# 9 Quadrilaterals

## Exercise 9.1

**Question 1.** The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

**Solution** Given, the ratio of the angles of quadrilateral are 3 : 5 : 9 : 13.

Let the angles of the quadrilateral are $3x$, $5x$, $9x$ and $13x$.

We know that, sum of angles of a quadrilateral = 360°

$\therefore \quad 3x + 5x + 9x + 13x = 360°$

$\Rightarrow \quad 30x = 360° \Rightarrow x = \dfrac{360°}{30} = 12°$

$\therefore$ Angles of the quadrilateral are $3x = 3 \times 12 = 36°$

$5x = 5 \times 12 = 60°$

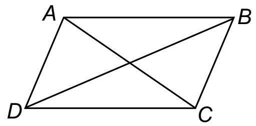
$9x = 9 \times 12 = 108°$

and $13x = 13 \times 12 = 156°$

**Question 2.** If the diagonals of a parallelogram are equal, then show that it is a rectangle.

**Solution** Let given parallelogram is *ABCD* whose diagonals *AC* and *BD* are equal. *i.e.*, $AC = BD$.

Now, we have to prove that *ABCD* is a rectangle.




**Proof.** In $\Delta ABC$ and $\Delta DCB$, we have

$AB = CD$ (Opposite sides of parallelogram)

$BC = CB$ (Common in both triangles)

and $AC = BD$ (Given)

$\therefore \quad \Delta ABC \cong \Delta DCB$ (By SSS rule)

$\therefore \quad \angle ABC = \angle DCB$ ...(i)

(Corresponding Part of Congruent Triangle)

But $DC \parallel AB$ and transversal *CB* intersect them.

$\therefore \quad \angle ABC + \angle DCB = 180°$

($\because$ Both are interior angles on the same side of the transversal)

$\Rightarrow \quad \angle ABC + \angle ABC = 180°$ [From Eq. (i)]

$\Rightarrow \quad 2\angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 90° = \angle DCB$

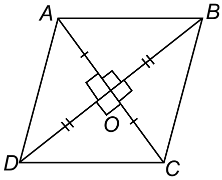
Thus, *ABCD* is a parallelogram and one of angles is 90°.

Hence, *ABCD* is a rectangle.

Hence proved.

**Question 3.** Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

**Solution** Given, a quadrilateral $ABCD$ whose diagonals $AC$ and $BD$ bisect each other at right angles.




*i.e.*, $OA = OC$ and $OB = OD$

and $\angle AOD = \angle AOB = \angle COD = \angle BOC = 90°$

**To prove**, $ABCD$ is a rhombus.

**Proof.** In $\Delta OAB$ and $\Delta ODC$, we have

$OA = OC$ and $OB = OD$ (Given)

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\therefore$ $\Delta OAB \cong \Delta OCD$ (By SAS rule)

$\therefore$ $AB = CD$ ...(i)

(Corresponding part of congruent triangles)

Again, in $\Delta OAD$ and $\Delta OBC$, we have

$OA = OC$ and $OD = OB$ (Given)

and $\angle AOD = \angle BOC$ (Vertically opposite angle)

$\therefore$ $\Delta\ OAD \cong \Delta\ OCB$ (By SAS rule)

$\therefore$ $AD = BC$ ...(ii)

(Corresponding part of congruent triangles)

Similarly, we can prove that

$AB = AD$

$CD = BC$ ...(iii)

Hence, from Eqs. (i), (ii) and (iii), we get

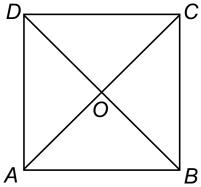
$AB = BC = AD = CD$

Hence, $ABCD$ is a rhombus.

Hence proved.

**Question 4.** Show that the diagonals of a square are equal and bisect each other at right angles.

**Solution** **Given** A square $ABCD$ whose diagonals $AC$ and $BD$ intersect at $O$.




**To prove** Diagonals are equal and bisect each other at right angles.

*i.e.*, $AC = BD$, $OD = OB$, $OA = OC$ and $AC \perp BD$

**Proof** In $\Delta$ *ABC* and $\Delta$ *BAD*, we have

$AB = BA$ (Common)

$BC = AD$ (Sides of a square)

$\angle ABC = \angle BAD = 90°$

$\therefore$ $\Delta ABC \cong \Delta BAD$ (By SAS rule)

Hence, $AC = BD$

(Corresponding Parts of Congruent Triangle)

In $\Delta OAB$ and $\Delta OCD$

$AB = DC$ (Side of square)

$\angle OAB = \angle DCO$

($\because AB \parallel CD$ and transversal $AC$ intersect)

and $\angle OBA = \angle BDC$

($\because AB \parallel CD$ and transversal $BD$ intersect)

$\therefore$ $\Delta OAB \cong \Delta OCD$

$\therefore$ $OA = OC$ and $OB = OD$

(Corresponding Parts of Congruent Triangle)

Now, in $\Delta AOB$ and $\Delta AOD$, we have

$OB = OD$ (Prove in above)

$AB = AD$ (Sides of a square)

$AO = OA$ (Common)

$\therefore$ $\Delta AOB \cong \Delta AOD$ (By SSS)

$\therefore$ $\angle AOB = \angle AOD$ (By CPCT)

But $\angle AOB + \angle AOD = 180°$ (Linear pair)

$\therefore$ $\angle AOB = \angle AOD = 90°$

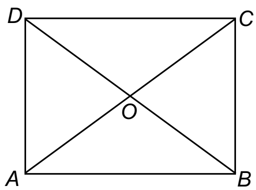
Thus, $AO \perp BD$ *i.e.*, $AC \perp BD$.

Hence, $AC = BD$, $OA = OC$, $OB = OD$ and $AC \perp BD$

Hence proved.

**Question 5.** Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

**Solution** **Given** A quadrilateral *ABCD* in which $AC = BD$ and $AC \perp BD$ such that $OA = OC$ and $OB = OD$. So, *ABCD* is a parallelogram.




**To prove** *ABCD* is a square.

**Proof** Let *AC* and *BD* intersect at a point *O*.

In $\Delta ABO$ and $\Delta ADO$, we have

$$BO = OD \quad \text{(Given)}$$
$$AO = OA \quad \text{(Common)}$$
$$\angle AOB = \angle AOD = 90° \quad \text{(Given)}$$
$\therefore$ $\Delta ABO \cong \Delta ADO$ (By SAS)

$\therefore$ $AB = AD$ (By CPCT)

Also, $AB = DC$

and $AD = BC$ (Opposite sides of parallelogram)

$\therefore$ $AB = BC = DC = AD$ ...(i)

Again, in $\Delta ABC$ and $\Delta BAD$, we have

$$AB = BA \quad \text{(Common)}$$
$$AC = BD \quad \text{(Given)}$$
$$BC = AD \quad \text{[From Eq. (i)]}$$
$\therefore$ $\Delta ABC \cong \Delta BAD$ (By SSS)

$\therefore$ $\angle ABC = \angle BAD$ ...(ii)

But $\angle ABC + \angle BAD = 180°$

(Sum of interior angles of a parallelogram)

$\therefore$ $\angle ABC = \angle BAD = 90°$ [From Eq. (ii)]

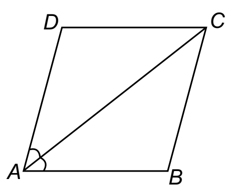
Thus, $AB = BC = CD = DA$ and $\angle A = 90°$

$\therefore$ $ABCD$ is a square.

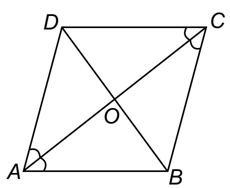
Hence proved.

**Question 6.** Diagonal $AC$ of a parallelogram $ABCD$ bisects $\angle A$ (see figure). Show that

(i) it bisects $\angle C$ also, (ii) $ABCD$ is a rhombus.




**Solution** Given, diagonal $AC$ of a parallelogram $ABCD$ bisects $\angle A$.




*i.e.*, $\angle DAC = \angle BAC = \frac{1}{2} \angle BAD$ ...(i)

Here, $AB \parallel CD$ and $AC$ is transversal.

$\therefore$ $\angle DCA = \angle CAB$ (Pair of alternate angle)...(ii)

and $\angle BCA = \angle DAC$ (Pair of alternate angle)...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\angle DAC = BCA = \angle BAC = \angle DCA$$

Now, $\angle BCD = \angle BCA + \angle DCA$

$= \angle DAC + \angle CAB$

$= \angle BAD$

$\therefore$ Diagonal $AC$ also bisects $\angle C$.

Again, in $\Delta\, OAD$ and $\Delta\, OCD$, we have

$OA = OC$ ($\because$ Diagonals bisect each other)

$OD = OD$ (Common)

$\angle AOD = \angle COD = 90°$

$\therefore$ $\Delta\, OAD \cong \Delta\, OCD$ (By SAS)

$\therefore$ $AD = CD$ (By CPCT)

Now, $AB = CD$

and $AD = BC$ } (Opposite sides of parallelogram)

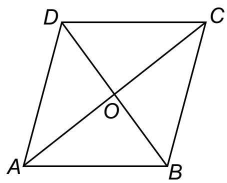
$\therefore$ $AB = CD = AD = BC$

Hence, $ABCD$ is a rhombus.

Hence proved.

**Question 7.** $ABCD$ is a rhombus. Show that diagonal $AC$ bisects $\angle A$ as well as $\angle C$ and diagonal $BD$ bisects $\angle B$ as well as $\angle D$.

**Solution** **Given** $ABCD$ is a rhombus.




$\therefore$ $AD = AB = BC = CD$ ...(i)

**To prove** (i) Diagonal $AC$ bisect $\angle A$ as well as $\angle C$.

(ii) Diagonal $BD$ bisects $\angle B$ as well as $\angle D$.

**Proof** (i) Let $AC$ and $BD$ are the diagonals of rhombus $ABCD$.

In $\Delta ABC$ and $\Delta ADC$,

$AD = AB$

$CD = BC$ [From Eq. (i)]

and $AC = CA$ (Common)

$\therefore$ $\Delta ABC \cong \Delta ADC$ (By SSS rule)

$\therefore$ $\angle DAC = \angle BAC$ (By CPCT)

and $\angle DCA = \angle BCA$

Also, $\angle DAC = \angle DCA$

and $\angle BAC = \angle BCA$

This shows that $AC$ bisect $\angle A$ as well as $\angle C$.

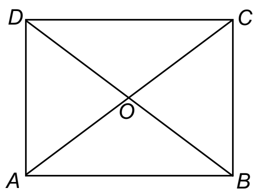
(ii) Again, in $\Delta BDC$ and $\Delta BDA$,

$$AB = BC$$
$$AD = CD$$
$$BD = BD \quad \text{(Common)}$$
$\therefore$ $$\Delta BDC \cong \Delta BDA \quad \text{(SSS rule)}$$
$\therefore$ $$\angle BDA = \angle BDC$$
and $$\angle DBA = \angle DBC$$
Also, $$\angle BDA = \angle DBA$$
and $$\angle BDC = \angle DBC$$

This shows that $BD$ bisect $\angle B$ as well as $\angle D$.

**Question 8.** $ABCD$ is a rectangle in which diagonal $AC$ bisects $\angle A$ as well as $\angle C$. Show that (i) $ABCD$ is a square (ii) diagonal $BD$ bisects $\angle B$ as well as $\angle D$.

**Solution** **Given** $ABCD$ is a rectangle.




$\therefore$ $$AB = CD \text{ and } BC = AD \quad \ldots\text{(i)}$$

**To Prove** (i) $ABCD$ is a square.

*i.e.,* $$AB = BC = CD = DA$$

(ii) Dioagonal $BD$ bisects $\angle B$ as well as $\angle D$.

**Proof** (i) In $\Delta ADC$ and $\Delta ABC$, we have

Since, $AB \parallel DC$ and $AC$ transversal intersect

$$\angle DAC = \angle BAC$$
$$\angle DCA = \angle BCA$$
and $$AC = CA \quad \text{(Common)}$$
$\therefore$ $$\Delta\ ADC \cong \Delta\ ABC \quad \text{(By ASA rule)}$$
$\therefore$ $$AD = AB \quad \text{(By CPCT)}$$
and $$CD = BC \quad \ldots\text{(ii)}$$

Hence, from Eqs. (i) and (ii), we get

$$AB = BC = AD = CD$$

$\therefore ABCD$ is a square.

(ii) In $\Delta AOB$ and $\Delta COB$, we have

$AB = BC$ (Side of square)

$BO = OB$ (Common)

$OA = OC$

($\because$ Diagonal of square bisect each other)

$\therefore$ $\Delta AOB \cong \Delta COB$ (By SSS rule)

$\therefore$ $\angle OBA = \angle OBC$

This shows that $BO$ or $BD$ bisect $\angle B$.

Similarly, in $\Delta$ $AOD$ and $\Delta$ $COD$, we have

$AD = CD$ (Side of square)

$OD = DO$ (Common)

and $OA = OC$

($\because$ Diagonal of square bisect each other)

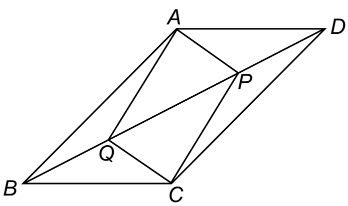
$\therefore$ $\Delta$ $AOD \cong \Delta$ $COD$ (By SSS rule)

$\therefore$ $\angle ADO = \angle CDO$

This shows that $DO$ or $DB$ bisect $\angle D$.

Hence proved.

**Question 9.** In parallelogram $ABCD$, two points $P$ and $Q$ are taken on diagonal $BD$ such that $DP = BQ$ (see figure). Show that

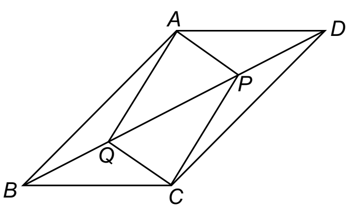



(i) $\Delta APD \cong \Delta CQB$ (ii) $AP = CQ$

(iii) $\Delta AQB \cong \Delta CPD$ (iv) $AQ = CP$

(v) $APCQ$ is a parallelogram.

**Solution** Given, $ABCD$ is a parallelogram and $P$ and $Q$ are lie on $BD$ such that




$DP = BQ$ ...(i)

(i) We have to show,

$$\Delta APD \cong \Delta CQB$$

Now, in $\Delta APD$ and $\Delta CQB$, we have

$DP = BQ$ (Given)

$AD = BC$ (Opposite sides are equal in parallelogram)

$\because AD \parallel BC$ and $BD$ is a transversal.

$\therefore \angle ADP = \angle QBC$ (Alternate interior angle)

$\therefore \Delta APD \cong \Delta CQB$ (By SAS)

(ii) Since, $\Delta APD \cong \Delta CQB$

$\therefore AP = CQ$

(iii) Here, we have to show, $\Delta AQB \cong \Delta CPD$

Now, in $\Delta AQB$ and $\Delta CPD$, we have

$BQ = DP$ (Given)

$AB = CD$ (Opposite sides of parallelogram)

$\because AB \parallel CD$ and $BD$ is a transversal.

$\therefore \angle ABQ = \angle CDP$ (Alternate interior angle)

$\therefore \Delta AQB \cong \Delta CPD$

(iv) Since, $\Delta AQB \cong \Delta CPD$

$\therefore AQ = CP$

(v) Now, in $\Delta APQ$ and $\Delta PCQ$, we have

$AQ = CP$ [From part (iv)]

$AP = CQ$ [From part (ii)]

$PQ = QP$ (Common)

$\therefore \Delta APQ \cong \Delta PCQ$ (By SSS)

$\therefore \angle APQ = \angle PQC$

and $\angle AQP = \angle CPQ$ (Vertically opposite)

Now, these equal angles form a pair of alternate angle when line segment $AP$ and $QC$ are intersected by a transversal $PQ$.

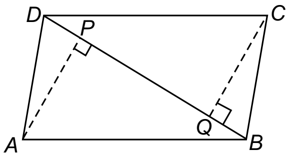
$\therefore AP \parallel CQ$ and $AQ \parallel CP$

Now, both pairs of opposite sides of quadrilateral $APCQ$ are parallel.

Hence, $APCQ$ is a parallelogram.

Hence proved.

**Question 10.** $ABCD$ is a parallelogram and $AP$ and $CQ$ are perpendiculars from vertices $A$ and $C$ on diagonal $BD$ (see figure). Show that




(i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$.

**Solution** Given, $ABCD$ is a parallelogram and $AP$ and $CQ$ are perpendicular from vertices $A$ and $C$ on diagonal $BD$.

$\because AB \parallel CD$ and $BD$ is a transversal.

$\therefore \quad \angle CDB = \angle DBA \quad \ldots$(i)

(i) Now, in $\Delta APB$ and $\Delta CQD$, we have

$CD = AB$ (Sides of parallelogram)

$\angle CQD = \angle APB = 90°$ (Given)

$\angle CDQ = \angle ABP$ [From Eq. (i)]

$\therefore \quad \Delta APB \cong \Delta CQD$ (By ASA rule)

(ii) $\because \quad \Delta APB \cong \Delta CQD$ (By CPCT)

$\therefore \quad AP = CQ$

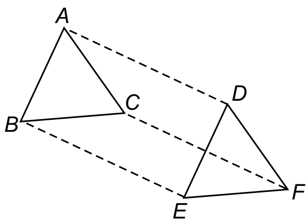
Hence proved.

**Question 11.** In $\Delta ABC$ and $\Delta DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices $A$, $B$ and $C$ are joined to vertices $D$, $E$ and $F$, respectively (see figure).

Show that

(i) quadrilateral $ABED$ is a parallelogram

(ii) quadrilateral $BEFC$ is a parallelogram




(iii) $AD \parallel CF$ and $AD = CF$

(iv) quadrilateral $ACFD$ is a parallelogram

(v) $AC = DF$

(vi) $\Delta ABC \cong \Delta DEF$

**Solution** Given, in $\Delta ABC$ and $\Delta DEF$,

$$AB = DE,\ AB \parallel DE$$

and $$BC = EF,\ BC \parallel EF$$

(i) Now, in quadrilateral $ABED$,

$AB = DE$ and $AB \parallel DE$ (Given)

$\Rightarrow ABED$ is a parallelogram.

($\because$ A pair of opposite sides is equal and parallel)

(ii) In quadrilateral $BEFC$,

$$BC = EF \text{ and } BC \parallel EF$$

$\Rightarrow BEFC$ is a parallelogram.

($\because$ A pair of opposite sides is equal and parallel)

(iii) Since, $ABED$ is a parallelogram.

$\therefore$ $AD \parallel BE$ and $AD = BE$ ...(i)

Also, $BEFC$ in a parallelogram.

$\therefore$ $CF \parallel BE$ and $CF = BE$ ...(ii)

From Eqs. (i) and (ii), we get

$$AD \parallel CF \text{ and } AD = CF$$

(iv) In quadrilateral $ACFD$, we have

$AD \parallel CF$ and $AD = CF$ [From part (iii)]

$\Rightarrow ACFD$ is a parallelogram.

(v) Since, $ACFD$ is a parallelogram.

$\therefore$ $AC = DF$ and $AC \parallel DF$

(vi) Now, in $\Delta ABC$ and $\Delta DEF$,

$AB = DE$ (Given)

$BC = EF$ (Given)

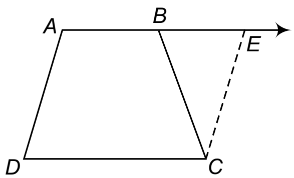
and $AC = DF$ [From part (v)]

$\therefore$ $\Delta ABC \cong \Delta DEF$ (By SSS rule)

**Question 12.** $ABCD$ is a trapezium in which $AB \parallel CD$ and $AD = BC$ (see figure). Show that

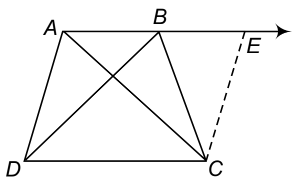
(i) $\angle A = \angle B$ (ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$ (iv) diagonal $AC$ = diagonal $BD$




[**Hint** Extend $AB$ and draw a line through $C$ parallel to $DA$ intersecting $AB$ produced at $E$].

**Solution** Given, $ABCD$ is a trapezium.




$AB \parallel CD$ and $AD = BC$

Now, extend $AB$ and draw a line through $C$ parallel to $DA$ intersecting $AB$ produced at $E$.

Now, $ADCE$ is a parallelogram.

$\therefore \quad AD \parallel CE$ and $AD = CE$

But $\quad AD = BC$

$\therefore \quad AD = BC = CE$

(i) We know that, $\angle A + \angle E = 180°$

($\because$ Interior angles on the same side of the transversal $AE$)

$\Rightarrow \quad \angle E = 180° - \angle A$

Since, $\quad BC = EC$

$\therefore \quad \angle E = \angle CBE = 180° - \angle A$

Also, $\quad \angle ABC = 180° - \angle CBE$ ($\because$ $ABE$ is straight line)

$= 180° - 180° + \angle A$

$\Rightarrow \quad \angle B = \angle A$ ...(i)

(ii) Now, $\quad \angle A + \angle D = 180°$

($\because$ Interior angles on the same side of the transversal $AD$)

$\Rightarrow \quad \angle D = 180° - \angle A$

$\Rightarrow \quad \angle D = 180° - \angle B$ [From Eq. (i)]...(ii)

Also, $\angle C + \angle B = 180°$

($\because$ Interior angles on the same side of the transversal $BC$)

$\Rightarrow \quad \angle C = 180° - \angle B$ ...(iii)

From Eqs. (ii) and (iii), we get

$$\angle C = \angle D$$

(iii) Now, in $\Delta$ $ABC$ and $\Delta$ $BAD$, we have

$AB = BA$ (Common)

$AD = BC$ (Given)

$\angle A = \angle B$ [From Eq. (i)]

$\therefore \quad \Delta ABC \cong \Delta BAD$ (By SAS)

(iv) Since, $\quad \Delta ABC \cong \Delta BAD$

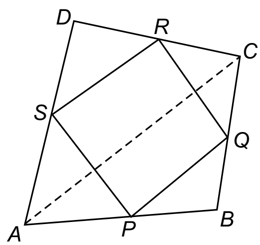
$\therefore \quad AC = BD$

Hence proved.

# 9 Quadrilaterals

## Exercise 9.2

**Question 1.** $ABCD$ is a quadrilateral in which $P, Q, R$ and $S$ are mid-points of the sides $AB, BC, CD$ and $DA$ (see figure). $AC$ is a diagonal. Show that




(i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

(ii) $PQ = SR$

(iii) $PQRS$ is a parallelogram.

**Solution** Given, $P$, $Q$, $R$ and $S$ are mid-points of the sides.

$\therefore$ $$AP = PB, BQ = CQ$$
$$CR = DR \text{ and } AS = DS$$

(i) In $\Delta ADC$, we have

$S$ is mid-point of $AD$ and $R$ is mid-point of the $DC$.

We know that, the line segment joining the mid-points of two sides of a triangle is parallel to the third side.

$\therefore$ $$SR \parallel AC \quad \text{...(i)}$$

Also, $$SR = \frac{1}{2} AC \quad \text{...(ii)}$$

(ii) Similarly, in $\Delta\, ABC$, we have

$$PQ \parallel AC \quad \text{...(iii)}$$

and $$PQ = \frac{1}{2} AC \quad \text{...(iv)}$$

Now, from Eqs. (ii) and (iv), we get

$$SR = PQ = \frac{1}{2} AC \quad \text{...(v)}$$

(iii) Now, from Eqs. (i) and (iii), we get

$$PQ \parallel SR$$

and from Eq. (v), $$PQ = SR$$

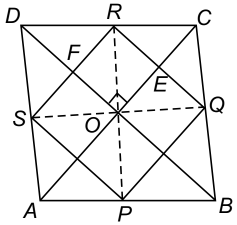
Since, a pair of opposite sides of a quadrilateral $PQRS$ is equal and parallel.

So, $PQRS$ is a parallelogram.

Hence proved.

**Question 2.** *ABCD* is a rhombus and *P*, *Q*, *R* and *S* are the mid-points of the sides *AB*, *BC*, *CD* and *DA*, respectively. Show that the quadrilateral *PQRS* is a rectangle.

**Solution** Given, *ABCD* is a rhombus and *P*, *Q*, *R* and *S* are mid-points of *AB*, *BC*, *CD* and *DA*.




By mid-point theorem,

In $\Delta ADC$, $SR \parallel AC$ and $SR = \frac{1}{2} AC$ ...(i)

In $\Delta ABC$, $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ...(ii)

From Eqs. (i) and (ii), we get

$$PQ \parallel SR \text{ and } PQ = SR = \frac{1}{2} AC$$

$\therefore$ *PQRS* is a parallelogram.

Now, we know that diagonals of a rhombus bisect each other at right angles.

$\therefore$ $\angle EOF = 90^\circ$

Now, $RQ \parallel BD$ (By mid-point theorem)

$\Rightarrow$ $RE \parallel OF$

Also, $SR \parallel AC$ [From Eq. (i)]

$\Rightarrow$ $FR \parallel OE$

$\therefore$ *OERF* is a parallelogram.

So, $\angle ERF = \angle EOF = 90^\circ$

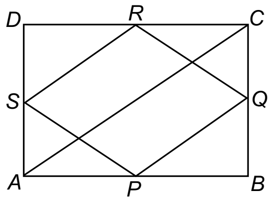
(Opposite angle of a quadrilateral is equal)

Thus, *PQRS* is a parallelogram with $\angle R = 90^\circ$.

Hence, *PQRS* is a rectangle.

**Question 3.** *ABCD* is a rectangle and *P*, *Q*, *R* ans *S* are mid-points of the sides *AB*, *BC*, *CD* and *DA*, respectively. Show that the quadrilateral *PQRS* is a rhombus.

**Solution** Given, *ABCD* is a rectangle.




$\therefore$ $\angle A = \angle B = \angle C = \angle D = 90^\circ$

and $AD = BC$, $AB = CD$

Also, given *P*, *Q*, *R* and *S* are mid-points of *AB*, *BC*, *CD* and *DA* ,respectively.

$\therefore PQ \parallel BD$ and $PQ = \frac{1}{2} BD$

and $SR \parallel AC$ and $SR = \frac{1}{2} AC$

and $SR \parallel AC$ and $SR = \frac{1}{2} AC$

In rectangle *ABCD*,

$$AC = BD$$

$\therefore$ $PQ = SR$ ...(i)

Now, in $\Delta ASP$ and $\Delta BQP$

$$AP = BP \quad \text{(Given)}$$

$$AS = BQ \quad \text{(Given)}$$

$$\angle A = \angle B \quad \text{(Given)}$$

$\therefore$ $\Delta ASP \cong \Delta BQP$ (By SAS)

$\therefore$ $SP = PQ$ (By CPCT)...(ii)

Similarly, in $\Delta RDS$ and $\Delta RCQ$,

$$SD = CQ \quad \text{(Given)}$$

$$DR = RC \quad \text{(Given)}$$

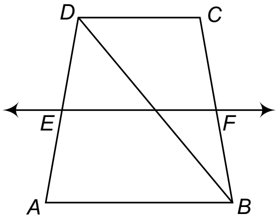
$$\angle C = \angle D \quad \text{(Given)}$$

$\therefore$ $\Delta RDS \cong \Delta RCQ$ (By SAS)

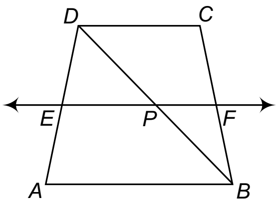
$\therefore$ $SR = RQ$ (By CPCT)...(iii)

From Eqs. (i), (ii) and (iii), it is clear that quadrilateral *PQRS* is a rhombus.

**Question 4.** *ABCD* is a trapezium in which $AB \parallel DC$, *BD* is a diagonal and *E* is the mid-point of *AD*. A line is drawn through *E* parallel to *AB* intersecting *BC* at *F* (see figure). Show that *F* is the mid-point of *BC*.




**Solution** Given, *ABCD* is a trapezium in which $AB \parallel CD$ and *E* is mid-point of *AD* and $EF \parallel AB$.




In $\Delta ABD$, we have

$$EP \parallel AB$$

and *E* is mid-point of *AD*.

So, by theorem, if a line drawn through the mid-point of one side of a triangle parallel to another side bisect the third side.

$\therefore$ *P* is mid-point of *BD*.

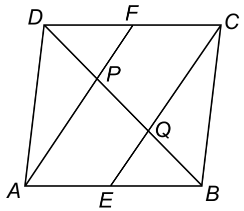
Similarly, in $\Delta BCD$, we have,

$$PF \parallel CD \quad \text{(Given)}$$

and *P* is mid-point of *BD*.

So, by converse of mid-point theorem, *F* is mid-point of *CB*.

**Question 5.** In a parallelogram *ABCD*, *E* and *F* are the mid-points of sides *AB* and *CD* respectively (see figure). Show that the line segments *AF* and *EC* trisect the diagonal *BD*.




**Solution** **Given** *ABCD* is a parallelogram and *E*, *F* are the mid-points of sides *AB* and *CD* respectively.

**To prove** Line segments *AF* and *EC* trisect the diagonal *BD*.

**Proof** Since, *ABCD* is a parallelogram.

$$AB \parallel DC$$

and $AB = DC$ (Opposite sides of a parallelogram)

$\Rightarrow$ $AE \parallel FC$ and $\frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow$ $AE \parallel FC$ and $AE = FC$

$\therefore$ *AECF* is a parallelogram.

$\therefore$ $AF \parallel EC$

$\Rightarrow$ $EQ \parallel AP$ and $FP \parallel CQ$

In $\Delta BAP$, *E* is the mid-point of *AB* and $EQ \parallel AP$, so *Q* is the mid-point of *BP*.

(By converse of mid-point theorem)

$\therefore$ $BQ = PQ$ ...(i)

Again, in $\Delta DQC$, *F* is the mid-point of *DC* and $FP \parallel CQ$, so *P* is the mid-point of *DQ*. (By converse of mid-point theorem)

$\therefore$ $QP = DP$ ...(ii)

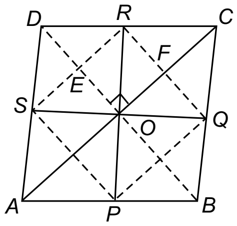
From Eqs. (i) and (ii), we get

$$BQ = PQ = PD$$

Hence, *CE* and *AF* trisect the diagonal *BD*.

**Question 6.** Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

**Solution** Let *ABCD* is a quadrilateral and *P*, *Q*, *R* and *S* are the mid-points of the sides *AB*, *BC*, *CD* and *DA*, respectively. *i.e.*, $AS = SD$, $AP = BP$, $BQ = CQ$ and $CR = DR$. We have to show that *PR* and *SQ* bisect each other *i.e.*, $SO = OQ$ and $PO = OR$.




Now, in $\Delta$ *ADC*, *S* and *R* are mid-points of *AD* and *CD*.

We know that, the line segment joining the mid-points of two sides of a triangle is parallel to the third side. (By mid-point theorem)

$\therefore$ $$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \ldots(i)$$

Similarly, in $\Delta$ *ABC*, *P* and *Q* are mid-points of *AB* and *BC*.

$\therefore$ $$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \text{(By mid-point theorem)}\ldots(ii)$$

From Eqs. (i) and (ii), we get

$$PQ \parallel SR$$

and $$PQ = SR = \frac{1}{2} AC$$

$\therefore$ Quadrilateral *PQRS* is a parallelogram whose diagonals are *SQ* and *PR*. Also, we know that diagonals of a parallelogram bisect each other. So, *SQ* and *PR* bisect each other.

**Question 7.** *ABC* is a triangle right angled at *C*. A line through the mid-point *M* of hypotenuse *AB* and parallel to *BC* intersects *AC* at *D*. Show that

(i) *D* is the mid-point of *AC* (ii) $MD \perp AC$

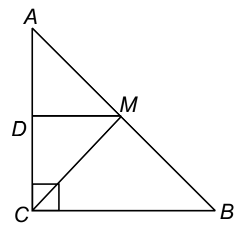
(iii) $CM = MA = \frac{1}{2} AB$

**Solution** Given, *ABC* is a right angled triangle.

$$\angle C = 90°$$

and *M* is the mid-point of *AB*.

Also, $$DM \parallel BC$$




(i) In $\Delta$ *ABC*, *BC* || *MD* and *M* is mid-point of *AB*.

$\therefore$ *D* is the mid-point of *AC*.

(By converse of mid-point theorem)

(ii) Since, $MD \parallel BC$ and $CD$ is transversal.

$\therefore \quad \angle ADM = \angle ACB$ (Corresponding angles)

But $\angle ACB = 90^\circ$

$\therefore \quad \angle ADM = 90^\circ \Rightarrow MD \perp AC$

(iii) Now, in $\Delta ADM$ and $\Delta CDM$, we have

$DM = MD$ (Common)

$AD = CD$ ($\because D$ is mid point of $AC$)

$\angle ADM = \angle MDC$ (Each equal to $90^\circ$)

$\therefore \quad \Delta ADM \cong \Delta CDM$ (By SAS)

$\therefore \quad CM = AM$ (By CPCT)...(i)

Also, $M$ is mid-point of $AB$.

$\therefore \quad AM = BM = \frac{1}{2} AB$ ...(ii)

From Eqs. (i) and (ii), we get

$$CM = AM = \frac{1}{2} AB$$

Hence proved.