$$\Rightarrow \quad 3\,(x^2 - 8x - 1280) = 0$$

$$\Rightarrow \quad x^2 - 8x - 1280 = 0$$

Since, it is of the form $ax^2 + bx + c = 0$, $a \neq 0$. Thus, it represents the required quadratic equation.

7 Quadratic Equations

Exercise 7.2

Question 1. Find the roots of the following quadratic equations by factorisation

(i) $x^3 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$ (iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Solution (i) Given equation is

$$x^2 - 3x - 10 = 0$$

$$\Rightarrow \quad x^2 - (5x - 2x) - 10 = 0$$

$$\Rightarrow \quad x^2 - 5x + 2x - 10 = 0$$

$$\Rightarrow \quad x(x - 5) + 2(x - 5) = 0$$

$$\Rightarrow \quad (x - 5)(x + 2) = 0$$

Now, $x - 5 = 0 \Rightarrow x = 5$

and $x + 2 = 0 \Rightarrow x = -2$

Hence, the roots of the equation $x^2 - 3x - 10 = 0$ are -2 and 5.

(ii) Given equation is

$$2x^2 + x - 6 = 0$$

$$\Rightarrow \quad 2x^2 + (4x - 3x) - 6 = 0$$

$$\Rightarrow \quad 2x^2 + 4x - 3x - 6 = 0$$

$$\Rightarrow \quad 2x(x + 2) - 3(x + 2) = 0$$

$$\Rightarrow \quad (x + 2)(2x - 3) = 0$$

Now, $x + 2 = 0 \Rightarrow x = -2$

and $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Hence, the roots of the equation $2x^2 + x - 6 = 0$ are -2 and $\frac{3}{2}$.

(iii) Given equation is

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\sqrt{2}x^2 + (5x + 2x) + 5\sqrt{2} = 0$$

$$\Rightarrow \quad \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

$$\Rightarrow \quad x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$

$$\Rightarrow \quad (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

Now, $\sqrt{2}x + 5 = 0 \Rightarrow x = -\frac{5}{\sqrt{2}}$

and $x + \sqrt{2} = 0 \Rightarrow x = -\sqrt{2}$

Hence, the roots of the equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ are $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$.

(iv) Given equation is $2x^2 - x + \frac{1}{8} = 0$

Multiplying on both sides by 8,

$$\Rightarrow \quad 16x^2 - 8x + 1 = 0$$
$$16x^2 - (4x + 4x) + 1 = 0$$
$$\Rightarrow \quad 16x^2 - 4x - 4x + 1 = 0$$
$$\Rightarrow \quad 4x(4x - 1) - 1(4x - 1) = 0$$
$$\Rightarrow \quad (4x - 1)(4x - 1) = 0$$

Now, $4x - 1 = 0 \Rightarrow x = \frac{1}{4}$

and $4x - 1 = 0 \Rightarrow x = \frac{1}{4}$

Hence, the roots of the equation $2x^2 - x + \frac{1}{8} = 0$ are $\frac{1}{4}$ and $\frac{1}{4}$.

(v) Given equation is

$$100x^2 - 20x + 1 = 0$$
$$100x^2 - (10x + 10x) + 1 = 0$$
$$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$$
$$\Rightarrow \quad 10x(10x - 1) - 1(10x - 1) = 0$$
$$\Rightarrow \quad (10x - 1)(10x - 1) = 0$$

Now, $10x - 1 = 0 \Rightarrow x = \frac{1}{10}$

and $10x - 1 = 0 \Rightarrow x = \frac{1}{10}$

Hence, the roots of the equation $100x^2 - 20x + 1 = 0$ are $\frac{1}{10}$ and $\frac{1}{10}$.

Question 2. Solve the following quadratic equations

(i) $x^2 - 45x + 324 = 0$

(ii) $x^2 - 55x + 750 = 0$

Solution (i) Given equation is

$$x^2 - 45x + 324 = 0$$
$$x^2 - (36x + 9x) + 324 = 0$$
$$\Rightarrow \quad x^2 - 36x - 9x + 324 = 0$$
$$\Rightarrow \quad x(x - 36) - 9(x - 36) = 0$$
$$\Rightarrow \quad (x - 36)(x - 9) = 0$$

Now, $x - 36 = 0 \Rightarrow x = 36$

and $x - 9 = 0 \Rightarrow x = 9$

Hence, the roots of the equation $x^2 - 45x + 324 = 0$ are 9 and 36.

(ii) Given equation is

$$x^2 - 55x + 750 = 0$$

$$\Rightarrow \quad x^2 - (30x + 25x) + 750 = 0$$

$$\Rightarrow \quad x^2 - 30x - 25x + 750 = 0$$

$$\Rightarrow \quad x(x - 30) - 25(x - 30) = 0$$

$$\Rightarrow \quad (x - 30)(x - 25) = 0$$

Now, $x - 30 = 0 \Rightarrow x = 30$

and $x - 25 = 0 \Rightarrow x = 25$

Hence, the roots of the equation $x^2 - 55x + 750 = 0$ are 25 and 30.

Question 3. Find two numbers whose sum is 27 and product is 182.

Solution Let the numbers be x and y.

According to question,

Sum of the numbers $= x + y = 27$ ...(i)

Product of the numbers $= xy = 182$ ...(ii)

From Eq. (ii), put $y = \frac{182}{x}$ in Eq. (i), we get

$$x + \frac{182}{x} = 27$$

On multiplying both sides by x

$$\Rightarrow \quad x^2 + 182 = 27x$$

$$\Rightarrow \quad x^2 - 27x + 182 = 0$$

$$x^2 - (14x + 13x) + 182 = 0$$

$$\Rightarrow \quad x^2 - 14x - 13x + 182 = 0$$

$$\Rightarrow \quad x(x - 14) - 13(x - 14) = 0$$

$$\Rightarrow \quad (x - 14)(x - 13) = 0$$

Now, $x - 14 = 0 \Rightarrow x = 14$

and $x - 13 = 0 \Rightarrow x = 13$

$\therefore$ $x = 13, 14$

Putting $x = 13$ in Eq. (i), we get $y = 14$

Putting $x = 14$ in Eq. (i), we get $y = 13$

Therefore, in both cases, the numbers are 13 and 14.

Question 4. Find two consecutive positive integers, sum of whose squares is 365.

Solution Let the two consecutive positive integers be x and $x + 1$.

According to the question,

Sum of the squares of two consecutive positive integers = 365

$$x^2 + (x + 1)^2 = 365$$

$$\Rightarrow \quad x^2 + x^2 + 2x + 1 = 365 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$

$$\Rightarrow \quad 2x^2 + 2x - 364 = 0$$

$$\Rightarrow \quad x^2 + x - 182 = 0$$

$$\Rightarrow \quad x^2 + (14x - 13x) - 182 = 0$$

$$\Rightarrow \quad x^2 + 14x - 13x - 182 = 0$$

$$\Rightarrow \quad x(x + 14) - 13(x + 14) = 0$$

$\Rightarrow \quad (x + 14)(x - 13) = 0$

Now, $\quad x + 14 = 0 \Rightarrow x = -14$

and $\quad x - 13 = 0 \Rightarrow x = 13$

$\therefore \quad x = -14, 13$

Since, x is a positive integer which cannot be negative, therefore $x = 13$.

Hence, the two consecutive positive integers are 13 and 14.

Question 5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

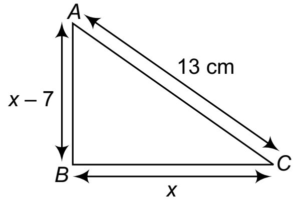
Solution Given that, hypotenuse of right triangle = 13 cm

Let the base of the right triangle = x

According to the question,

Altitude of the triangle = 7 cm

Less than its base = $(x - 7)$ cm



By Pythagoras theorem, we have

In ΔABC, $\quad AC^2 = BC^2 + AB^2$

$\Rightarrow \quad (13)^2 = x^2 + (x - 7)^2$

$\Rightarrow \quad 169 = x^2 + x^2 - 14x + 49 \quad [\because (a - b)^2 = a^2 - 2ab + b^2]$

$\Rightarrow \quad 2x^2 - 14x - 120 = 0$

$\Rightarrow \quad x^2 - 7x - 60 = 0$

$x^2 - (12x - 5x) - 60 = 0$

$\Rightarrow \quad x^2 - 12x + 5x - 60 = 0$

$\Rightarrow \quad x(x - 12) + 5(x - 12) = 0$

$\Rightarrow \quad (x - 12)(x + 5) = 0$

Now, $\quad x - 12 = 0 \Rightarrow x = 12$

and $\quad x + 5 = 0 \Rightarrow x = -5$

Since, altitude of the triangle cannot be negative, hence $x \neq -5$

Hence, base of the triangle = 12 cm and altitude of the triangle = 12 – 7 = 5 cm

Question 6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in ₹) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

Solution Let the number of pottery articles produced on a particular day = x

$\therefore$ Cost of production of each article = $2x + 3$ (By given condition)

So, the total cost of production = Number of pottery articles × Cost of production of each article

$= x(2x + 3)$

According to the question,

$x(2x + 3) = 90$

$\Rightarrow \quad 2x^2 + 3x = 90$

$$\Rightarrow \quad 2x^2 + 3x - 90 = 0$$
$$\Rightarrow \quad 2x^2 + (15x - 12x) - 90 = 0$$
$$\Rightarrow \quad 2x^2 + 15x - 12x - 90 = 0$$
$$\Rightarrow \quad x(2x + 15) - 6(2x + 15) = 0$$
$$\Rightarrow \quad (2x + 15)(x - 6) = 0$$

Now, $2x + 15 = 0 \Rightarrow x = -\frac{15}{2}$

and $x - 6 = 0 \Rightarrow x = 6$

$\therefore \quad x = -\frac{15}{2}$ and 6

But x cannot be negative. *i.e.,* number of pottery articles should be positive.

$\therefore \quad x = 6$

Hence, the number of articles produced is 6 and the cost of each article is

$(2 \times 6 + 3) = ₹\ 15$

7 Quadratic Equations

Exercise 7.3

Question 1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Solution (i) Given equation is

$$2x^2 - 7x + 3 = 0$$

On dividing both sides by 2

$$\Rightarrow \quad x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Adding $\left(\frac{1}{2} \text{ coefficient of } x\right)^2$ *i.e.*, $\left(\frac{1}{2} \times \frac{7}{2}\right)^2 = \frac{49}{16}$ on both sides, we get

$$x^2 - \frac{7}{2}x + \frac{49}{16} = -\frac{3}{2} + \frac{49}{16}$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \qquad [\because a^2 - 2ab + b^2 = (a-b)^2]$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \left(\frac{5}{4}\right)^2$$

$$\therefore \quad x - \frac{7}{4} = \pm \frac{5}{4}$$

$$\Rightarrow \quad x = \frac{7}{4} \pm \frac{5}{4} = \frac{7 \pm 5}{4}$$

$$= \frac{12}{4}, \frac{2}{4} = 3, \frac{1}{2}$$

Hence, the roots of equation $2x^2 - 7x + 3 = 0$ are $\frac{1}{2}$ and 3.

(ii) Given equation is

$$2x^2 + x - 4 = 0$$

On dividing both sides by 2

$$\Rightarrow \quad x^2 + \frac{1}{2}x - 2 = 0$$

$$\Rightarrow \quad x^2 + \frac{1}{2}x = 2$$

Adding $\left(\frac{1}{2} \text{ coefficient of } x\right)^2$ *i.e.*, $\left(\frac{1}{2} \times \frac{1}{2}\right)^2 = \frac{1}{16}$ on both sides, we get

$$x^2 + \frac{1}{2}x + \frac{1}{16} = 2 + \frac{1}{16}$$

$$\Rightarrow \quad x^2 + \frac{1}{2}x + \frac{1}{16} = \frac{33}{16}$$

$$\Rightarrow \quad \left(x+\frac{1}{4}\right)^2 = \left(\frac{\sqrt{33}}{4}\right)^2 \qquad [\because a^2+2ab+b^2=(a+b)^2]$$

$$\Rightarrow \quad x+\frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow \quad x = -\frac{1}{4} \pm \frac{\sqrt{33}}{4} = \frac{-1\pm\sqrt{33}}{4}$$

$$\Rightarrow \quad x = \frac{\sqrt{33}-1}{4}, \frac{-\sqrt{33}+1}{4}$$

Hence, the roots of equation $2x^2+x-4=0$ are $\frac{-\sqrt{33}+1}{4}$ and $\frac{\sqrt{33}-1}{4}$.

(iii) Given equation is

$$4x^2+4\sqrt{3}x+3=0$$

On dividing both sides by 4

$$\Rightarrow \quad x^2+\sqrt{3}x+\frac{3}{4}=0 \Rightarrow x^2+\sqrt{3}x=-\frac{3}{4}$$

Adding $\left(\frac{1}{2}\text{ coefficient of } x\right)^2$ *i.e.*, $\left(\frac{1}{2}\sqrt{3}\right)^2 = \frac{3}{4}$ on both sides, we get

$$x^2+\sqrt{3}x+\frac{3}{4} = -\frac{3}{4}+\frac{3}{4}$$

$$\Rightarrow \quad \left(x+\frac{\sqrt{3}}{2}\right)^2 = 0 \qquad [\because a^2+2ab+b^2=(a+b)^2]$$

$$\therefore \quad x+\frac{\sqrt{3}}{2}=0 \quad \text{and} \quad x+\frac{\sqrt{3}}{2}=0$$

$$\Rightarrow \quad x=-\frac{\sqrt{3}}{2} \quad \text{and} \quad x=-\frac{\sqrt{3}}{2}$$

Hence, the roots of equation $4x^2+4\sqrt{3}x+3=0$ are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

(iv) Given equation is $2x^2+x+4=0$

On dividing both sides by 2

$$\Rightarrow \quad x^2+\frac{1}{2}x+2=0 \Rightarrow x^2+\frac{1}{2}x=-2$$

Adding $\left(\frac{1}{2}\text{ coefficient of } x\right)^2$ *i.e.*, $\left(\frac{1}{2}\times\frac{1}{2}\right)^2 = \frac{1}{16}$ on both sides, we get

$$x^2+\frac{1}{2}x+\frac{1}{16} = -2+\frac{1}{16}$$

$\therefore \left(x+\frac{1}{4}\right)^2 = -\left(\frac{\sqrt{31}}{4}\right)^2$ which is not possible as the square of a real number cannot be negative. Therefore, the real roots of the equation $2x^2+x+4=0$ do not exist.

Question 2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Solution (i) Given equation is $2x^2 - 7x + 3 = 0$

On comparing with $ax^2 + bx + c = 0$

$a = 2, b = -7$ and $c = 3$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3$

$= 49 - 24 = 25 > 0$

So, the given equation has real and different roots.

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-7) \pm \sqrt{25}}{2 \times 2}$$

$$= \frac{7 \pm 5}{4} = \frac{12}{4}, \frac{2}{4} = 3, \frac{1}{2}$$

(ii) Given equation is $2x^2 + x - 4 = 0$

On comparing with $ax^2 + bx + c = 0$

$a = 2, b = 1$ and $c = -4$

$\therefore$ Discriminant, $D = b^2 - 4ac = (1)^2 - 4\,(2)\,(-4)$

$= 1 + 32 = 33 > 0$

So, the given equation has real and different roots.

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{33}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{33}}{4}$$

$$= \frac{\sqrt{33} - 1}{4}, \frac{-\sqrt{33} - 1}{4}$$

(iii) Given equation is

$$4x^2 + 4\sqrt{3}x + 3 = 0$$

On comparing with $ax^2 + bx + c = 0$

$a = 4, b = 4\sqrt{3}, c = 3$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4 \times 4 \times 3$

$= 48 - 48 = 0$

So, the given equation has equal and real roots.

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a} = -\frac{b}{2a} = -\frac{4\sqrt{3}}{2 \times 4}$$

$$= -\frac{\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$$

(iv) Given equation is

$$2x^2 + x + 4 = 0$$

On comparing with $ax^2 + bx + c = 0$

$a = 2, b = 1$ and $c = 4$

$\therefore$ Discriminant, $D = b^2 - 4ac = (1)^2 - 4 \times 2 \times 4 = 1 - 32 = -31 < 0$

So, the given equation has no real roots. *i.e.,* it has imaginary roots.

Therefore, the real roots of the equation $2x^2 + x + 4 = 0$ do not exist

Question 3. Find the roots of the following equations

(i) $x - \frac{1}{x} = 3, x \neq 0$ (ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Solution (i) Given equation is

$$x - \frac{1}{x} = 3, x \neq 0$$

$$\Rightarrow \quad \frac{x^2 - 1}{x} = 3$$

$$\Rightarrow \quad x^2 - 3x - 1 = 0$$

On comparing with $ax^2 + bx + c = 0$

$a = 1, b = -3$ and $c = -1$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)} \quad \text{(By quadratic formula)}$$

$$= \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

$$= \frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2}$$

Hence, the roots of equation $x - \frac{1}{x} = 3$ are $\frac{3 + \sqrt{13}}{2}$ and $\frac{3 - \sqrt{13}}{2}$.

(ii) Given equation is

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$$

$$\Rightarrow \quad \frac{(x-7) - (x+4)}{(x+4)(x-7)} = \frac{11}{30} \Rightarrow \frac{x - 7 - x - 4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \quad \frac{-11}{x^2 - 3x - 28} = \frac{11}{30} \Rightarrow \frac{-1}{x^2 - 3x - 28} = \frac{1}{30}$$

$$\Rightarrow \quad -30 = x^2 - 3x - 28$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0$$

On comparing with $ax^2 + bx + c = 0$

Here, $a = 1$, $b = -3$ and $c = 2$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(By quadratic formula)}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2 \times 1} = \frac{3 \pm \sqrt{9 - 8}}{2}$$

$$= \frac{3 \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2} = \frac{4}{2}, \frac{2}{2} = 2, 1$$

Hence, the roots of equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ are 2 and 1.

Question 4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Solution Let the present age of Rehman = x year

$\therefore$ Rehman's age, 3 yr ago = $(x - 3)$ yr

Rehman's age, after 5 yr = $(x + 5)$ yr

According to the question, $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$\Rightarrow \quad \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \Rightarrow \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$

$\therefore \quad 6x + 6 = x^2 + 2x - 15$

$\Rightarrow \quad x^2 + 2x - 6x - 15 - 6 = 0$

$x^2 - 4x - 21 = 0$

$x^2 - (7x - 3x) - 21 = 0$

$\Rightarrow \quad x^2 - 7x + 3x - 21 = 0$ (By factorisation method)

$\Rightarrow \quad x(x - 7) + 3(x - 7) = 0$

$\Rightarrow \quad (x - 7)(x + 3) = 0$

$\therefore x - 7 = 0 \Rightarrow x = 7$ or $x + 3 = 0 \Rightarrow x = -3$

which is not possible because age cannot be negative.

So, present age of Rehman = 7 yr

Question 5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Solution Let Shefali's marks in Mathematics be x. Then, her marks in English will be $(30 - x)$ because given that the sum of Shefali's marks in Mathematics and English is 30.

According to the question,

(Marks in Mathematics + 2) × (Marks in English – 3) = 210

$(x + 2) \times [(30 - x) - 3] = 210$

$\Rightarrow \quad (x + 2)(27 - x) = 210$

$\Rightarrow \quad 27x - x^2 + 54 - 2x = 210$

$\Rightarrow \quad x^2 - 25x + 156 = 0$

$x^2 - (13x + 12x) + 156 = 0$

$\Rightarrow \quad x^2 - 13x - 12x + 156 = 0$

$\Rightarrow \quad x(x - 13) - 12(x - 13) = 0$ (By factorisation method)

$\Rightarrow \quad (x - 13)(x - 12) = 0$

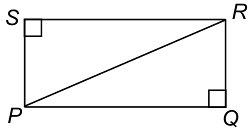
$\Rightarrow \quad x = 12$ and 13

When $x = 12$, marks in Mathematics = $x = 12$ and marks in English = $30 - 12 = 18$

When $x = 13$, marks in Mathematics = $x = 13$ and marks in English = $30 - 13 = 17$

Question 6. The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 30 m more than the shorter side, find the sides of the field.

Solution Let $PQRS$ be the rectangular field. Let the shorter side QR of the rectangle $= x$ m.



According to the question,

Diagonal of the rectangle $PR = 60$ m more than the shorter side $= (x + 60)$ m

Side of the rectangle $PQ = 30$ m more than the shorter side $= (x + 30)$ m

$\therefore$ By Pythagoras theorem,

In ΔPQR, $$PR^2 = PQ^2 + QR^2$$

($\because$ In rectangle every adjacent side makes an angle 90° to each other.)

$\Rightarrow$ $$(x + 60)^2 = (x + 30)^2 + x^2$$

$\Rightarrow$ $$x^2 + 120x + 3600 = x^2 + 60x + 900 + x^2$$

$[\because (a + b)^2 = a^2 + 2ab + b^2]$

$\Rightarrow$ $$(2x^2 - x^2) + (60x - 120x) + (900 - 3600) = 0$$

$\Rightarrow$ $$x^2 - 60x - 2700 = 0$$

$$x^2 - (90x - 30x) - 2700 = 0$$

$\Rightarrow$ $$x^2 - 90x + 30x - 2700 = 0$$ (By factorisation method)

$\Rightarrow$ $$x(x - 90) + 30(x - 90) = 0$$

$\Rightarrow$ $$(x - 90)(x + 30) = 0$$

$\therefore$ Either $x - 90 = 0 \Rightarrow x = 90$ or $x + 30 = 0 \Rightarrow x = -30$ which is not possible because side cannot be negative.

$\therefore$ $$x = 90$$

So, breadth of the rectangle = 90 m and length of the rectangle

$= 90 + 30 = 120$ m

Question 7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Solution Let the required numbers be x and y, where $x > y$

Difference of squares of two numbers = 180

Given, $$x^2 - y^2 = 180 \quad \ldots\text{(i)}$$

Square of smaller number = 8 × Larger number

$\Rightarrow$ $$y^2 = 8x \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$x^2 - 8x = 180$$

$\Rightarrow$ $$x^2 - 8x - 180 = 0$$

$$x^2 - (18x - 10x) - 180 = 0$$

$\Rightarrow \quad x^2 - 18x + 10x - 180 = 0$ (By factorisation method)

$\Rightarrow \quad x(x - 18) + 10(x - 18) = 0$

$\Rightarrow \quad (x - 18)(x + 10) = 0$

$\therefore \quad x - 18 = 0$ or $x + 10 = 0$

$\therefore \quad x = 18$ or $x = -10$

Now, $x = 18$

$\Rightarrow \quad y^2 = 8 \times 18 = 144$

$\Rightarrow \quad y = \pm 12$

$\Rightarrow \quad y = 12$ or -12 [From Eq. (ii)]

Again, $x = -10 \Rightarrow y^2 = [8 \times (-10)] = -80$ which is not possible *i.e.,* imaginary value.

Hence, the numbers are (18 and 12) or (18 and –12).

Question 8. **A train travels 360 km at a uniform speed. If the speed had been 5 kmh^{-1} more, if would have taken 1 hour less for the same journey. Find the speed of the train.**

Solution Let the uniform speed of the train be x kmh^{-1}. So, time taken to cover 360 km $= \frac{360}{x}$ h and time taken to cover 360 km when the speed is increased as 5 km/h $= \frac{360}{x+5}$ h. $\left(\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$

According to the question,

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$\Rightarrow \quad 360(x + 5) - 360x = x(x + 5)$

$\Rightarrow \quad 360x + 1800 - 360x = x^2 + 5x$

$\Rightarrow \quad x^2 + 5x - 1800 = 0$

$x^2 + (45x - 40x) - 1800 = 0$

$\Rightarrow \quad x^2 + 45x - 40x - 1800 = 0$ (By factorisation method)

$\Rightarrow \quad x(x + 45) - 40(x + 45) = 0$

$\Rightarrow \quad (x + 45)(x - 40) = 0$

$\Rightarrow \quad x + 45 = 0 \Rightarrow x = -45$

or $\quad x - 40 = 0 \Rightarrow x = 40$

Since, x cannot be negative because speed of train cannot be negative.

$\therefore \quad x = 40$

Hence, the original speed of the train is 40 km/h.

Question 9. Two water taps together can fill a tank in $9\frac{3}{8}$ h. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution Let the time taken by the larger tap to fill the tank = x h

$\therefore$ Time taken by the smaller tap to fill the tank = $(x + 10)$ h (By condition)

$\therefore$ Portion of the tank filled by the larger tap in 1 h = $\frac{1}{x}$

Portion of the tank filled by the smaller tap in 1 h = $\frac{1}{x+10}$

Portion of the tank filled by both tap in 1 h = $\frac{1}{x} + \frac{1}{x+10} = \frac{x + 10 + x}{x(x+10)}$

$$= \frac{2x+10}{x(x+10)} = \frac{2(x+5)}{x(x+10)} \quad \text{...(i)}$$

According to the question,

Portion of the tank filled by both taps in 1 h = $\frac{1}{\left(\frac{75}{8}\right)} = \frac{8}{75}$ ...(ii)

From Eqs. (i) and (ii), we get $\frac{2(x+5)}{x(x+10)} = \frac{8}{75}$

$$\Rightarrow \quad \frac{x+5}{x^2+10x} = \frac{4}{75}$$

$$\Rightarrow \quad 75x + 375 = 4x^2 + 40x$$

$$\Rightarrow \quad 4x^2 + 40x - 75x - 375 = 0$$

$$\Rightarrow \quad 4x^2 - 35x - 375 = 0$$

On comparing with $ax^2 + bx + c = 0$, we get, $a = 4$, $b = -35$ and $c = -375$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(By quadratic formula)}$$

$$= \frac{35 \pm \sqrt{(-35)^2 - 4 \times 4 \times (-375)}}{8}$$

$$= \frac{35 \pm \sqrt{25 \times 49 + 16 \times 25 \times 15}}{8}$$

$$= \frac{35 \pm \sqrt{25 \times 289}}{8} = \frac{35 \pm 5 \times 17}{8}$$

$$= \frac{35 \pm 85}{8} = \frac{120}{8}, \frac{-50}{8} = 15, \frac{-25}{4}$$

$$= 15, -6\frac{1}{4}$$

But negative sign is not possible because time cannot be negative.

Hence, $x = 15$

Thus, time taken by each tap to fill the tank separately are 15 h and 15 + 10 = 25 h.

Question 10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bengaloru (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 kmh^{-1} more than that of the passenger train, find the average speed of the two trains.

Solution Let the average speed of the passenger train = x kmh^{-1}

Then, the average speed of the express train = $(x + 11)$ kmh^{-1}

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ h $\left(\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$

Time taken by express train to cover 132 km = $\frac{132}{(x + 11)}$ h

According to the question,

$$\frac{132}{x} - \frac{132}{x + 11} = 1$$

On dividing both sides by 132

$$\Rightarrow \quad \frac{1}{x} - \frac{1}{x + 11} = \frac{1}{132}$$

$$\Rightarrow \quad \frac{(x + 11) - x}{x(x + 11)} = \frac{1}{132}$$

$$\Rightarrow \quad x(x + 11) = 132 \times 11$$

$$\Rightarrow \quad x^2 + 11x - 1452 = 0$$

$$x^2 + (44 - 33)x - 1452 = 0$$

$$\Rightarrow \quad x^2 + 44x - 33x - 1452 = 0 \quad \text{(By factorisation method)}$$

$$\Rightarrow \quad x(x + 44) - 33(x + 44) = 0$$

$$\Rightarrow \quad (x + 44)(x - 33) = 0$$

$$\therefore \quad x + 44 = 0$$

$$\Rightarrow \quad x = -44 \text{ or } x - 33 = 0$$

$$\Rightarrow \quad x = 33$$

Since, $x \neq -44$ because speed never negative.

Hence, the average speed of the passenger train = 33 km/h and the average speed of the express train = (33 + 11) km/h = 44 km/h

Question 11. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Solution Suppose the side of the two squares be x metre and y metre.

Then, area of the first square = x^2 m^2 [$\because$ Area of the square = $(\text{Side})^2$]

Area of the second square = y^2 m^2

Perimeter of the first square = $4x$ metre

Perimeter of the second square = $4y$ metre

[$\because$ Perimeter of the square = 4 × (Side)]

According to the question, sum of areas of two squares = 468 m^2

$$x^2 + y^2 = 468 \quad \text{...(i)}$$

and difference of their perimeters = 24 m

$\Rightarrow \quad 4x - 4y = 24$ or $x - y = 6$ ...(ii)

From Eq. (ii), $y = x - 6$

Putting the value of y in Eq. (i), we get

$$x^2 + (x - 6)^2 = 468$$

$\Rightarrow \quad x^2 + x^2 - 12x + 36 - 468 = 0 \quad [\because (a - b)^2 = a^2 - 2ab + b^2]$

$\Rightarrow \quad 2x^2 - 12x - 432 = 0$

$\Rightarrow \quad x^2 - 6x - 216 = 0$

$x^2 - (18x - 12x) - 216 = 0$

$\Rightarrow \quad x^2 - 18x + 12x - 216 = 0$ (By factorisation method)

$\Rightarrow \quad x(x - 18) + 12(x - 18) = 0$

$\Rightarrow \quad (x - 18)(x + 12) = 0$

Either $x - 18 = 0$ or $x + 12 = 0$

$\Rightarrow \quad x = 18$ or $x = -12$

Since, the negative sign is not possible because side never negative.

$\therefore \quad x = 18$

$\therefore$ Side of the first square $= x$ m $= 18$ m

and side of the second square $= y$ m $= (18 - 6)$ m $= 12$ m

7 Quadratic Equations

Exercise 7.4

Question 1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them

(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Solution (i) Given equation is

$$2x^2 - 3x + 5 = 0$$

On comparing with $ax^2 + bx + c = 0$

Here, $a = 2, b = -3$ and $c = 5$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-3)^2 - 4(2)(5)$

$= 9 - 40 = -31 < 0$

Hence, the equation $2x^2 - 3x + 5 = 0$ has no real roots *i.e.,* imaginary roots.

(ii) Given equation is

$$3x^2 - 4\sqrt{3}x + 4 = 0 \quad \text{...(i)}$$

On comparing with $ax^2 + bx + c = 0$

Here, $a = 3, b = -4\sqrt{3}$ and $c = 4$

$\therefore$ Discriminant, $D = b^2 - 4ac = (-4\sqrt{3})^2 - 4 \times 3(4)$

$= 48 - 48 = 0$

Hence, the equation $3x^2 - 4\sqrt{3}x + 4 = 0$ has two equal and real roots.

Eq. (i) can be written as $(\sqrt{3}x)^2 - 2(\sqrt{3}x)(2) + (2)^2 = 0$

$\Rightarrow$ $(\sqrt{3}x - 2)^2 = 0$ $[\because a^2 - 2ab + b^2 = (a - b)^2]$

$\therefore$ $(\sqrt{3}x - 2)(\sqrt{3}x - 2) = 0$

$\Rightarrow$ $x = \dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}$

Hence, the equal roots are $\dfrac{2}{\sqrt{3}}$ and $\dfrac{2}{\sqrt{3}}$.

(iii) Given equation is

$$2x^2 - 6x + 3 = 0$$

On comparing with $ax^2 + bx + c = 0$

Here, $a = 2, b = -6$ and $c = 3$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (-6)^2 - 4(2)(3)$

$= 36 - 24 = 12 > 0$

Hence, the equation $2x^2 - 6x + 3 = 0$ has two distinct real roots.

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Hence, the real roots are $\dfrac{3 + \sqrt{3}}{2}$ and $\dfrac{3 - \sqrt{3}}{2}$.

Question 2. Find the values of k for each of the following quadratic equations so that they have two equal roots

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x-2) + 6 = 0$

Solution (i) The given equation is

$$2x^2 + kx + 3 = 0$$

This equation is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = k$ and $c = 3$

$\therefore$ Discriminant, $D = b^2 - 4ac = (k)^2 - 4 \times 2 \times 3$

$= k^2 - 24$

For equal roots, $D = 0 \Rightarrow k^2 - 24 = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$

$\therefore k = \pm 2\sqrt{6}$

(ii) The given equation is

$$kx(x-2) + 6 = 0 \quad \text{...(i)}$$

Eq. (i) can be written as $kx^2 - 2kx + 6 = 0$ ...(ii)

On comparing Eq. (ii) with equation $ax^2 + bx + c = 0$, we get

$a = k$, $b = -2k$ and $c = 6$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (-2k)^2 - 4 \times k \times 6$

$= 4k^2 - 24k = 4k(k-6)$

For equal roots, $D = 0 \Rightarrow 4k(k-6) = 0$

$\Rightarrow k = 0$ or $k = 6$

Question 3. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2? If so, find its length and breadth.

Solution Let breadth of a rectangular mango grove = x metre

$\therefore$ Length of a rectangular mango grove = $2x$ metre (By given condition)

$\therefore$ According to the question,

Area of rectangular mango grove = 800 m^2

$\Rightarrow$ Length $\times$ Breadth = $2x(x) = 800$

$\Rightarrow 2x^2 = 800 \Rightarrow x^2 = 400$

$\Rightarrow x = \pm 20$

Since, negative sign is not possible because breadth never be negative.

$\therefore$ Length = $2x$ = 40 m and Breadth = 20 m

Question 4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution Let age of one of two friends = x yr

Then, age of other friend $= (20 - x)$ yr

($\because$ The sum of the ages of two friends is 20 yr)

4 yr ago age of one of two friends = $(x - 4)$ yr

4 yr ago age of the other friend = $(20 - x - 4)$ yr = $(16 - x)$ yr

According to the question,

$$(x - 4)(16 - x) = 48$$

$$\Rightarrow \quad 16x - x^2 - 64 + 4x = 48$$

$$\Rightarrow \quad x^2 - 20x + 112 = 0$$

On comparing the above equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -20 \text{ and } c = 112$$

$$\therefore \quad \text{Discriminant, } D = b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112$$

$$= 400 - 448 = 48 < 0$$

which implies that the real roots are not possible because this condition represents an imaginary roots. So, the solution does not exist.

Question 5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Solution Let the breadth fo the park = x metre

Then, according to the question,

$$\text{Perimeter of a rectangular park} = 80 \text{ m}$$

$$\Rightarrow \quad 2 (\text{Length} + \text{Breadth}) = 80 \text{ m}$$

$$\Rightarrow \quad \text{Length} + \text{Breadth} = 40 \text{ m}$$

$$\Rightarrow \quad \text{Length} = (40 - x) \text{ m}$$

$\therefore$ Area of a rectangular park = Length $\times$ Breadth = $(40 - x)x$ m^2

But according to the question,

Area of the rectangular park is 400 m^2.

$$\therefore \quad (40 - x)x = 400$$

$$\Rightarrow \quad x^2 - 40x + 400 = 0$$

$$\Rightarrow \quad x^2 - 2x \times 20 + (20)^2 = 0$$

$$\Rightarrow \quad (x - 20)^2 = 0 \qquad [\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$\Rightarrow \quad x = 20$$

Thus, breadth of the park = 20 m

and length of the park = $(40 - 20)$ m = 20 m

So, it is possible to design the rectangular park having equal length and breadth *i.e.,* 20 m.