# 13 Surface Areas and Volumes

## Exercise 13.1

**Question 1.** A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring 1 $m^2$ costs ₹ 20.

**Solution** We have a plastic box of

$$l = \text{length} = 1.5 \text{ m}$$
$$b = \text{width} = 1.25 \text{ m}$$
$$h = \text{depth} = 65 \text{ cm}$$
$$= \frac{65}{100} \text{ m} = 0.65 \text{ m} \qquad (\because 1 \text{ m} = 100 \text{ cm})$$

Surface area of the box $= 2(lb + bh + hl)$

$$= 2(1.5 \times 1.25 + 1.25 \times 0.65 + 0.65 \times 1.5)$$
$$= 2(1.875 + 0.8125 + 0.975) = 2\,(3.6625)$$
$$= 7.325 \text{ m}^2$$

(i) Area of the sheet required for making the box

$$= 7.325 - l \times b \qquad (\because \text{Box is opened at the top})$$
$$= 7.325 - 1.5 \times 1.25 = 7.325 - 1.875 = 5.45 \text{ m}^2$$

(ii) A sheet measuring 1 $m^2$ costs = ₹ 20

$\therefore$ Sheet measuring 5.45 $m^2$ costs = ₹ 20 × 5.45 = ₹ 109

**Question 2.** The length, breadth and height of a room are 5 m, 4 m and 3 m, respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 7.50 per $m^2$.

**Solution** We have a room of $l = 5$m

$$b = 4\text{m}$$
$$h = 3\text{m}$$

Required area for white washing

$$= \text{Area of the four walls} + \text{Area of ceiling}$$
$$= 2(l + b) \times h + (l \times b)$$
$$= 2(5 + 4) \times 3 + (5 \times 4)$$
$$= 2 \times 9 \times 3 + 20$$
$$= 54 + 20$$
$$= 74 \text{ m}^2$$

White washing 1 $m^2$ costs = ₹ 7.50

White washing 74 $m^2$ costs = ₹ 7.50 × 74 = ₹ 555

**Question 3.** The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹10 per $m^2$ is ₹ 15000, find the height of the hall.

[**Hint** Area of the four walls = Lateral surface area]

**Solution** Let the rectangular hall of length = $l$, breadth = $b$, height = $h$

$$\text{Now, Area of four walls} = \frac{\text{Cost of painting the four walls}}{\text{Cost of painting per m}^2}$$

$$= \frac{₹15000}{₹10}$$

$$= 1500 \text{ m}^2$$

We have, perimeter of the hall = $2(l + b) = 250$ m

$\because$ Area of four walls = $1500 \text{ m}^2$ (Given)

$\therefore$ $2(l + b) \times h = 1500$

$\Rightarrow$ $250 \times h = 1500$

$\Rightarrow$ $h = \frac{1500}{250}$

$\Rightarrow$ $h = 6$ m

Hence, the height of the hall is 6 m.

**Question 4.** The paint in a certain container is sufficient to paint an area equal to 9.375 $m^2$. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container.

**Solution** Given, dimensions of a brick

$l = 22.5$ cm, $b = 10$ cm

and $h = 7.5$ cm

$$\text{Total surface area of bricks} = 2(l \times b + b \times h + h \times l)$$

$$= 2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)$$

$$= 2(225 + 75 + 168.75)$$

$$= 2 \times 468.75 \text{ cm}^2$$

$$= 937.5 \text{ cm}^2$$

$$= \frac{937.5}{100 \times 100} \text{ m}^2 \qquad (\because 1 \text{ m} = 100 \text{ cm})$$

Number of bricks that painted out of this container

$$= \frac{\text{Total surface area of container}}{\text{Total surface area of bricks}}$$

$$= \frac{9.375}{\frac{937.5}{100 \times 100}}$$

$$= \frac{9.375 \times 100 \times 100}{937.5}$$

$$= \frac{937500}{9375} = 100$$

**Question 5.** A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

**Solution** We have $l_1$ for cubical box = 10 cm

For cuboidal box $l = 12.5$ cm

$b = 10$ cm

$h = 8$ cm

(i) Lateral surface area of cubical box $= 4l^2 = 4(10)^2$

$= 4 \times 100$

$= 400 \text{ cm}^2$

Lateral surface area of cuboidal box $= 4(l + b) \times h$

$= 2(12.5 + 10) \times 8$

$= 2(22.5) \times 8$

$= 45 \times 8 = 360 \text{ cm}^2$

($\because$ Lateral surface area of cuboidal box) > (Lateral surface area of cuboidal box)

($\because 400 > 360$)

$\therefore$ Required area $= (400 - 360) \text{ cm}^2 = 40 \text{ cm}^2$

(ii) Total surface area of cubical box $= 6l^2 = 6(10)^2 = 6 \times 100 = 600 \text{ cm}^2$

Total surface area of cuboidal box $= 2(l \times b + b \times h + h \times l)$

$= 2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5)$

$= 2(125 + 80 + 100)$

$= 2 \times 305$

$= 610 \text{ cm}^2$

$\therefore$ (Area of cuboidal box) > (Area of cubical box) ($\because 610 > 600$)

Required area $= (610 - 600) \text{ cm}^2 = 10 \text{ cm}^2$

**Question 6.** A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 edges?

**Solution** Dimension for herbarium are

$l = 30$ cm, $b = 25$ cm and $h = 25$ cm

Area of the glass $= 2(l \times b + b \times h + h \times l)$

$= 2(30 \times 25 + 25 \times 25 + 25 \times 30)$

$= 2(750 + 625 + 750) = 2(2125) = 4250 \text{ cm}^2$

$\therefore$ Length of the tape $= 4(l + b + h) = 4(30 + 25 + 25)$

[$\because$ Herbarium is a shape of cuboid length $= 4(l + b + h)$]

$= 4 \times 80 = 320$ cm

**Question 7.** Shanti Sweets Stalll was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is ₹ 4 for 1000 $cm^2$, find the cost of cardboard required for supplying 250 boxes of each kind.

**Solution** Dimension for bigger box, $l = 25$ cm, $b = 20$ cm and $h = 5$ cm

Total surface area of the bigger size box

$$= 2\,(l \times b + b \times h + h \times l)$$
$$= 2(25 \times 20 + 20 \times 5 + 5 \times 25)$$
$$= 2(500 + 100 + 125)$$
$$= 2(725) = 1450\ cm^2$$

Dimension for smaller box, $l = 15$ cm, $b = 12$ cm and $h = 5$ cm

Total surface area of the smaller size box $= 2(15 \times 12 + 12 \times 5 + 5 \times 15)$

$$= 2\,(180 + 60 + 75) = 2\,(315) = 630\,cm^2$$

Area for all the overlaps $= 5\% \times 2080\ cm^2 = \frac{5}{100} \times 2080\ cm^2 = 104\ cm^2$

Total surface area of both boxes and area of overlaps

$$= (2080 + 104)\ cm^2 = 2184\,cm^2$$

Total surface area for 250 boxes $= 2184 \times 250\ cm^2$

Cost of the cardboard for 1000 $cm^2$ = ₹4

Costs of the cardboard for 1 $cm^2$ = ₹$\frac{4}{1000}$

Cost of the cardboard for $2184 \times 250\ cm^2$ = ₹$\frac{4 \times 2184 \times 250}{1000}$ = ₹2184

**Question 8.** Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

**Solution** Dimension for shetter, $l = 4$ m, $b = 3$ m and $h = 2.5$ cm

Required area of tarpaulin to make the shelter

= (Area of 4 sides + Area of the top) of the car

$$= 2(l + b) \times h + (l \times b)$$
$$= 2(4 + 3) \times 2.5 + (4 \times 3)$$
$$= (2 \times 7 \times 2.5) + 12 = 35 + 12 = 47\ m^2$$

# 13 Surface Areas and Volumes

## Exercise 13.2

- *Assume* $\pi = \frac{22}{7}$, *unless stated otherwise.*

**Question 1.** The curved surface area of a right circular cylinder of height 14 cm is 88 $cm^2$. Find the diameter of the base of the cylinder.

**Solution** We have, height = 14 cm

Curved surface area of a right circular cylinder = 88 $cm^2$

$$2\pi rh = 88$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 14 = 88$$

$$\Rightarrow \quad r = \frac{88 \times 7}{2 \times 22 \times 14}$$

$$r = 1 \text{ cm}$$

Diameter = 2 × Radius = 2 × 1 = 2 cm

**Question 2.** It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

**Solution** Let $r$ be the radius and $h$ be the height of the cylinder.

Given, $r = \frac{140}{2} = 70 \text{ cm} = 0.70 \text{ m}$

and $h = 1$ m

Metal sheet required to make a closed cylindrical tank

$$= \text{Total surface area}$$

$$= 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 0.7(1 + 0.70)$$

$$= 2 \times 22 \times 0.1 \times 1.70$$

$$= 7.48 \text{ m}^2$$

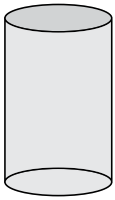
Hence, the sheet required to make a closed cylindrical tank = 7.48 $m^2$

**Question 3.** A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see figure). Find its

(i) inner curved surface area.

(ii) outer curved surface area.

(iii) total surface area.



**Solution** We have, $h = 77$ cm

Outer diameter ($d_1$) = 4.4 cm

and inner diameter $(d_2) = 4$ cm

$\therefore$ Outer radius $(r_1) = 2.2$ cm

Inner radius $(r_2) = 2$ cm

(i) Inner curved surface area $= 2\pi r_2 h = 2 \times \frac{22}{7} \times 2 \times 77 = 88 \times 11$

$= 968\ \text{cm}^2$

(ii) Outer curved surface area $= 2\pi r_1 h$

$$= 2 \times \frac{22}{7} \times 2.2 \times 77$$

$$= 44 \times 2.2 \times 11 = 1064.8\ \text{cm}^2$$

(iii) Total surface area = Inner curved surface area

+ Outer curved surface area + Areas of two bases

$$= 968 + 1064.8 + 2\pi\,(r_1^2 - r^2)$$

$$= 968 + 1064.8 + 2 \times \frac{22}{7}\,[(2.2)^2 - 2]$$

$$= [2032.8 + 2 \times \frac{22}{7}(4.84 - 4)]$$

$$= 2032.8 + \frac{44}{7} \times 0.84 = 2032.8 + 44 \times 0.12$$

$$= 2032.8 + 5.28\ \text{cm}^2 = 2038.08\ \text{cm}^2$$

**Question 4.** The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in $\text{m}^2$.

**Solution** We have, diameter of a roller = 84 cm

$r$ = radius of a roller = 42 cm

$h$ = 120 cm

To cover 1 revolution = Curved surface area of roller

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 42 \times 120$$

$$= 44 \times 720\ \text{cm}^2$$

$$= 31680\ \text{cm}^2$$

$$= \frac{31680}{100 \times 100}\ \text{m}^2 = 3.168\ \text{m}^2 \qquad (\because 1\ \text{m} = 100\ \text{cm})$$

Area of the playground = Takes 500 complete revolutions

$$= 500 \times 3.168\ \text{m}^2 = 1584\ \text{m}^2$$

**Question 5.** A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹ 12.50 per $m^2$.

**Solution** Given, Diameter = 50 cm

$\therefore$ Radius, $r = \frac{50}{2 \times 100}$ m $= 0.25$ m $\quad (\because 1 \text{ cm} = \frac{1}{100} \text{ m})$

and $h = 3.5$ m

Curved surface area of the pillar $= 2\pi rh = 2 \times \frac{22}{7} \times 0.25 \times 3.5$

$= 2 \times 22 \times 0.25 \times 0.5 = 5.5 \text{ m}^2$

Cost of painting per $m^2$ = ₹ 12.50

Cost of painting 5.5 $m^2$ = ₹ $12.50 \times 5.5$ = ₹ 68.75

**Question 6.** Curved surface area of a right circular cylinder is 4.4 $m^2$. If the radius of the base of the cylinder is 0.7 m, find its height.

**Solution** We have, curved surface area of a right circular cylinder = $4.4 \text{m}^2$

$\therefore \quad 2\pi rh = 4.4$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 0.7 \times h = 4.4$

$\Rightarrow \quad h = \frac{44}{44}$

$\Rightarrow \quad h = 1$ m

Hence, the height of the right circular cylinder is 1 m.

**Question 7.** The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

(i) its inner curved surface area.

(ii) the cost of plastering this curved surface at the rate of ₹ 40 per $m^2$.

**Solution** We have, inner diameter = 3.5 m

$\therefore$ inner radius $= \frac{3.5}{2}$ m

and $h = 10$ m

(i) Inner curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10 = 22 \times 5 = 110 \text{ m}^2$

(ii) Cost of plastering per $m^2$ = ₹ 40

Cost of plastering 110 $m^2$ = ₹ $40 \times 110$ = ₹ 4400

**Question 8.** In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

**Solution** We have, $h = 28$ m

Diameter = 5 cm

$\therefore$ Radius, $r = \frac{5}{2}$ cm $= 2.5$ cm $= \frac{2.5}{100}$ m $= 0.025$ m $\quad (\because 1 \text{ cm} = \frac{1}{100} \text{ m})$

Total radiating surface in the system

= Curved surface area of the cylindrical pipe

$= 2\pi rh = 2 \times \frac{22}{7} \times 0.025 \times 28 = 4.4 \text{ m}^2$

**Question 9.** Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) how much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank?

**Solution** (i) We have, diameter = 4.2 m

$\therefore$ Radius, $r = \frac{4.2}{2} = 2.1$ m

and $h = 4.5$m

Curved surface area of a closed cylindrical petrol storage tank

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2.1 \times 4.5$

$= 44 \times 0.3 \times 4.5$

$= 59.4 \text{ m}^2$

(ii) Now, total surface area = Curved surface area + $2\pi r^2$

$= 59.4 + 2 \times \frac{22}{7} \times (2.1)^2$

$= 59.4 + 27.72$

$= 87.12 \text{ m}^2$

Since, $\frac{1}{12}$ of the actual steel used was wasted, therefore the area of the steel which was actually used for making the tank

$= \left(1 - \frac{1}{12}\right)$ of $x$

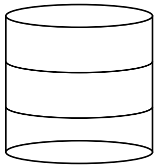
$= \frac{11}{12}$ of $x = \frac{11x}{12}$

$\therefore \quad 87.12 = \frac{11x}{12}$

$\Rightarrow \quad x = \frac{87.12 \times 12}{11} = 1045.44 = 95.04 \text{ m}^2$

$\therefore$ Actually steel used = $95.04 \text{ m}^2$

**Question 10.** In figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



**Solution** Given,
$$r = \frac{20}{2} \text{ cm} = 10 \text{ cm}$$
$$h = 30 \text{ cm}$$

Since, a margin of 2.5 cm is used for folding it over the top and bottom so the total height of frame,
$$h_1 = 30 + 2.5 + 2.5$$
$$h_1 = 35 \text{ cm}$$

$\therefore$ Cloth required for covering the lampshade = Its curved surface area
$$= 2\pi r\,(h_1) = 2 \times \frac{22}{7} \times 10\,(35)$$
$$= \frac{440}{7} \times 35 = 440 \times 5$$
$$= 2200 \text{ cm}^2$$

**Question 11.** The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

**Solution** Cardboard required by each competitor
$$= \text{Base area} + \text{Curved surface area of one penholder}$$
$$= \pi r^2 + 2\pi rh \qquad [\text{Given, } h = 10.5 \text{ cm}, r = 13 \text{ cm}]$$
$$= \frac{22}{7} \times (3)^2 + 2 \times \frac{22}{7} \times 3 \times 10.5$$
$$= (28.28 + 198) \text{ cm}^2$$
$$= 226.28 \text{ cm}^2$$

For 35 competitors cardboard required = $35 \times 226.28 = 7920 \text{ cm}^2$

Hence, $7920 \text{ cm}^2$ of cardboard was required to be bought for the competition.

# 13 Surface Areas and Volumes

## Exercise 13.3

- *Assume $\pi = \frac{22}{7}$, unless stated otherwise.*

**Question 1.** Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

**Solution** We have, diameter = 10.5 cm

$$\text{Radius } (r) = \frac{10.5}{2} = 5.25 \text{ cm}$$

and slant height $l = 10$ cm

$$\text{Curved surface area} = \pi r l = \frac{22}{7} \times 5.25 \times 10 = 165 \text{ cm}^2$$

**Question 2.** Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

**Solution** We have, slant height $l = 21$ m

and diameter = 24 cm

$\therefore$ $$r = \frac{24}{2} = 12 \text{ m}$$

$$\begin{aligned} \text{Total surface area} &= \pi r(l + r) \\ &= \pi \times 12(21 + 12) \\ &= \pi \times 12 \times 33 \\ &= 396\pi \\ &= 396 \times \frac{22}{7} \\ &= \frac{8712}{7} = 1244\frac{4}{7} \text{ m}^2 \end{aligned}$$

**Question 3.** Curved surface area of a cone is 308 $cm^2$ and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

**Solution** We have, slant height, $l = 14$ cm

Curved surface area of a cone = 308 $cm^2$

$\therefore$ $$\pi r l = 308$$

$\Rightarrow$ $$\frac{22}{7} \times r \times 14 = 308$$

$\Rightarrow$ $$r = \frac{308}{22 \times 2}$$

$\Rightarrow$ $$r = 7 \text{ cm}$$

(i) Radius = 7 cm

(ii) Total surface area of a cone $= \pi r(r + l) = \frac{22}{7} \times 7(7 + 14)$

$$= 22 \times 21$$
$$= 462 \text{ cm}^2$$

**Question 4.** A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 $m^2$ canvas is ₹ 70.

**Solution** We have, $h = 10 \text{ m}$

$$r = 24 \text{ m}$$

(i) $\because$ $l = \sqrt{r^2 + h^2}$

$\therefore$ $l = \sqrt{(24)^2 + 10^2}$

$$= \sqrt{576 + 100}$$
$$= \sqrt{676}$$
$$= 26 \text{ m}$$

Hence, the slant height of the canvas tent is 26 m.

(ii) Canvas required to make the tent = Curved surface area of tent

$$= \pi r l$$
$$= \pi \times 24 \times 26 = 624\pi \text{ m}^2$$

$\because$ Cost of 1 $m^2$ canvas = ₹ 70

Cost of $624\pi$ $m^2$ canvas = ₹ $70 \times 624\pi$

$$= ₹\ 70 \times 624 \times \frac{22}{7}$$
$$= ₹\ 10 \times 624 \times 22$$
$$= ₹\ 137280$$

Hence, the cost of the canvas is ₹ 137280.

**Question 5.** What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (Use $\pi = 3.14$)

**Solution** Let $r, h$ and $l$ be the radius, height and slant height of the tent, respectively.

Given, $r = 6 \text{ m}, h = 8 \text{ m}$

$\because$ $l = \sqrt{r^2 + h^2}$

$\therefore$ $l = \sqrt{6^2 + 8^2}$

$$= \sqrt{36 + 64}$$
$$= \sqrt{100} = 10 \text{ m}$$

Area of the canvas used for the tent

= Curve surface area of the cone

$$= \pi r l$$

$$= 3.14 \times 6 \times 10$$
$$= 188.4 \text{ m}^2$$

$\therefore$ Length of tarpaulin required $= \dfrac{\text{Area of tarpaulin required}}{\text{Width of tarpaulin}}$

$$= \frac{188.4}{3} \quad (\because \text{Width of tarpaulin} = 3, \text{ given})$$
$$= 62.8$$

The extra material required for stitching margins and cutting = 20 cm = 0.2 m

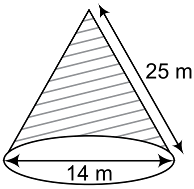
Hence, the total length of tarpaulin required = 62.8 + 0.2 = 63 m

**Question 6.** The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹210 per 100 $m^2$.

**Solution** We have, slant height, $l = 25$ m

and diameter = 14 m

$\therefore$ Radius, $r = 7$ m




Curved surface area of the conical tomb $= \pi r l = \dfrac{22}{7} \times 7 \times 25$

$$= 22 \times 25 = 550 \text{ m}^2$$

Cost of white washing per 100 $m^2$ = ₹ 210

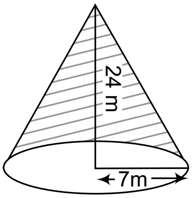
Cost of white washing per 1 $m^2$ = ₹ $\dfrac{210}{100}$

Cost of white washing 550 $m^2$ = ₹ $\dfrac{210 \times 550}{100}$

= ₹1155

**Question 7.** A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

**Solution** We have, $r = 7$ cm, $h = 24$ cm




We know that, $l = \sqrt{h^2 + r^2}$

$\therefore$ $l = \sqrt{(24)^2 + 7^2} = \sqrt{576 + 49} = \sqrt{625}$

$l = 25$ cm

Curved surface area of joker's cap $= \pi r l = \dfrac{22}{7} \times 7 \times 25$

$$= 22 \times 25 = 550 \text{ cm}^2$$

$\because$ The sheet required to make 1 cap = 550 $cm^2$

$\therefore$ The sheet required to make 10 caps = 550 × 10 = 5500 $cm^2$

**Question 8.** A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per $m^2$, what will be the cost of painting all these cones? (Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

**Solution** We have, diameter = 40 cm

$\therefore$ Radius, $r = \dfrac{40}{2} = 20$ cm

$= \dfrac{20}{100}$ m $= 0.2$ m $\quad \left(\because 1 \text{ cm} = \dfrac{1}{100} \text{ m}\right)$

and height = 1 m

$\because \quad l = \sqrt{r^2 + h^2}$

$\therefore \quad l = \sqrt{(0.2)^2 + 1^2} = \sqrt{0.04 + 1}$

$= \sqrt{1.04} = 1.02$ m

Curved surface area of a cone $= \pi r l = 3.14 \times 0.2 \times 1.02 = 0.64056 \text{ m}^2$

Cost of painting per $m^2$ = ₹12

Cost of painting $0.64056 \text{ m}^2$ = ₹12 × 0.64056 = ₹ 7.68672

Cost of painting for 1 cone = ₹ 7.68672

Cost of painting 50 cones = ₹ 7.68672 × 50 = ₹ 384.336 = ₹ 384.34

# 13 Surface Areas and Volumes

## Exercise 13.4

**Question 1.** Find the surface area of a sphere of radius
(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

**Solution** (i) We have, $r = 10.5$ cm

$$\text{Surface area of a sphere} = 4\pi r^2$$
$$= 4 \times \frac{22}{7} \times 10.5 \times 10.5$$
$$= 88 \times 1.5 \times 10.5$$
$$= 1386 \text{ cm}^2$$

(ii) We have, $r$ = radius of the sphere = 5.6 cm

$$\therefore \quad \text{Surface area} = 4\pi r^2$$
$$= 4 \times \frac{22}{7} \times 5.6 \times 5.6 = 88 \times 0.8 \times 5.6$$
$$= 394.24 \text{ cm}^2$$

(iii) We have, $r$ = radius of the sphere = 14 cm

$$\therefore \quad \text{Surface area} = 4\pi r^2$$
$$= 4 \times \frac{22}{7} \times 14 \times 14$$
$$= 88 \times 2 \times 14 = 2464 \text{ cm}^2$$

**Question 2.** Find the surface area of a sphere of diameter
(i) 14 cm (ii) 21 cm (iii) 3.5 m

**Solution** (i) We have, diameter = 14 cm

$$\therefore \quad \text{Radius, } r = 7 \text{ cm}$$

$$\text{Surface area of a sphere} = 4\pi r^2 = 4 \times \frac{22}{7} \times 7^2 = 4 \times 22 \times 7 = 616 \text{ cm}^2$$

(ii) We have, diameter = 21 cm

$$r = \frac{21}{2} = 10.5 \text{ cm}$$

$$\therefore \quad \text{Surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 10.5 \times 10.5$$
$$= 88 \times 1.5 \times 10.5$$
$$= 1386 \text{ cm}^2$$

(iii) We have, diameter = 3.5 m

$$\therefore \quad r = \frac{3.5}{2} = 1.75 \text{ cm}$$

$$\therefore \quad \text{Surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 1.75 \times 1.75 = \frac{269.50}{7}$$
$$= 38.5 \text{ cm}^2$$

**Question 3.** Find the total surface area of a hemisphere of radius 10 cm. (Use $\pi = 3.14$)

**Solution** We have, $r = 10$ cm

Total surface area of a hemisphere $= 3\pi r^2$

$$= 3 \times 3.14 \times (10)^2$$
$$= 9.42 \times 100$$
$$= 942 \text{ cm}^2$$

**Question 4.** The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

**Solution** Let initial radius, $r_1 = 7$ cm

After increases, $r_2 = 14$ cm

Surface area for initial balloon $= 4\pi r_1^2 = 4 \times \frac{22}{7} \times 7 \times 7 = 88 \times 7$

$$A_1 = 616 \text{ cm}^2$$

Surface area for increasing balloon $= 4\pi r_2^2 = 4 \times \frac{22}{7} \times 14 \times 14 = 88 \times 28$

$$A_2 = 2464 \text{ cm}^2$$

$\therefore$ Required ratio $= A_1 : A_2 = 616 : 2464 = 1 : 4$

**Question 5.** A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹ 16 per 100 cm$^2$.

**Solution** We have, inner diameter = 10.5 cm

Inner radius $= \frac{10.5}{2} \text{ cm} = 5.25 \text{ cm}$

Curved surface area of hemispherical bowl of inner side $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times (5.25)^2$$
$$= 2 \times \frac{22}{7} \times 5.25 \times 5.25 = 173.25 \text{ cm}^2$$

$\because$ Cost of tin-plating on inside for 100 cm$^2$ = ₹ 16

$\because$ Cost of tin-plating on the inside for 173.25 cm$^2$ = ₹ $\frac{16 \times 173.25}{100}$ = ₹ 27.72

**Question 6.** Find the radius of a sphere whose surface area is 154 cm$^2$.

**Solution** Surface area of a sphere = 154 cm$^2$

$\therefore$ $4\pi r^2 = 154$

$\Rightarrow$ $4 \times \frac{22}{7} \times r^2 = 154$

$$\Rightarrow \quad r^2 = \frac{154 \times 7}{22 \times 4} = 12.25$$

$$\Rightarrow \quad r = \sqrt{12.25}$$

$$\Rightarrow \quad r = 3.5 \text{ cm}$$

Hence, the radius of the sphere is 3.5 cm.

**Question 7.** The diameter of the Moon is approximately one-fourth of the diameter of the Earth. Find the ratio of their surface areas.

**Solution** Let diameter of the Earth = $d_1$

Then, diameter of the Moon $= \frac{1}{4} d_1$

$\therefore$ Radius of the Earth, $r_1 = \frac{d_1}{2}$

Radius of the Moon, $r_2 = \frac{d_1}{2 \times 4} = \frac{d_1}{8}$

Surface area of the Earth $= 4\pi r_1^2$

$$S_1 = 4\pi \left(\frac{d_1}{2}\right)^2 = \pi d_1^2$$

Surface area of the Moon $= 4\pi \left(\frac{d_1}{8}\right)^2 = 4\pi \frac{d_1^2}{64}$

$$S_2 = \frac{\pi d_1^2}{16}$$

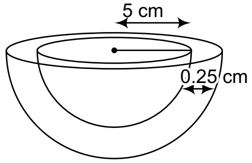
Hence, $S_1 : S_2 = \frac{\pi d_1^2}{1} : \frac{\pi d_1^2}{16}$

$$\Rightarrow \quad S_1 : S_2 = 16 : 1$$

**Question 8.** A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

**Solution** Outer radius of the bowl = (Inner radius + Thickness)

$= (5 + 0.25) \text{ cm} = 5.25 \text{ cm}$




Outer curved surface area of the bowl $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$

$= 173.25 \text{ cm}^2$

**Question 9.** A right circular cylinder just encloses a sphere of radius $r$ (see figure). Find

(i) surface area of the sphere,

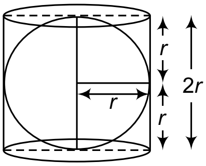
(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).

**Solution** The radius of the sphere $= r$

Radius of the cylinder = Radius of the sphere $= r$

Height of the cylinder = Diameter $= 2r$




(i) Surface area of the sphere $A_1 = 4\pi r^2$

(ii) Curved surface area of the cylinder $= 2\pi rh$

$$A_2 = 2\pi \times r \times 2r$$
$$A_2 = 4\pi r^2$$

(iii) Required ratio $= A_1 : A_2 = 4\pi r^2 : 4\pi r^2 = 1 : 1$

# 13 Surface Areas and Volumes

## Exercise 13.5

**Question 1.** A matchbox measures $4\,cm \times 2.5\,cm \times 1.5\,cm$. What will be the volume of a packet containing 12 such boxes?

**Solution** Volume of a match box $= 4\,cm \times 2.5\,cm \times 1.5\,cm = 15\,cm^3$

Volume of a packet $= 12 \times 15\,cm^3 = 180\,cm^3$

**Question 2.** A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? $(1\,m^3 = 1000\,L)$

**Solution** Volume of a cuboidal water tank $= 6\,m \times 5\,m \times 4.5\,m$

$= 30 \times 4.5\,m^3 = 135\,m^3$

$= 135 \times 1000\,L = 135000\,L$ $\quad (\because 1\,m^3 = 1000\,L)$

**Question 3.** A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

**Solution** Volume of a cuboidal vessel = hold liquid

$\therefore \quad l \times b \times h = 380\,m^3$

$\Rightarrow \quad 10 \times 8 \times h = 380$

$\Rightarrow \quad h = \frac{380}{80}$

$\Rightarrow \quad h = 4.75\,m$

Hence, the cuboidal vessel must be made 4.75 m high.

**Question 4.** Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹ 30 per $m^3$.

**Solution** Volume of a cuboidal pit $= l \times b \times h = (8 \times 6 \times 3)\,m^3 = 144\,m^3$

$\because$ Cost of digging per $m^3$ = ₹ 30

$\therefore$ Cost of digging $144\,m^3$ = ₹ $30 \times 144$ = ₹ 4320

**Question 5.** The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are 2.5 m and 10 m, respectively.

**Solution** Given, $l = 2.5\,m$ and $h = 10\,m$.

Let breadth of tank be $b$.

Capacity of a cuboidal tank $= 50000\,L = \frac{50000}{1000}\,m^3 = 50\,m^3$ $\quad \left(\because 1\,L = \frac{1}{1000}\,m^3\right)$

$\because \quad$ Volume of a cuboidal tank $= 50\,m^3$ $\quad$ (Given)

$\therefore \quad l \times b \times h = 50$

$\Rightarrow \quad 2.5 \times b \times 10 = 50$

$\Rightarrow \quad b = \frac{50}{25}$

$\Rightarrow$ $b = 2\text{ m}$

Hence, breadth of the cuboidal tank is 2 m.

**Question 6.** A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

**Solution** Given,

$$l = 20\text{ m}, b = 15\text{ m and } h = 6\text{ m}$$

$\therefore$ Capacity of the tank = Volume of the tank = $lbh$

$$= (20 \times 15 \times 6)\text{ m}^3$$

$$= 1800\text{ m}^3$$

$\because$ Water requirement per person per day = 150 L

Water required for 4000 persons per day = (4000 × 150) L

$$= \left(\frac{4000 \times 150}{1000}\right)\text{m}^3 \qquad \left(\because 1\text{ L} = \frac{1}{1000}\text{ m}\right)$$

$$= 600\text{ m}^3$$

$\therefore$ Number of days the water will last $= \dfrac{\text{Capacity of tank}}{\text{Total water required per day}}$

$$= \frac{1800}{600} = 30$$

Hence, the water will last for 30 days.

**Question 7.** A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

**Solution** Dimension for godown, $l = 40\text{ m}$, $b = 25\text{ m}$ and $c = 10\text{ m}$

Volume of the godown = $l \times b \times h = 40\text{ m} \times 25\text{ m} \times 10\text{ m}$

Dimension for wooden gate $l = 1.5\text{ m}$, $b = 1.25\text{ m}$, $b = 1.25\text{ m}$, $h = 0.5\text{ m}$

Volume of wooden gates = $l \times b \times h = 1.5\text{ m} \times 1.25\text{ m} \times 0.5\text{m}$

Number of wooden gates $= \dfrac{\text{Volume of the godown}}{\text{Volume of wooden gates}}$

$$= \frac{40\text{ m} \times 25\text{ m} \times 10\text{ m}}{1.5\text{ m} \times 1.25\text{ m} \times 0.5\text{ m}}$$

$$= \frac{10000}{0.9375} = 10666$$

**Question 8.** A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

**Solution** Volume of a solid cube $= 12\text{ cm} \times 12\text{ cm} \times 12\text{ cm} = 1728\text{ cm}^3$

The solid cube is cut into eight cubes of equal volume.

Hence, the volume of the new cube $= \dfrac{1728}{8}\text{ cm}^3 = 216\text{ cm}^3$

$$(\text{side})^3 = 216\text{cm}^3 \qquad \text{(Taking cube root)}$$

$$\text{side} = 6\text{cm}$$

Hence, side of the new cube $= 6$ cm

$$\text{Surface area of solid cube} = 6\,(\text{side})^2 = 6\,(12)^2\text{ cm}^2$$

$$S_1 = 6 \times 144 = 864\text{ cm}^2$$

$$\text{Surface area of new cube} = 6\,(\text{side})^2 = 6(6^2)\text{cm}^2$$

$$S_2 = 216\text{cm}^2$$

$\therefore$ Required ratio $= S_1 : S_2$

$= 864 : 216$

$= 4 : 1$

**Question 9.** A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

**Solution** Given, $l = 2\text{ km} = 2 \times 1000\text{ m} = 2000\text{ m}$

$$b = 40\text{ m}$$

and $h = 3\text{ m}$

Since, the water flows at the rate of 2 km $\text{h}^{-1}$, *i.e.,* the water from 2 km of river flows into the sea in one hour.

The volume of water flowing into the sea in one hour

$= \text{Volume of the cuboid}$

$= l \times b \times h = (2000 \times 40 \times 3)\text{ m}^3$

$= 240000\text{ m}^3$

$\therefore$ The volume of water flowing into the sea in one minute

$= \dfrac{240000}{60}\text{ m}^3$

$= 4000\text{ m}^3$

# 13 Surface Areas and Volumes

## Exercise 13.6

- *Assume* $\pi = \frac{22}{7}$, *unless stated otherwise.*

**Question 1.** The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? ($1000\,\text{cm}^3 = 1\,\text{L}$.)

**Solution** We have, circumference of the base = 132 cm

$\therefore \quad 2\pi r = 132$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$

$$r = \frac{132 \times 7}{22 \times 2}$$

$$r = 21\,\text{cm}$$

$$\text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 34650\ \text{cm}^3$$

$$\left(\text{Given, } 1000\ \text{cm}^3 = 1\ \text{L},\ 1\ \text{cm}^3 = \frac{1}{1000}\ \text{L}\right)$$

$$= 34650\ \text{cm}^3 = \frac{34650}{1000}\ \text{L}$$

$$= 34.65\ \text{L}$$

**Question 2.** The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 $\text{cm}^3$ of wood has a mass of 0.6 g.

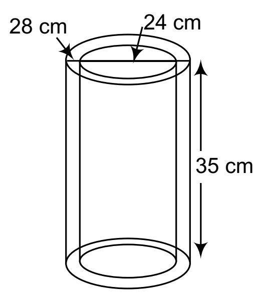
**Solution** Given, inner diameter = 24 cm

Inner radius $(r_1) = 12\,\text{cm}$

Outer diameter = 28 cm

Outer radius $(r_2) = 14\,\text{cm}$

$h = 35\,\text{cm}$



$$\text{Volume of cylindrical wooden pipe} = \pi(r_2^2 - r_1^2)h$$
$$= \frac{22}{7}(14^2 - 12^2) \times 35$$
$$= \frac{22}{7}(14 + 12)(14 - 12) \times 35$$
$$= 22 \times 26 \times 2 \times 5 = 5720 \text{ cm}^3$$

$\because$ 1 $cm^3$ of wood has a mass = 0.6 g

$\therefore$ 5720 $cm^3$ of wood have a mass $= 0.6 \times 5720 \text{ g} = 3432 \text{ g}$

$$= \frac{3432}{1000} \text{ kg} \qquad (\because 1 \text{ kg} = 1000 \text{ g})$$
$$= 3.432 \text{ kg}$$

**Question 3.** A soft drink is available in two packs

(i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm.

(ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

**Solution** (i) We have, $h = 15 \text{ cm}$

$$l = 5 \text{ cm}, \ b = 4 \text{ cm}, \ h = 15 \text{ cm}$$

$$\text{Volume of cuboidical body} = l \times b \times h = 5 \times 4 \times 15 = 300 \text{ cm}^3 \qquad \ldots\text{(i)}$$

(ii) We have, Diameter = 7 cm

$$\text{Radius, } r = \frac{7}{2} \text{ cm}$$

$$\text{Height, } h = 10 \text{ cm}$$

$$\text{Then, volume of a plastic cylinder} = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10$$
$$= 77 \times 5 = 385 \text{ cm}^3 \qquad \ldots\text{(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we see that volume of a plastic cylinder has greater capacity and its capacity is $385 - 300 = 85 \text{ cm}^3$ is more than the tin can.

**Question 4.** If the lateral surface of a cylinder is 94.2 $cm^2$ and its height is 5 cm, then find

(i) radius of its base,

(ii) its volume. (Use $\pi = 3.14$)

**Solution** We have, lateral surface of a cylinder = 94.2 $cm^2$ and $h = 5$ cm

$\therefore$ $2\pi rh = 94.2$

$\Rightarrow$ $2 \times 3.14 \times r \times 5 = 94.2$

$\Rightarrow$ $r = \frac{94.2}{31.4}$

$\Rightarrow$ $r = 3 \text{ cm}$

(i) Hence, radius of base, $r = 3$ cm

(ii) Volume of a cylinder $= \pi r^2 h = 3.14(3)^2 \times 5$

$$= 3.14 \times 9 \times 5$$
$$= 141.3 \text{ cm}^3$$

**Question 5.** It costs ₹ 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹ 20 per $m^2$, find

(i) inner curved surface area of the vessel,
(ii) radius of the base,
(iii) capacity of the vessel.

**Solution** We have, cost to paint the inner curved surface = ₹ 2200

Cost to paint per $m^2$ = ₹ 20

$\because$ Inner curved surface area = $\dfrac{\text{Cost to paint the inner curved surface}}{\text{Cost to paint per m}^2}$

$$2\pi rh = \frac{₹\,2200}{₹\,20} = 110\,m^2$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 10 = 110$$

$$r = \frac{110 \times 7}{2 \times 220}$$

$$r = \frac{7}{4} = 1.75\ m$$

(i) Inner curved surface area of the vessel = $110\,m^2$
(ii) Hence, radius of the base is 1.75 m.
(iii) Capacity of the vessel = Volume of the vessel

$$= \pi r^2 h$$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 10 = \frac{77}{8}$$

$$= 96.25\ m^3$$

**Question 6.** The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

**Solution** Capacity of a closed cylindrical vessel = 15.4 L

$$= 15.4 \times 1000\,cm^3 \qquad (\because 1L = 1000\ cm^2)$$

$$\therefore \quad \pi r^2 h = 15400\ cm^3$$

$$\pi r^2 \times 100 = 15400$$

$$r^2 = 154 \times \frac{7}{22} \qquad (\because h = 1\,m = 100\,cm)$$

$$\Rightarrow \quad r^2 = 7 \times 7$$

$$\Rightarrow \quad r = 7\,cm$$

Total surface area of closed cylindrical vessel = $2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 7(7 + 100)$$

$$= 44 \times 107 = 4708\,cm^2$$

The metal sheet must be required = $\dfrac{4708}{100 \times 100}\ m^2 \qquad (\because 1\,cm = \dfrac{1}{100}\,m)$

$$= 0.4708\ m^2$$

**Question 7.** A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

**Solution** Diameter of the graphite cylinder $= 1 \text{ mm} = \frac{1}{10} \text{ cm} \quad \left(\because 1\text{mm} = \frac{1}{10} \text{ cm}\right)$

$$\Rightarrow \qquad \text{Radius, } r = \frac{1}{20} \text{ cm}$$

Length of the graphite, $h = 14 \text{ cm}$

$$\text{Volume of the graphite cylinder} = \pi r^2 h = \left(\frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14\right) \text{cm}^3$$
$$= 0.11 \text{cm}^3$$

Also, $\quad$ diameter of the pencil $= 7 \text{ mm} = \frac{7}{10} \text{ cm}$

$\therefore \quad$ Radius of the pencil, $r = \frac{7}{20} \text{ cm}$

and $\quad$ length of the pencil, $h = 14 \text{ cm}$

$$\therefore \qquad \text{Volume of the pencil} = \pi r^2 h = \left(\frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14\right) \text{cm}^3 = 5.39 \text{ m}^3$$

Hence, volume of wood = Volume of the pencil − Volume of the graphite

$$= (5.39 - 0.11) \text{ cm}^3$$
$$= 5.28 \text{ cm}^3$$

**Question 8.** A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

**Solution** We have, diameter $= 7 \text{ cm}$

$$\text{Radius, } r = \frac{7}{2} \text{ cm}$$
$$h = 4 \text{ cm}$$

$$\text{Capacity of the cylindrical bowl} = \text{Volume of the cylinder} = \pi r^2 h$$
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4$$
$$= 22 \times 7 = 154 \text{ cm}^3$$

Volume of soup to be prepared daily to serve 250 patients

$$= 154 \times 250 \text{ cm}^3 = 38500 \text{ cm}^3$$
$$= \frac{38500}{1000} \qquad \left(\because 1 \text{ cm}^3 = \frac{1}{1000} \text{ L}\right)$$
$$= 38.5 \text{ L}$$

# 13 Surface Areas and Volumes

## Exercise 13.7

- *Assume* $\pi = \frac{22}{7}$, *unless stated otherwise.*

**Question 1.** Find the volume of the right circular cone with
(i) radius 6 cm, height 7 cm
(ii) radius 3.5 cm, height 12 cm

**Solution** (i) We have, $r = 6$ cm and $h = 7$ cm

Then, volume of the right circular cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 6^2 \times 7$

$$= \frac{22}{3} \times 6 \times 6$$
$$= 22 \times 12$$
$$= 264 \text{ cm}^3$$

(ii) Given, $r = 3.5$ cm and $h = 12$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12\right) \text{cm}^3$$
$$= (22 \times 0.5 \times 3.5 \times 4) \text{ cm}^3$$
$$= 154 \text{ cm}^3$$

**Question 2.** Find the capacity in litres of a conical vessel with
(i) radius 7 cm, slant height 25 cm
(ii) height 12 cm, slant height 13 cm

**Solution** (i) We have, $r = 7$ cm and $l = 25$ cm

We know that $l^2 = r^2 + h^2$

$\Rightarrow$ $h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2}$

$$= \sqrt{625 - 49} = \sqrt{576} = 24 \text{ cm}$$

Capacity of conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$
$$= 22 \times 7 \times 8 = 1232 \text{ cm}^3$$
$$= \frac{1232}{1000} \text{L} = 1.232 \text{ L} \qquad \left(\because 1 \text{ cm}^3 = \frac{1}{1000} \text{L}\right)$$

(ii) We have, $h = 12 \text{ cm}$ and $l = 13 \text{ cm}$

We know that, $l^2 = h^2 + r^2$

$$\Rightarrow \quad r = \sqrt{l^2 - h^2} = \sqrt{13^2 - 12^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = 5 \text{ cm}$$

Capacity of conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12$$

$$= \frac{22 \times 100}{7} \text{ cm}^3 = \frac{22 \times 100}{7 \times 1000} \text{L} = 0.314 \text{L}$$

$$\left(\because 1 \text{ cm}^3 = \frac{1}{1000} \text{L}\right)$$

**Question 3.** The height of a cone is 15 cm. If its volume is 1570 $\text{cm}^3$, find the radius of the base. (Use $\pi = 3.14$)

**Solution** We have, volume of a cone = 1570 $\text{cm}^3$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 1570$$

$$\Rightarrow \quad \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$$

$$\therefore \quad r^2 = \frac{1570}{3.14 \times 5} = \frac{15700}{157}$$

$$r = 10 \text{ cm}$$

Hence, radius of the base = 10 cm

**Question 4.** If the volume of a right circular cone of height 9 cm is $48\pi \text{ cm}^3$, find the diameter of its base.

**Solution** We have, volume of a right circular cone = $48\pi \text{ cm}^3$

$$\therefore \quad \frac{1}{3}\pi r^2 h = 48\pi \Rightarrow \frac{1}{3} r^2 \times 9 = 48 \qquad (\because h = 9 \text{ cm, given})$$

$$\Rightarrow \quad r^2 = 16 \Rightarrow r = 4 \text{ cm}$$

Hence, diameter of the base $= 2r = 2 \times 4 = 8$ cm

**Question 5.** A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

**Solution** We have, diameter = 3.5 m

$\therefore$ radius, $r = \frac{3.5}{2}$ m

and $h = 12$ m

Capacity of a conical pit $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{3.5}{2}\right)^2 \times 12$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 12$$

$$= 22 \times 0.5 \times 3.5$$

$$= 38.5 \text{ m}^3 \qquad (1 \text{ m}^3 = 1 \text{kL})$$

$$= 38.5 \text{ kL}$$

**Question 6.** The volume of a right circular cone is 9856 $cm^3$. If the diameter of the base is 28 cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

**Solution** We have, $d = 28 \text{ cm} \Rightarrow r = 14$ cm

$\because$ Volume of a right circular cone = 9856 $cm^3$

$\therefore$ $\frac{1}{3}\pi r^2 h = 9856$

$\Rightarrow$ $\frac{1}{3} \times \frac{22}{7} \times 14^2 \times h = 9856$

$\Rightarrow$ $h = \frac{9856 \times 3 \times 7}{22 \times 14 \times 14}$

$\Rightarrow$ $h = \frac{448 \times 3}{28} = 48$ cm

(i) Height of the cone = 48 cm

(ii) We have, $h = 48$ cm

and $r = 14$ cm

$\because$ $l = \sqrt{h^2 + r^2}$

$$= \sqrt{48^2 + 14^2}$$

$$= \sqrt{2304 + 196}$$

$$= \sqrt{2500}$$

$\Rightarrow$ $l = 50$ cm

(i) Hence, slant height of the cone is 50 cm.

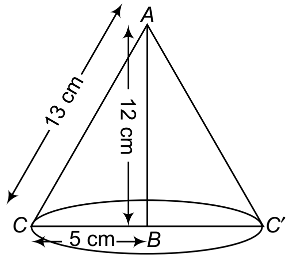
(iii) Curved surface area of the cone $= \pi r l = \frac{22}{7} \times 14 \times 50$

$= 44 \times 50$

$= 2200 \text{ cm}^2$

**Question 7.** A right triangle *ABC* with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

**Solution** On revolving the right $\Delta ABC$ about the side *AB*, we get a cone as shown in the adjoining figure.




$\therefore$ Volume of the solid so obtained $= \frac{1}{3}\pi r^2 h$

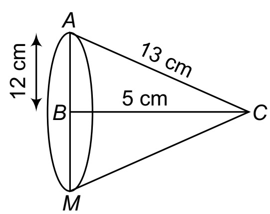
$= \frac{1}{3} \times \pi \times 5 \times 5 \times 12$

$= \pi \times 5 \times 5 \times 4 = 100\pi \text{ cm}^3$

Hence, the volume of the solid so obtained is $100\ \pi\ \text{cm}^2$.

**Question 8.** If the triangle *ABC* in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

**Solution** On revolving the right $\Delta ABC$ about the side $BC(= 5\text{ cm})$, we get a cone as shown in the adjoining figure.




Volume of solid so obtained $V_1 = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 12 \times 12 \times 5$

$= 240\pi \text{ cm}^3$

In above Question 7, the volume $V_2$ obtained by revolving the $\Delta ABC$ about the side 12 cm *i.e.,* $V_2 = 100\pi\,\text{cm}^3$.

Hence, $$\frac{V_2}{V_1} = \frac{100\pi}{240\pi} = \frac{10}{24} = \frac{5}{12}$$

Hence, the required ratio = 5:12

**Question 9.** A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

**Solution** We have, $d = 10.5$ m, $r = \frac{10.5}{2} = 5.25$ m and $h = 3$ m

$$\because \quad l = \sqrt{r^2 + h^2}$$
$$= \sqrt{(5.25)^2 + 3^2}$$
$$= \sqrt{27.5625 + 9}$$
$$= \sqrt{36.5625}$$
$$= 6.046$$
$$= 6.05 \text{ m}$$

Volume of the heap of cone of wheat $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 3$$
$$= 86.625 \text{ m}^2$$

Area of the canvas required = curved surface area of the heap

$$= \pi r l$$
$$= \frac{22}{7} \times 5.25 \times 6.05 \text{ m}^2$$
$$= 22 \times 0.75 \times 6.05 \text{ m}^2$$
$$= 99.825 \text{ m}^2$$

# 13 Surface Areas and Volumes

## Exercise 13.8

- *Assume* $\pi = \frac{22}{7}$, *unless stated otherwise.*

**Question 1.** Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

**Solution** (i) We have, $r = 7$ cm

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (7)^3 = \frac{88 \times 7 \times 49}{3 \times 7}$$
$$= \frac{4312}{3} = 1437\frac{1}{3} \text{ cm}^2$$

(ii) We have, $r = 0.63$ m

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7}(0.63)^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 0.63 \times 0.63 \times 0.63$$
$$= 88 \times 0.9 \times 0.21 \times 0.63$$
$$= 10.478 \text{ m}^3 = 10.48 \text{ m}^3$$

**Question 2.** Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

**Solution** (i) We have, $d = 28$ cm $\Rightarrow r = 14$ cm

$$\text{Water displaced by a solid spherical ball} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$
$$= 11498.66 \text{ cm}^3$$

(ii) Diameter of the spherical ball = 0.21 m

$$\therefore \quad \text{Radius} = \left(\frac{0.21}{2}\right) \text{m} = 0.105 \text{ m}$$

Amount of water displaced by the spherical ball = Volume of sphere

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 0.105 \times 0.105 \times 0.105$$
$$= 0.004851 \text{ m}^3$$

**Question 3.** The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per $cm^3$?

**Solution** We have, $d = 4.2$ cm $\Rightarrow r = 2.1$ cm

$$\text{Volume of the metallic ball} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$

$$= 38.808 \text{ cm}^3$$

The density of the metal per $cm^3$ = 8.9 g

The density of the metal 38.808 $cm^3$ = 38.808 × 8.9 g = 345.39 g

Hence, the mass of the ball = 345.39 g

**Question 4.** The diameter of the Moon is approximately one-fourth of the diameter of the Earth. What fraction of the volume of the Earth is the volume of the Moon?

**Solution** Let diameter of the Earth is $d_2$.

$$\text{We have, diameter of Moon } (d_1) = \frac{1}{4} \text{ diameter of the Earth}$$

$$d_1 = \frac{1}{4} d_2$$

Hence, the radius of the Moon $= \frac{d_1}{2} = \frac{d_2}{8}$

The radius of the Earth $= \frac{d_2}{2}$

$\therefore$ $$\text{Volume of the Earth } (V_1) = \frac{4}{3}\pi\left(\frac{d_2}{2}\right)^3 = \frac{4}{3}\pi \times \frac{d_2^{\ 3}}{8}$$

$\therefore$ $$\text{Volume of the Moon } (V_2) = \frac{4}{3}\pi\left(\frac{d_2}{8}\right)^3$$

$$= \frac{4}{3}\pi \frac{d_2^{\ 3}}{512}$$

$\therefore$ $$\text{Required fraction} = \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi\left(\frac{d_2^3}{8}\right)}{\frac{4}{3}\pi\left(\frac{d_2^3}{512}\right)}$$

$$\frac{V_1}{V_2} = \frac{512}{8}$$

$\Rightarrow$ $$\frac{V_1}{V_2} = \frac{64}{1}$$

**Question 5.** How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

**Solution** Diameter = 10.5 cm

$\Rightarrow$ Radius, $r = 5.25$ cm

$$\text{Volume of a hemisphere} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25$$

$$= 303.1875 \text{ cm}^3$$

$$\text{Hemispherical bowl can hold milk} = \frac{303.1875}{1000} \text{ L} \qquad \left(\because 1 \text{ cm}^3 = \frac{1}{1000} \text{ L}\right)$$

$$= 0.303 \text{ L (Approx.)}$$

**Question 6.** A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

**Solution** Inner radius $(r_1) = 1 \text{ m} = 100 \text{ cm}$

$$\text{Outer radius } (r_2) = (100 + 1) \text{ cm} \qquad (\because \text{Inner radius + Thickness})$$

$$= 101 \text{ cm}$$

$$\text{Outer volume} = \frac{2}{3}\pi(101)^3 \qquad \left(\because \text{Volume} = \frac{4}{3}\pi r^3\right)$$

$$\text{Inner volume} = \frac{2}{3}\pi(100)^3$$

Hence, volume of the iron used to make the tank

$$= (\text{Outer volume} - \text{Inner volume})$$

$$= \frac{2}{3}\pi[(101)^3 - (100)^3]$$

$$= \frac{2}{3} \times \frac{22}{7}(101 - 100)\,[(101)^2 + 101 \times 100 + (100)^2]$$

$$= \frac{2}{3} \times \frac{22}{7}(10201 + 10100 + 10000)$$

$$= \frac{44}{21}(30301)$$

$$= 63487.80 \text{ cm}^3$$

$$= \frac{63487.80}{100 \times 100 \times 100} \text{ m}^3$$

$$= 0.063 \text{ m}^2 \text{(Approx.)}$$

**Question 7.** Find the volume of a sphere whose surface area is 154 $cm^2$.

**Solution** We have, surface area of a sphere = 154 $cm^2$

$$\therefore \quad 4\pi r^2 = 154$$

$$4 \times \frac{22}{7} r^2 = 154$$

$$\Rightarrow \quad r^2 = \frac{154 \times 7}{22 \times 4}$$

$$\Rightarrow \quad r = \frac{7}{2} \text{ cm}$$

Hence, volume of a sphere $= \frac{4}{3} \pi \left(\frac{7}{2}\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$$= \frac{11 \times 7 \times 7}{3}$$

$$= 179.666 \text{ cm}^3 = 179.67 \text{ cm}^3$$

**Question 8.** A dome of a building is in the form of a hemisphere. From inside, it was white washed at the cost of ₹ 498.96. If the cost of white washing is ₹ 2.00 per square metre, find the

(i) inside surface area of the dome,

(ii) volume of the air inside the dome.

**Solution** Cost of white washed = ₹ 498.96

Cost of white washing per square metre = ₹ 2.00

$$\text{Surface area of the dome} = \frac{\text{Cost of white washed}}{\text{Cost of white washing per square metre}}$$

$$= \frac{₹\ 498.96}{₹\ 2.00}$$

$$= 249.48 \text{ sq m}$$

(i) Required surface area = 249.48 sq m

$$\therefore \quad 2\pi r^2 = 249.48$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r^2 = 249.48$$

$$\Rightarrow \quad r^2 = \frac{249.48 \times 7}{44}$$

$$\Rightarrow \quad r^2 = 36.69$$

$$\Rightarrow \quad r = 6.3 \text{ m}$$

(ii) Volume of the air inside the dome $= \frac{2}{3} \pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times (6.3)^3$$

$$= 523.908 \text{ Cum (Approx.)}$$

**Question 9.** Twenty seven solid iron spheres, each of radius $r$ and surface area $S$ are melted to form a sphere with surface area $S'$. Find the

(i) radius $r'$ of the new sphere. (ii) ratio of $S$ and $S'$.

**Solution** (i) Volume of solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of new sphere $= 27 \times \frac{4}{3}\pi r^3$

$$\therefore \quad \frac{4}{3}\pi r'^3 = 36\pi r^3$$

$$r'^3 = \frac{36 \times 3r^3}{4} = 27\,r^3 = (3r)^3$$

$$\Rightarrow \quad r' = 3r$$

Radius $r'$ of the new sphere $= 3r$

(ii) Surface area ($S$) of solid iron sphere $= 4\pi r^2$

Surface area ($S'$) of new sphere $= 4\pi(r')^2$

$$= 4\pi(3r)^2 = 36\pi r^2$$

$$\therefore \quad \text{Required ratio} = S:S' = 4\pi r^2 : 36\pi r^2 = 1:9$$

**Question 10.** A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in $mm^3$) is needed to fill this capsule?

**Solution** We have, diameter = 3.5 mm

$$\Rightarrow \quad \text{Radius, } r = \frac{3.5}{2} \text{ mm}$$

Volume of capsule $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times \frac{3.5}{2} = \frac{11 \times 0.5 \times 3.5 \times 3.5}{3}$$

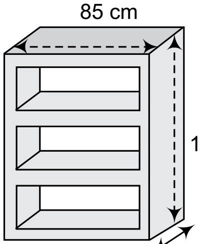
$$= 22.45833$$

$$= 22.46 \text{ mm}^3$$

# 13 Surface Areas and Volumes

## Exercise 13.9 (Optional)

**Question 1.** A wooden bookshelf has external dimensions as follows : Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see figure). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per $cm^2$ and the rate of pointing is 10 paise per $cm^2$, find the total expenses required for palishing and painting the surface of the bookshelf.




**Solution** Here, $l = 85$ cm, $b = 25$ cm and $h = 110$ cm

Area of the bookshelf of outer surface $= 2\,lb + 2bh + hl$

$= [2(85 \times 25) + 2(110 \times 25) + 85 \times 110]\ cm^2$

$= (4250 + 5500 + 9350)\ cm^2$

$= 19100\ cm^2$

Cost of polishing of the outer surface of bookshelf

$$= 19100 \times \frac{20}{100} = ₹\ 3820$$

Thickness of the plank = 5 cm

∴ Internal height of bookshelf = $(110 - 2 \times 5) = 100$ cm

Internal depth of bookshelf = $(25 - 5) = 20$ cm

Internal breadth of bookshelf = $85 - 2 \times 5 = 75$ cm

Hence, area of the internal surface of bookshelf

$= 2(75 \times 20) + 2(100 \times 20) + 75 \times 100$

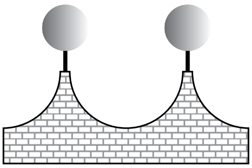
$= 3000 + 4000 + 7500 = 14500\ cm^2$

So, cost of painting of internal surface of bookshelf

$$= 14500 \times \frac{10}{100} = ₹\ 1450$$

Hence, total costing of polishing and painting = $3820 + 1450 =$ ₹ 5270

**Question 2.** The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per $cm^2$ and black paint costs 5 paise per $cm^2$.



**Solution** It is obvious, we have to subtract the cost of the sphere that is resting on the supports while calculating the cost of silver paint.

Surface area to be silver paint

$= 8$ (Curved surface area of the sphere − Area of circle on which sphere is resting)

$= 8\,(4\pi R^2 - \pi r^2)$

$= 8\pi(4R^2 - r^2)$ (Given, $R = \frac{21}{2}$ cm and $r = 1.5$ cm)

$$= 8\pi\left(4 \times \left(\frac{21}{2}\right)^2 - (1.5)^2\right)$$

$$= 8\pi\left(4 \times \frac{441}{4} - 2.25\right)$$

$= 8\pi(438.75)\,\text{cm}^2$

Therefore, the cost of silver paint at the rate of 25 paise per $cm^2$

$$= \left(8 \times \frac{22}{7} \times 438.75 \times \frac{25}{100}\right)$$

$= \frac{19305}{7} =$ ₹ 2757.86 (Approx.)

Hence, surface area to be black painted

$= 8 \times$ Curved surface area of cylinder

$= 8 \times 2\pi rh$

$$= 8 \times 2 \times \frac{22}{7} \times 1.5 \times 7$$

$= 528\,\text{cm}^2$

$\therefore$ Cost of black paint at the rate of 5 paise per $cm^2$ $= \left(528 \times \frac{5}{100}\right)$

= ₹ 26.40

Hence, total costing of painting = 2757.86 + 26.40

= ₹ 2784.26 (Approx.)

**Question 3.** The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

**Solution** Let $d$ be the diameter of the sphere.

$$\therefore \quad \text{Surface area} = 4\pi\left(\frac{d}{2}\right)^2 \qquad \left(\because r = \frac{d}{2}\right)$$

$$= \pi d^2$$

On decreasing its diameter by 25%, the new diameter

$$d_1 = d - \frac{25d}{100} = \frac{75}{100} \times d = \frac{3d}{4}$$

Hence, the new surface area $= 4\pi\left(\frac{d_1}{2}\right)^2 = 4\pi\left(\frac{1}{2} \times \frac{3d}{4}\right)^2$

$$= 4\pi\frac{9d^2}{64} = \frac{\pi d^2 9}{16}$$

$$\text{Decrease in surface area} = \pi d^2\left(1 - \frac{9}{16}\right) = \pi d^2\left(\frac{7}{16}\right)$$

$\therefore$ Percentage decrease in surface area

$$= \left(\pi d^2 \times \frac{7}{16} \times \frac{1}{\pi d^2} \times 100\right)\%$$

$$= \left(\frac{700}{16}\right)\%$$

$$= 43.75\%$$