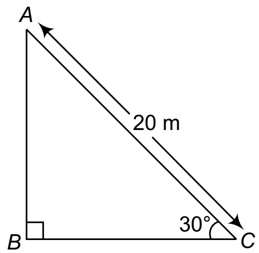
11 Some Applications of Trigonometry

Exercise 11.1

Question 1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°(see figure).



Solution In the given figure, AB be the height of the pole and $AC = 20$ m be the length of rope which is tied from the top of the pole.

We have to determine the height $AB = ?$

In ΔABC,

$$\sin 30° = \frac{AB}{AC} = \frac{AB}{20}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB}{20}$$

$$\Rightarrow \quad AB = \frac{20}{2} = 10\,\text{m}$$

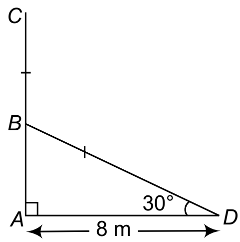
Hence, the height of the pole be 10 m.

Question 2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solution Let the initial height of the tree be AC. When the storm cone, the tree broke from point B. The broken part of the tree BC touches the ground at point D, making an angle 30° on the ground.

Also, given $AD = 8$ m

In right ΔABD,



$$\tan 30° = \frac{AB}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$\Rightarrow \quad AB = \frac{8}{\sqrt{3}}\text{ m}$$

Again in ΔABD,

$$\cos 30^\circ = \frac{AD}{BD} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{BD}$$

$$\Rightarrow \quad BD = \frac{16}{\sqrt{3}}$$

$$\therefore \quad AC = AB + BC = AB + BD \qquad (\because BC = BD)$$

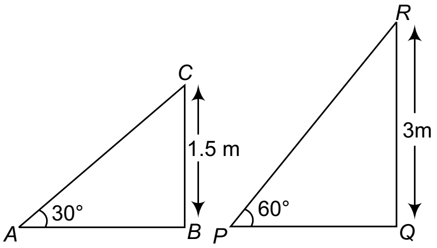
$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

Hence, the height of the tree is $8\sqrt{3}$ m.

Question 3. A contractor plans to install two slides for the children to play in a park, for the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Solution (i) In Fig. (a), it is the slide for the children below the age of 5 years.

Let $BC = 1.5$ m be the height of the slides and slide AC is inclined at $\angle CAB = 30^\circ$ to the ground.



(a) (b)

In right angled ΔABC,

$$\sin 30^\circ = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{AC} \Rightarrow AC = 3\text{m}$$

(ii) In Fig. (b), it is the slide for the elder children. Let $PQ = 3$ m be the height of the slides and slides PR is inclined at an $\angle RPQ = 60^\circ$ to the ground.

In right angled ΔQPR,

$$\sin 60^\circ = \frac{RQ}{PR} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{PR}$$

$$\Rightarrow \quad PR = \frac{3 \times 2}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

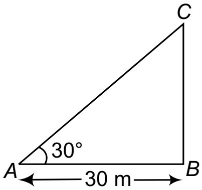
Hence, length of the slides in each case are 3 m and $2\sqrt{3}$ m.

Question 4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Solution Let BC be the height of the tower which is standing on the ground AB. Let A be a point on the ground which makes an elevation of the top of the tower.

Also, $AB = 30\text{m}$

In right angled ΔABC,



$$\tan 30° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BC}{30}$$

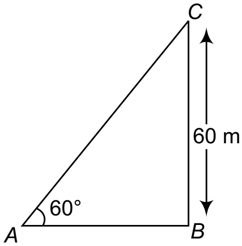
$$\Rightarrow \quad BC = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Hence, height of the tower is $10\sqrt{3}$ m.

Question 5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution Let C be the portion of the kite. AC be the length of the string which makes, an angle of 60° on the ground. The height of the kite on the ground is $BC = 60\,\text{m}$.

In right angled ΔABC,



$$\sin 60° = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$$\Rightarrow \quad AC = \frac{60 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{120\sqrt{3}}{3} = 40\sqrt{3} \text{ m}$$

Hence, length of the string is $40\sqrt{3}$ m.

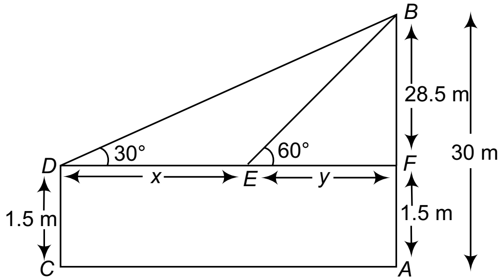
Question 6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution Let $AB = 30$ m be the height of the building, $DC = 1.5$ m be the length of the man. The point D be the man eyes. The angle of elevations are $\angle BDF = 30°$ and $\angle BEF = 60°$.

Let $DE = x$ and $EF = y$

Now, $BF = AB - AF$

$= 30 - 1.5 = 28.5$



In right angled ΔBDF,

$$\tan 30° = \frac{BF}{DF}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{28.5}{x + y}$$

$$\Rightarrow \quad x + y = 28.5\sqrt{3} \quad \ldots(i)$$

Again, in right angled ΔBEF,

$$\tan 60° = \frac{BF}{EF}$$

$$\Rightarrow \quad \sqrt{3} = \frac{28.5}{y}$$

$$\Rightarrow \quad y = \frac{28.5}{\sqrt{3}} \text{ m} \quad \ldots(ii)$$

Putting $y = \frac{28.5}{\sqrt{3}}$ in Eq. (i), we get

$$x + \frac{28.5}{\sqrt{3}} = 28.5\sqrt{3}$$

$$\Rightarrow \quad x = 28.5\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 28.5\left(\frac{3-1}{\sqrt{3}}\right)$$

$$= \frac{28.5 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

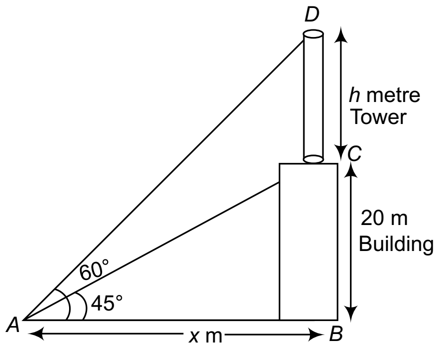
$$= \frac{57\sqrt{3}}{3} = 19\sqrt{3} \text{ m}$$

Hence, the distance between two elevations is $19\sqrt{3}$ m.

Question 7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Solution Let $BC = 20$ m be the height of the building and $DC = h$ metre be height of the tower, which is standing on the building. A be a fixed point on the ground. From a fixed point A, the angles of elevation of the bottom and the top of the transmission tower are

$$\angle BAC = 45° \text{ and } \angle BAD = 60°$$



Also, let $AB = x$ m

In right angled ΔABC,

$$\tan 45° = \frac{BC}{AB} \Rightarrow 1 = \frac{20}{x} \Rightarrow x = 20 \text{ m} \quad \ldots\text{(i)}$$

Again, in right angled ΔABD,

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20 + h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20 + h}{20} \quad \text{[From Eq. (i), } x = 20 \text{ m]}$$

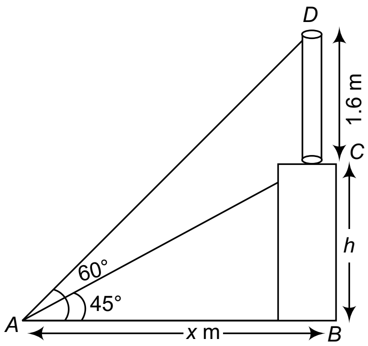
$$\Rightarrow \quad 20 + h = 20\sqrt{3} \quad \Rightarrow \quad h = 20(\sqrt{3} - 1) \text{ m}$$

Hence, the height of the tower is $20(\sqrt{3} - 1)$ m.

Question 8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Solution Let $BC = h$ metre be the height of the pedestal and $CD = 1.6$ m be the length of the statue, which is standing on the pedestal. Point A be a fixed point on the ground. From the point A, the angle of elevations of the top of the statue and bottom of the statue are

$$\angle DAB = 60° \text{ and } \angle CAB = 45°$$



Also, let $AB = x$ metre

In right angled ΔBAD, $\tan 60° = \dfrac{BD}{AB}$

$$\Rightarrow \quad \sqrt{3} = \frac{CD + CB}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1.6 + h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}\,x - 1.6 \qquad \text{...(i)}$$

In right ΔCAB

$$\tan 45° = \frac{BC}{AB} \Rightarrow 1 = \frac{h}{x}$$

$$\Rightarrow \quad x = h$$

Putting $x = h$ in Eq. (i), we get

$$h = \sqrt{3}\,h - 1.6$$

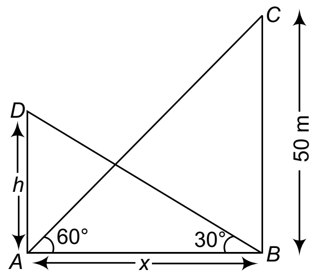
$$\Rightarrow \quad h(\sqrt{3} - 1) = 1.6$$

$$\Rightarrow \quad h = \frac{1.6}{(\sqrt{3} - 1)} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{1.6}{2}(\sqrt{3} + 1) = 0.8(\sqrt{3} + 1)\,\text{m}$$

Hence, the length of the pedestal is $0.8(\sqrt{3} + 1)$ m.

Question 9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.



Solution Let $BC = 50$ m be the height of the tower and $AD = h$ metre be the height of the building, angle of elevation, from the bottom of building and tower as well are

$$\angle BAC = 60° \text{ and } \angle ABD = 30°$$

Also, let $AB = x$ be the distance between foot of the tower and building.
In right angled ΔABD,

$$\tan 30° = \frac{AD}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad h = \frac{x}{\sqrt{3}} \quad \ldots\text{(i)}$$

Again, in right angled ΔBAC,

$$\tan 60° = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{50}{x}$$

$$\Rightarrow \quad x = \frac{50}{\sqrt{3}} \text{ m}$$

Putting $x = \frac{50}{\sqrt{3}}$ in Eq. (i), we get

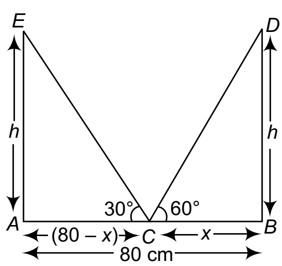
$$h = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{50}{3} = 16\frac{2}{3} \text{ m}$$

Hence, the height of the building is $16\frac{2}{3}$ m.

Question 10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

Solution Let $AB = 80$ m be the width of the road. On both sides of the road poles $AE = BD = h$ metre are standing. Let C be any point on AB such that point C makes an elevations are $\angle BCD = 60°$ and $\angle ACE = 30°$.



Let $BC = x$, then $AC = AB - BC = (80 - x)$ m
In right angled ΔACE,

$$\tan 30° = \frac{AE}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80 - x}$$

$$\Rightarrow \quad 80 - x = h\sqrt{3}$$

$$\Rightarrow \quad h\sqrt{3} + x = 80 \quad \ldots\text{(i)}$$

Again, in right angled ΔBCD,

$$\tan 60° = \frac{BD}{BC}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x \qquad \text{...(ii)}$$

Putting $h = \sqrt{3}\,x$ in Eq. (i), we get

$$\sqrt{3}x \times (\sqrt{3}) + x = 80$$

$$\Rightarrow \qquad 3x + x = 80$$

$$\Rightarrow \qquad 4x = 80$$

$$\Rightarrow \qquad x = 20\,\text{m}$$

Putting $x = 20\,\text{m}$ in Eq. (ii), we get

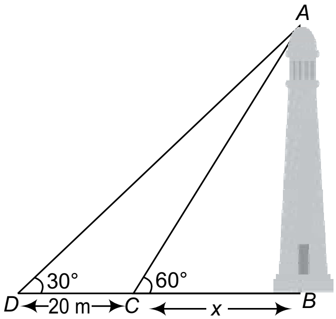
$$h = 20\sqrt{3}\ \text{m}$$

Also, $$AC = 80 - x$$

$$= 80 - 20 = 60\,\text{m}$$

Hence, height of the poles be $20\sqrt{3}$ m and the distances of the point from the poles are 60 m and 20 m.

Question 11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°(see figure). Find the height of the tower and the width of the canal.



Solution Let $BC = x$ metre be the width of the canal and $AB = h$ metre be the height of the tower.

In right angled ΔADB,

$$\tan 30° = \frac{AB}{DB}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{DC + CB}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{20 + x}$$

$$\Rightarrow \quad 20 + x = \sqrt{3}\,h \quad \text{...(i)}$$

Again in right angled ΔACB,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}x \quad \text{...(ii)}$$

Putting $h = \sqrt{3}x$ in Eq. (i), we get

$$20 + x = \sqrt{3}\,(\sqrt{3}\,x)$$

$$\Rightarrow \quad 20 + x = 3x$$

$$\Rightarrow \quad 2x = 20$$

$$\Rightarrow \quad x = 10\,\text{m}$$

Putting $x = 10$ m in Eq. (ii), we get

$$h = \sqrt{3}\,(10)$$

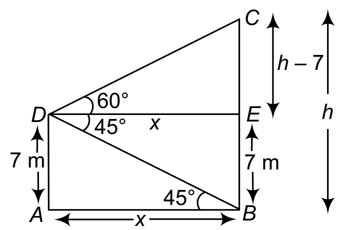
$$\Rightarrow \quad h = 10\sqrt{3}\,\text{m}$$

Hence, the height of the tower is $10\sqrt{3}$ m and width of the canal is 10 m.

Question 12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution Let $AD = 7$ m be the height of the building and $BC = h$ metre be the height of the cable tower. From the top of the building D, the angles of elevation and depression are $\angle CDE = 60^\circ$ and $\angle EDB = 45^\circ$.

From the point D, a perpendicular line DE is drawn on BC.



As $\quad DE \parallel AB$

$$\therefore \quad \angle EDB = \angle ABD = 45^\circ \quad \text{(alternate angle)}$$

Also, let $AB = DE = x$ metre be the distance between building and tower.

In right angled ΔABD,

$$\tan 45^\circ = \frac{AD}{AB} \Rightarrow 1 = \frac{7}{x}$$

$$\Rightarrow \quad x = 7\,\text{m} \quad \text{...(i)}$$

In right angled ΔCDE,

$$\tan 60^\circ = \frac{CE}{DE} \Rightarrow \sqrt{3} = \frac{h-7}{x}$$

$$\Rightarrow \quad h - 7 = x\sqrt{3}$$

$$\Rightarrow \quad h = x\sqrt{3} + 7$$

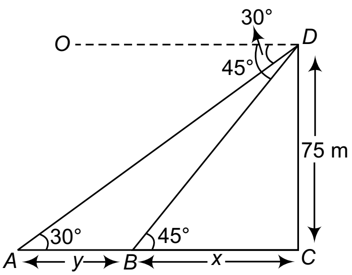
$$\Rightarrow \quad h = 7\sqrt{3} + 7 \quad \text{[From Eq. (i), } x = 7\text{ m]}$$

$$\Rightarrow \quad h = 7\,(\sqrt{3} + 1)\,\text{m}$$

Hence, the height of the tower is $7\,(\sqrt{3} + 1)$ m.

Question 13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Solution Let $CD = 75$ m be the height of the lighthouse from the sea level AC. Let A and B be the position of two ships on the sea-level.



From point D of a lighthouse the angle of depression of two ships A and B are

$\angle ODA = 30°$ and $\angle ODB = 45°$

$\Rightarrow \angle CAD = 30°$ and $\angle CBD = 45°$ (Alternate angle)

Let distance between two ships $AB = y$ metre and $BC = x$ metre

In right angled ΔACD,

$$\tan 30° = \frac{CD}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{AB + BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{y + x} \Rightarrow x + y = 75\sqrt{3} \quad \ldots\text{(i)}$$

In right angled ΔDBC,

$$\tan 45° = \frac{CD}{BC}$$

$$\Rightarrow \quad 1 = \frac{75}{x}$$

$$\Rightarrow \quad x = 75 \text{ m}$$

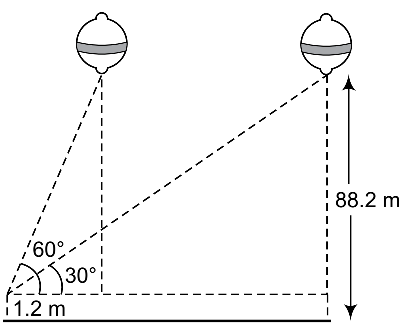
Putting $x = 75$ m in Eq.(i), we get

$$75 + y = 75\sqrt{3}$$

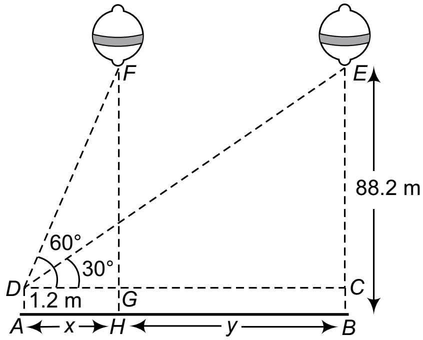
$$\Rightarrow \quad y = 75(\sqrt{3} - 1) \text{ m}$$

Hence, the distance between two ships is $75(\sqrt{3} - 1)$ m.

Question 14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval.



Solution Let $AD = 1.2$ m be the tall girl standing on the horizontal line AB. Let $FH = EB = 88.2$ m be the height of balloon from the line AB. At the eye of the girl D, the angle of elevations are $\angle FDC = 60°$ and $\angle EDC = 30°$.



Now, $$FG = EC = 88.2 - 1.2 = 87 \text{ m}$$

Let the distance travelled by the balloons $HB = y$ metre and $AH = x$ metre .

$\therefore$ $$DG = x \text{ metre and } GC = y \text{ metre}$$

In right angled ΔFDG,

$$\tan 60° = \frac{FG}{DG}$$

$$\Rightarrow \quad \sqrt{3} = \frac{87}{x}$$

$$\Rightarrow \quad x = \frac{87}{\sqrt{3}} \quad \ldots(i)$$

In right angled ΔEDC,

$$\tan 30° = \frac{EC}{DC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{DG + GC}$$

$$\Rightarrow \quad x + y = 87\sqrt{3} \quad \ldots(ii)$$

$\therefore$ From Eq. (i), putting $x = \dfrac{87}{\sqrt{3}}$ in Eq (ii), we get

$$\frac{87}{\sqrt{3}} + y = 87\sqrt{3}$$

$$\Rightarrow \quad y = 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= \frac{87\,(3-1)}{\sqrt{3}}$$

$$= \frac{87 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87 \times 2\sqrt{3}}{3} = 29 \times 2\sqrt{3}$$

$$= 58\sqrt{3} \text{ m}$$

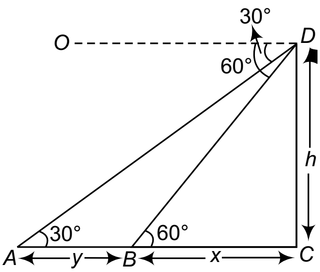
Hence, the distance between two balloons are $58\sqrt{3}$ m.

Question 15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Solution Let $CD = h$ metre be the height of the tower. At point D a man is standing on the tower and observe that a car at an angle of depression of 30°. After six second the angle of depression of the car is 60°.

i.e., $\angle ODA = 30^\circ$ and $\angle ODB = 60^\circ$

$\Rightarrow$ $\angle DAC = 30^\circ$ and $\angle DBC = 60^\circ$ (Alternate angle)



Let $AB = y$ and $BC = x$

In right angled ΔBCD,

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}\,x$$

In right angled ΔACD,

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x + y}$$

$$\Rightarrow \quad x + y = h\sqrt{3}$$

$$\Rightarrow \quad x + y = \sqrt{3}x\,(\sqrt{3})$$

$$\Rightarrow \quad x + y = 3x \qquad \ldots\text{(i)}$$

It is given, car moves from point A to B in six seconds. Let its speed be k km/s.

$$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow \quad 6 = \frac{y}{k}$$

$$\Rightarrow \quad y = 6\,k$$

$\therefore$ From Eq. (i), we get

$$x + 6k = 3x$$

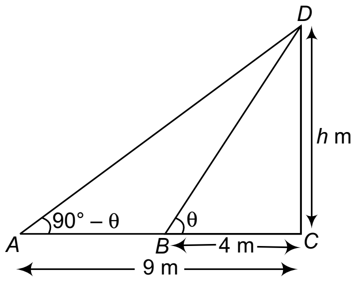
$$\Rightarrow \quad 6k = 2x$$

$$\Rightarrow \quad x = 3k$$

Hence, the car moves from point B to C in 3 s.

Question 16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution Let $CD = h$ metre be the height of the tower. AC be a horizontal line on a ground. A and B be the two points on a line at a distance of 9 m and 4 m from the base of the tower.



Let $\angle CBD = \theta$, then $\angle CAD = 90° - \theta$

(The complementary means the sum of two angles are 90°)

In right angled ΔCAD,

$$\tan(90° - \theta) = \frac{CD}{AC}$$

$$\Rightarrow \quad \cot\theta = \frac{h}{9} \quad \ldots(i)$$

And in right angled ΔCBD,

$$\tan\theta = \frac{CD}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{4} \quad \ldots(ii)$$

On multiplying Eqs. (i) and (ii), we get

$$\cot\theta \times \tan\theta = \frac{h}{9} \times \frac{h}{4}$$

$$\Rightarrow \quad 1 = \frac{h^2}{36}$$

$$\Rightarrow \quad h^2 = 36 \Rightarrow h = 6\,\text{m}$$

Hence, the height of the tower is 6 m.

Hence proved.