1 Real Numbers

Exercise 1.1

Question 1. Use Euclid's division algorithm to find the HCF of

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 255

Solution (i) We start with the larger number 225.

By Euclid's division algorithm, we have

$$\text{Divident} = (\text{Divisor} \times \text{Quotient} + \text{Remainder})$$

$$225 = 135 \times 1 + 90$$

$$\begin{array}{r} 1 \\ 135\overline{)225(} \\ \underline{135} \\ 90 \end{array}$$

We apply Euclid's division algorithm on divisor 135 and the remainder 90.

$$\text{Divident} = (\text{Divisor} \times \text{Quotient} + \text{Remainder})$$

$$\begin{array}{r} 1 \\ 90\overline{)135(} \\ \underline{90} \\ 45 \end{array}$$

$$135 = 90 \times 1 + 45$$

Again, we apply Euclid's division algorithm on divisor 90 and remainder 45.

$$\begin{array}{r} 2 \\ 45\overline{)90(} \\ \underline{90} \\ 0 \end{array}$$

$$90 = 45 \times 2 + 0$$

HCF (135, 225) = 45

(ii) We have,

Division = 38200

and Divisor = 196

$$\text{Divident} = (\text{Divisor} \times \text{Quotient} + \text{Remainder})$$

$$38220 = 196 \times 195 + 0$$

$$\begin{array}{r} 195 \\ 196\overline{)38220} \\ \underline{196} \\ 1862 \\ \underline{1764} \\ 980 \\ \underline{980} \\ \underline{0} \end{array}$$

Hence, HCF (196, 38220) = 196

(iii) By Euclid's division algorithm, we have

$$\text{Divident} = (\text{Divisor} \times \text{Quotient} + \text{Remainder})$$

$$867 = 255 \times 3 + 102$$

$$\begin{array}{r} 3 \\ 255 \overline{)\,867} \\ \underline{765} \\ \underline{102} \end{array}$$

We apply Euclid's division algorithm on the divisor 255 and the remainder 102.

$$\text{Divident} = (\text{Divisor} \times \text{Quotient} + \text{Remainder})$$

$$255 = 102 \times 2 + 51$$

$$\begin{array}{r} 2 \\ 102 \overline{)\,255} \\ \underline{204} \\ \underline{51} \end{array}$$

Again, we apply Euclid's division algorithm on the divisor 102 and the remainder 51.

$$\text{Divident} = (\text{Divisor} \times \text{Quotient} + \text{Remainder})$$

$$102 = 51 \times 2 + 0$$

$$\text{HCF}\,(867, 255) = 51$$

$$\begin{array}{r} 2 \\ 51 \overline{)\,102} \\ \underline{102} \\ \underline{0} \end{array}$$

Question 2. Show that any positive odd integer is of the form $6q + 1$, $6q + 3$ or $6q + 5$, where q is some integer.

Solution By Euclid division algorithm, we have

$$a = bq + r \quad \ldots\text{(i)}$$

$$0 \le r < b$$

On putting, $b = 6$ in Eq. (i), we get

$$a = 6q + r \; [0 \le r < 6 \; i.e., \; r = 0, 1, 2, 3, 4, 5]$$

If $r = 0$, $a = 6q$, $6q$ is divisible by $6 \Rightarrow 6q$ is even.

If $r = 1$, $a = 6q + 1$, $6q + 1$ is not divisible by 2.

If $r = 2$, $a = 6q + 2$, $6q + 2$ is divisible by $2 \Rightarrow 6q + 2$ is even.

If $r = 3$, $a = 6q + 3$, $6q + 3$ is not divisible by 2.

If $r = 4$, $a = 6q + 4$, $6q + 4$ is divisible by $2 \Rightarrow 6q + 4$ is even.

If $r = 5$, $a = 6q + 5$, $6q + 5$ is not divisible by 2.

Since, $6q$, $6q + 2$, $6q + 4$ are even.

Hence, the remaining integers $6q + 1$, $6q + 3$ and $6q + 5$ are odd.

Question 3. An army contingent of 616 members is to march behind an army bond of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution To find the maximum number of columns, we have to find the HCF of 616 and 32. By Euclid's division algorithm

$$\begin{array}{r|l} & 19 \\ \hline 32 & 616 \\ & 32 \\ \hline & 296 \\ & 288 \\ \hline & 8 \end{array} \qquad \begin{array}{r|l} & 4 \\ \hline 8 & 32 \\ & 32 \\ \hline & 0 \end{array}$$

i.e., $616 = 32 \times 19 + 8$ *i.e.*, $32 = 8 \times 4 + 0$

$\therefore$ The HCF of 616 and 32 is 8.

Hence, maximum number of columns is 8.

Question 4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

[**Hint** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now, square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Solution By Euclid's division algorithm, we have $a = bq + r$ ...(i)

On putting $b = 3$ in Eq. (i), we get $a = 3q + r$, $[0 \le r < 3$, *i.e.*, $r = 0, 1, 2]$

If $r = 0 \Rightarrow a = 3q \Rightarrow a^2 = 9q^2$ ...(ii)

If $r = 1 \Rightarrow a = 3q + 1 \Rightarrow a^2 = 9q^2 + 6q + 1$ ...(iii)

If $r = 2 \Rightarrow a = 3q + 2 \Rightarrow a^2 = 9q^2 + 12q + 4$ ...(iv)

From Eq. (ii), $9q^2$ is a square of the form $3m$, where $m = 3q^2$

From Eq. (iii), $9q^2 + 6q + 1$ *i.e.*, $3(3q^2 + 2q) + 1$ is a square which is of the form $3m + 1$, where $m = 3q^2 + 2q$

From Eq. (iv) $9q^2 + 12q + 4$ *i.e.*, $3(3q^2 + 4q + 1) + 1$ is a square which is of the form $3m + 1$, where $m = 3q^2 + 4q + 1$.

$\therefore$ The square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

Question 5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Solution Let q be any positive integer. Then, it is of the form $3q$, $3q + 1$ or $3q + 2$.

Now, we have to prove that the cube of each of these can be rewritten in the form

$$9m,\ 9m + 1 \text{ or } 9m + 8$$

Now, $(3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 3q^3$

$$(3q + 1)^3 = (3q)^3 + 3(3q)^2 \cdot 1 + 3(3q) \cdot 1^2 + 1$$
$$= 27q^3 + 27q^2 + 9q + 1$$
$$= 9(3q^3 + 3q^2 + q) + 1 = 9m + 1$$

where $m = 3q^3 + 3q^2 + q$

and $(3q + 2)^3 = (3q)^3 + 3(3q)^2 \cdot 2 + 3(3q) \cdot 2^2 + 8$

$$= 27q^3 + 54q^2 + 36q + 8$$
$$= 9(3q^3 + 6q^2 + 4q) + 8$$
$$= 9m + 8,$$

where $m = 3q^3 + 6q^2 + 4q$

1 Real Numbers

Exercise 1.2

Question 1. Express each number as a product of its prime factors

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Solution (i) We use division method as shown below

$$\begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore \quad 140 = 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$

(ii) We use the division method as shown below

$$\begin{array}{r|l} 2 & 156 \\ \hline 2 & 78 \\ \hline 3 & 39 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$\therefore \quad 156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) We use the division method as shown below

$$\begin{array}{r|l} 3 & 3825 \\ \hline 3 & 1275 \\ \hline 5 & 425 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$\therefore \quad 3825 = 3 \times 3 \times 5 \times 5 \times 17$

$= 3^2 \times 5^2 \times 17$

(iv) We use the division method as shown below

$$\begin{array}{r|l} 5 & 5005 \\ \hline 7 & 1001 \\ \hline 11 & 143 \\ \hline 13 & 11 \\ \hline & 1 \end{array}$$

$\therefore \quad 5005 = 5 \times 7 \times 11 \times 13$

(v) We use the division method as shown below

$$\begin{array}{r|l} 17 & 7429 \\ \hline 19 & 437 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$\therefore \quad 7429 = 17 \times 19 \times 23$

Question 2. Find the LCM and HCF of the following pairs of integers and verify that LCM $\times$ HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92

(iii) 336 and 54

Solution (i) 26 and 91

$$\begin{array}{r|l} 2 & 26 \\ \hline 13 & 13 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$26 = 2 \times 13 \text{ and } 91 = 7 \times 13$$

$\therefore$ LCM of 26 and 91 $= 2 \times 7 \times 13 = 182$

and HCF of 26 and 91 = 13

Now, $182 \times 13 = 2366$ and $26 \times 91 = 2366$

Hence, $182 \times 13 = 26 \times 91$

(ii) 510 and 92

$$\begin{array}{r|l} 2 & 510 \\ \hline 3 & 255 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 92 \\ \hline 2 & 46 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$$510 = 2 \times 3 \times 5 \times 17$$

and $92 = 2 \times 2 \times 23$

$\therefore$ LCM of 510 and 92

$$= 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$$

and HCF of 510 and 92 = 2

Now, $23460 \times 2 = 46920$

and $510 \times 92 = 46920$

Hence, $23460 \times 2 = 510 \times 92$

(iii) 336 and 54

$$\begin{array}{r|l} 2 & 336 \\ \hline 2 & 168 \\ \hline 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$$

and $54 = 2 \times 3 \times 3 \times 3$

$\therefore$ LCM of 336 and 54 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$

and HCF of 336 and 54 $= 2 \times 3 = 6$

Now, $3024 \times 6 = 18144$ and $336 \times 54 = 18144$

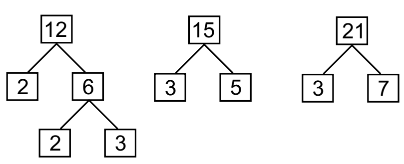
Hence, $3024 \times 6 = 336 \times 54$

Question 3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12,15 and 21 (ii) 17, 23 and 29

(iii) 8, 9 and 25

Solution (i)



$$12 = 2 \times 2 \times 3$$

$$15 = 3 \times 5$$
$$21 = 3 \times 7$$

Here, 3 is common prime factor of the given numbers.

Hence, $\quad \text{HCF } (12, 15, 21) = 3$

LCM is product of the prime factors $2 \times 2 \times 3 \times 7 \times 5$.

The common factor 3 is repeated three times, but 3 in multiplication is written once.

$$\text{LCM } (12, 15, 21) = 420$$

(ii) $\boxed{17}\ \boxed{23}\ \boxed{29}$

There is no common factor as 17, 23, 29 they are primes. Hence, HCF is 1.

LCM is the product of all prime factor 17, 23 and 29.

$$17 \times 23 \times 29 = 11339$$

Hence, HCF (17, 23, 29) = 1, LCM (17, 23, 29) = 11339

(iii) First we write the prime factorisation of each of the given numbers.

$8 = 2 \times 2 \times 2 = 2^3$, $9 = 3 \times 3 = 3^2$, $25 = 5 \times 5 = 5^2$

$\therefore \quad \text{LCM} = 2^3 \times 3^2 \times 5^2$

$\quad = 8 \times 9 \times 25 = 1800$

and $\quad \text{HCF} = 1$

Question 4. Given that HCF (306,657) = 9, find LCM (306, 657).

Solution We have, HCF (306, 657) = 9

We know that,

Product of LCM and HCF = Product of two numbers

$\Rightarrow \quad \text{LCM} \times 9 = 306 \times 657$

$\Rightarrow \quad \text{LCM} = \dfrac{306 \times 657}{9} = 22338$

Hence, $\quad$ LCM (306, 657) = 22338

Question 5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution If the number 6^n ends with the digits zero. Then, it is divisible by 5. Therefore, the prime factorisation of 6^n contains the prime 5. This is not possible because the only primes in the factorisation of 6^n are 2 and 3 and the uniqueness of the fundamental theorem of arithmetic guarantees that there are no other prime in the factorisation of 6^n.

So, there is no value of n in natural numbers for which 6^n ends with the digit zero.

Question 6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers?

Solution We have, $7 \times 11 \times 13 + 13 = 1001 + 13 = 1014$

$$1014 = 2 \times 3 \times 13 \times 13$$

So, it is the product of more than two prime numbers. 2, 3 and 13.

Hence, it is a composite number.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 2040 + 5 = 5045$$

$$\Rightarrow \qquad 5045 = 5 \times 1009$$

It is the product of prime factor 5 and 1009.

Hence, it is a composite number.

Question 7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution They will be again at the starting point atleast common multiples of 18 and 12 min. To find the LCM of 18 and 12, we have

$$18 = 2 \times 3 \times 3 \text{ and } 12 = 2 \times 2 \times 3$$

$$\text{LCM of 18 and 12} = 2 \times 2 \times 3 \times 3 = 36$$

So, Sonia and Ravi will meet again at the starting point after 36 min.

1 Real Numbers

Exercise 1.3

Question 1. Prove that $\sqrt{5}$ is irrational.

Solution Suppose, $\sqrt{5}$ represent a rational number. Then, $\sqrt{5}$ can expressed in the form $\frac{p}{q}$, where p, q are integer and have no common factor, $q \neq 0$. $\sqrt{5} = \frac{p}{q}$

On squaring both sides, we get

$$5 = \frac{p^2}{q^2} \Rightarrow p^2 = 5q^2 \qquad \ldots\text{(i)}$$

$\Rightarrow$ 5 divides $p^2 \Rightarrow$ 5 divides p (Concept)…(ii)

Let $p = 5m \Rightarrow p^2 = 25m^2$

On putting the value of p^2 in Eq. (i), we get $25m^2 = 5q^2 \Rightarrow 5m^2 = q^2$

$\Rightarrow$ 5 divides $q^2 \Rightarrow$ 5 divides q (Concept)…(iii)

Thus, from Eq. (ii), 5 divides p and from Eq. (iii), 5 also divides q. It means 5 is a common factor of p and q. This contradicts the supposition so there is no common factor of p and q.

Hence, $\sqrt{5}$ is an irrational number.

Hence proved.

Question 2. Prove that $3 + 2\sqrt{5}$ is irrational.

Solution Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is a rational number.

Now, let $3 + 2\sqrt{5} = \frac{a}{b}$, where a and b are coprime and $b \neq 0$.

So, $$2\sqrt{5} = \frac{a}{b} - 3$$

or $$\sqrt{5} = \frac{a}{2b} - \frac{3}{2}$$

Since, a and b are integer, therefore $\frac{a}{2b} - \frac{3}{2}$ is a rational number.

$\therefore \sqrt{5}$ is an rational number.

But $\sqrt{5}$ is an irrational number.

This show that our assumption is incorrect.

So, $3 + 2\sqrt{5}$ is an irrational number.

Hence proved.

Question 3. Prove that the following are irrationals

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Solution (i) Let us assume, to the contrary, that $\frac{1}{\sqrt{2}}$ is rational. That is, we can find coprime integers p and q $(q \neq 0)$.

Such that $$\frac{1}{\sqrt{2}} = \frac{p}{q} \text{ or } \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{p}{q}$$

or $$\frac{\sqrt{2}}{2} = \frac{p}{q} \text{ or } \sqrt{2} = \frac{2p}{q}$$

Since, p and q are integers $\frac{2p}{q}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $\frac{1}{\sqrt{2}}$ is an irrational.

Hence proved.

(ii) Let us assume, to the contrary, that $7\sqrt{5}$ is rational. That is, we can find coprime integers p and q $(q \neq 0)$ such that $7\sqrt{5} = \frac{p}{q}$.

So, $$\sqrt{5} = \frac{p}{7q}$$

Since, p and q are integers, $\frac{p}{7q}$ is rational and so is $\sqrt{5}$.

But this contradicts the fact $\sqrt{5}$ is irrational. So, we conclude that $7\sqrt{5}$ is an irrational.

Hence proved.

(iii) Let us assume, to the contrary, that $6 + \sqrt{2}$ is rational. That is, we can find integers p and q $(q \neq 0)$ such that

$$6 + \sqrt{2} = \frac{p}{q} \text{ or } \frac{p}{q} - 6 = \sqrt{2}$$

or $$\sqrt{2} = \frac{p}{q} - 6$$

Since, p and q are integers, we get $\frac{p - 6q}{q}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $6 + \sqrt{2}$ is an irrational.

Hence proved.

1 Real Numbers

Exercise 1.4

Question 1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$ (v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Solution (i) Since, the factors of the denominator 3125 are $2^0 \times 5^5$. Therefore, $\frac{13}{3125}$ is a terminating decimal.

(ii) Since, the factors of the denominator 8 are $2^3 \times 5^0$. So, $\frac{17}{8}$ is a terminating decimal.

(iii) Since, the factors of the denominator 455 is not in the form $2^n \times 5^m$. So, $\frac{64}{455}$ is non-terminating repeating decimal.

(iv) Since, the factors of the denominator 1600 are $2^6 \times 5^2$. So, $\frac{15}{1600}$ is a terminating decimal.

(v) Since, the factors of the denominator 343 is not of the form $2^n \times 5^m$. So, it is non-terminating repeating decimal.

(vi) Since, the denominator is of the form $2^3 \times 5^2$. So, $\frac{23}{2^3 \times 5^2}$ is a terminating decimal.

(vii) Since, the factors of the denominator $2^2 5^7 7^5$ is not of the form $2^n \times 5^m$. So, $\frac{129}{2^2 5^7 7^5}$ is non-terminating repeating decimal.

(viii) $\frac{6}{15} = \frac{2}{5}$ here the factors of the denominator 5 is of the form $2^0 \times 5^1$. So, $\frac{6}{15}$ is a terminating decimal.

(ix) Since, the factors of the denominator 50 is of the form $2^1 \times 5^2$. So, $\frac{35}{50}$ is terminating decimal.

(x) Since, the factors of the denominator 210 is not of the form $2^n \times 5^m$. So, $\frac{77}{210}$ is non-terminating repeating decimal.

Question 2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Solution (i) $\frac{13}{3125} = \frac{13}{5\times5\times5\times5\times5} = \frac{13\times2\times2\times2\times2\times2}{5\times2\times5\times2\times5\times2\times5\times2\times5\times2}$

$$= \frac{13\times32}{10\times10\times10\times10\times10} = \frac{416}{100000} = 0.00046$$

(ii) $\frac{17}{8} = \frac{17\times5^3}{2^3\times5^3} = \frac{17\times5^3}{10^3} = \frac{2125}{1000} = 2.125$

(iii) Non-terminating repeating.

(iv) $\frac{15}{1600} = \frac{15}{2^6\times5^2} = \frac{15}{2^4\times2^2\times5^2} = \frac{15}{2^4\times10^2}$

$$= \frac{15\times5^4}{2^4\times5^4\times10^2} = \frac{15\times625}{10^4\times10^2} = \frac{9375}{1000000} = 0.009375$$

(v) Non-terminating repeating.

(vi) $\frac{23}{2^3\cdot5^2} = \frac{23}{2\cdot2^2\cdot5^2} = \frac{23}{2\cdot10^2} = \frac{23\times5}{2\times5\times10^2} = \frac{115}{10\times10^2} = \frac{115}{1000} = 0.115$

(vii) Non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2}{5} = \frac{4}{10} = 0.4$

(ix) $\frac{35}{50} = \frac{35\times2}{50\times2} = \frac{70}{100} = 0.70$

(x) Non-terminating repeating.

Question 3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000…

(iii) $43.\overline{123456789}$

Solution (i) 43.123456789 is terminating. So, it represents a rational number.

Thus, $$43.123456789 = \frac{43123456789}{1000000000} = \frac{p}{q}$$

Thus, $$q = 10^9$$

(ii) 0.120120012000120000… is non-terminating and non-repeating. So, it is an irrational.

(iii) $43.\overline{123456789}$ is non-terminating but repeating. So it is a rational.

Thus, $$43.\overline{123456789} = \frac{43123456789 - 43}{999999999}$$

$$= \frac{4312345646}{999999999} = \frac{p}{q}$$

Thus $$q = 999999999$$