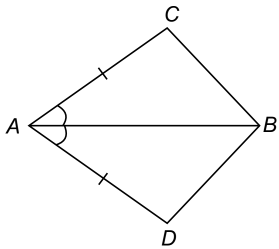
# 5 Triangles

## Exercise 5.1

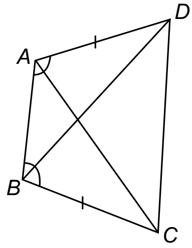
**Question 1.** In quadrilateral $ACBD$, $AC = AD$ and $AB$ bisects $\angle A$ (see figure). Show that $\Delta ABC \cong \Delta ABD$. What can you say about $BC$ and $BD$?




**Solution** In $\Delta ABC$ and $\Delta ABD$, we have

| | | |
|---|---|---|
| | $AC = AD$ | (Given) |
| | $\angle CAB = \angle DAB$ | ($\because$ $AB$ bisects $\angle A$) |
| and | $AB = AB$ | (Common) |
| $\therefore$ | $\Delta ABC \cong \Delta ABD$ | (By SAS congruence axiom) |
| $\therefore$ | $BC = BD$ | (By CPCT) |

**Question 2.** $ABCD$ is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ (see figure). Prove that




(i) $\Delta ABD \cong \Delta BAC$ (ii) $BD = AC$

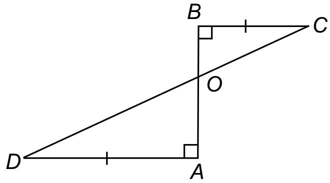
(iii) $\angle ABD = \angle BAC$

**Solution** In $\Delta ABC$ and $\Delta BAC$, we have

| | | |
|---|---|---|
| | $AD = BC$ | (Given) |
| | $\angle DAB = \angle CBA$ | (Given) |
| and | $AB = AB$ | (Common) |
| $\therefore$ | $\Delta ABD \cong \Delta BAC$ | (By SAS congruence axiom) |
| Hence, | $BD = AC$ | (By CPCT) |

and $\angle ABD = \angle BAC$ (By CPCT)

**Question 3.** $AD$ and $BC$ are equal perpendiculars to a line segment $AB$ (see figure). Show that $CD$ bisects $AB$.




**Solution** In $\Delta AOD$ and $\Delta BOC$, we have

$$\angle AOD = \angle BOC$$

$\because$ $AB$ and $CD$ intersects at $O$.

$\therefore$ Which are vertically opposite angle

$\because$ $\angle DAO = \angle CBO = 90°$

and $AD = BC$ (Given)

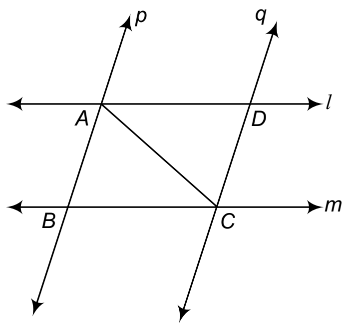
$\therefore$ $\Delta AOD \cong \Delta BOC$ (By AAS congruence axiom)

Hence, $OA = OB$ (By CPCT)

$\Rightarrow O$ is the mid-pont of $AB$.

Hence, $CD$ bisects $AB$.

**Question 4.** $l$ and $m$ are two parallel lines intersected by another pair of parallel lines $p$ and $q$ (see figure). Show that $\Delta ABC \cong \Delta CDA$.




**Solution** From figure, we have

$\angle 1 = \angle 2$ (Vertically opposite angles)...(i)

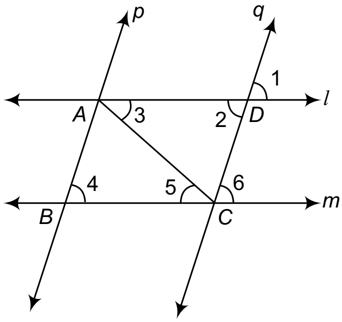
$\angle 1 = \angle 6$ (Corresponding angles)...(ii)

$\angle 6 = \angle 4$ (Corresponding angles) ...(iii)

From Eqs. (i) (ii) and (iii), we have

$$\angle 1 = \angle 4$$

and $\angle 2 = \angle 4$ ...(iv)




In $\Delta$ *ABC* and $\Delta$ *CDA*, we have

$\angle 4 = \angle 2$ [From Eq. (iv)]

$\angle 5 = \angle 3$ (Alternate interior angles)

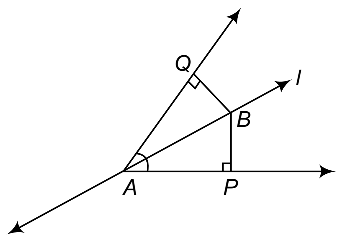
and $AC = AC$ (Common)

$\therefore$ $\Delta ABC \cong \Delta CDA$ (By AAS congruence axiom)

**Question 5.** Line $l$ is the bisector of an $\angle A$ and $\angle B$ is any point on $l$. *BP* and *BQ* are perpendiculars from *B* to the arms of $\angle A$ (see figure). Show that

(i) $\Delta APB \cong \Delta AQB$

(ii) $BP = BQ$ or $B$ is equidistant from the arms of $\angle A$.




**Solution** In $\Delta$ *APB* and $\Delta$ *AQB*, we have

$\angle APB = \angle AQB = 90°$

$\angle PAB = \angle QAB$ ($\because$ *AB* bisects $\angle PAQ$)

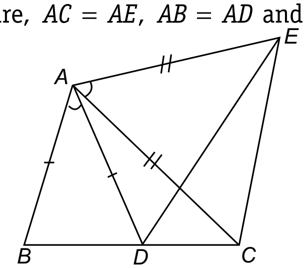
and $AB = AB$ (Common)

$\therefore$ $\Delta APB \cong \Delta AQB$ (By AAS congruence axiom)

$\Rightarrow$ $BP = BQ$ (By CPCT)

$\Rightarrow$ *B* is equidistant from the arms of $\angle A$.

**Question 6.** In figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.



**Solution** In $\Delta ABC$ and $\Delta ADE$, we have

$$AB = AD \quad \text{(Given)}$$

$$\angle BAD = \angle EAC \quad \text{(Given)...(i)}$$

On adding $\angle DAC$ on both sides in Eq. (i)

$\Rightarrow \quad \angle BAD + \angle DAC = \angle EAC + \angle DAC$

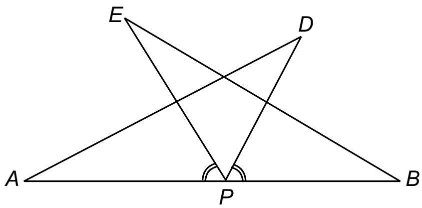
$\Rightarrow \quad \angle BAC = \angle DAE$

and $\quad AC = AE \quad$ (Given)

$\therefore \quad \Delta ABC \cong \Delta ADE \quad$ (By AAS congruence axiom)

$\Rightarrow \quad BC = DE \quad$ (By CPCT)

**Question 7.** $AB$ is a line segment and $P$ is its mid-point. $D$ and $E$ are points on the same side of $AB$ such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. (see figure). Show that




(i) $\Delta DAP \cong \Delta EBP$

(ii) $AD = BE$

**Solution** We have,

$AP = BP \quad [\because P$ is the mid-point of $AB$ (Given)]...(i)

$\angle EPA = \angle DPB \quad$ (Given)...(ii)

$\angle BAD = \angle ABE \quad$ (Given)...(iii)

On adding $\angle EPD$ on both sides in Eq. (ii), we have

$\Rightarrow \quad \angle EPA + \angle EPD = \angle DPB + \angle EPD$

$\Rightarrow \quad \angle DPA = \angle EPB \quad$ ...(iv)

Now, In $\Delta DAP$ and $\Delta EBP$, we have

$\angle DPA = \angle EPB \quad$ [From Eq. (iv)]

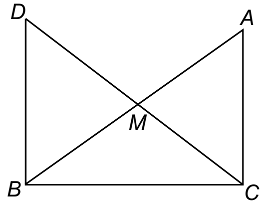
$\angle DAP = \angle EBP \quad$ (Given)

and $\quad AP = BP \quad$ [From Eq. (i)]

$\therefore \quad \Delta DAP \cong \Delta EBP \quad$ (By ASA congruence axiom)

Hence, $\quad AD = BE \quad$ (By CPCT)

**Question 8.** In right triangle $ABC$, right angled at $C$, $M$ is the mid-point of hypotenuse $AB$. $C$ is joined to $M$ and produced to a point $D$ such that $DM = CM$. Point $D$ is joined to point $B$ (see figure). Show that

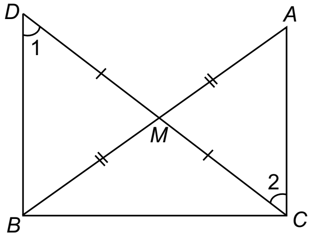



(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle

(iii) $\Delta DBC \cong \Delta ACB$ (iv) $CM = \frac{1}{2} AB$

**Solution** **Given** $\Delta\, ACB$ in which $\angle C = 90°$ and $M$ is the mid-point of $AB$.




**To prove** (i) $\Delta\, AMC \cong \Delta\, BMD$ (ii) $\angle DBC$ is a right angle

(iii) $\Delta\, DBC \cong \Delta\, ACB$ (iv) $CM = \frac{1}{2} AB$

**Construction** Produce $CM$ to $D$, such that $CM = MD$. Join $DB$.

**Proof** In $\Delta\, AMC$ and $\Delta\, BMD$, we have

$AM = BM$ ($M$ is the mid-point of $AB$)

$CM = DM$ (Given)

and $\angle AMC = \angle BMD$ (Vertically opposite angles)

$\therefore$ $\Delta\, AMC \cong \Delta\, BMD$ (By SAS congruence axiom)

$\Rightarrow$ $AC = DB$ (By CPCT) ...(i)

and $\angle 1 = \angle 2$ (By CPCT)

Which are alternate angles

$\therefore$ $BD \parallel CA$

Now, $BD \parallel CA$ and $BC$ is transversal

$\therefore$ $\angle ACB + \angle CBD = 180°$

$\Rightarrow$ $90° + \angle CBD = 180°$

$\Rightarrow$ $\angle CBD = 90°$

$\Rightarrow$ $\angle DBC = 90°$ [Which is part (ii)]

In $\Delta\, DBC$ and $\Delta\, ACB$, we have

$CB = BC$ (Common)

$DB = AC$ [Using part (i)]

and $\angle CBD = \angle BCA$ (Each 90°)

$\therefore$ $\Delta\, DBC \cong \Delta ACB$ (By SSA congruence axiom)

[Which is part (iii)]

$\Rightarrow$ $DC = AB$ (by CPCT)

$\Rightarrow$ $\frac{1}{2} DC = \frac{1}{2} AB$

$\Rightarrow$ $CM = \frac{1}{2} AB$ $\left(\because CM = \frac{1}{2} DC\right)$ [Which is part (iv)]

# 5 Triangles

## Exercise 5.2

**Question 1.** In an isosceles triangle $ABC$, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at $O$. Join $A$ to $O$. Show that

(i) $OB = OC$

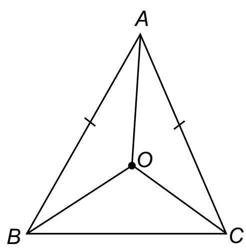
(ii) $AO$ bisects $\angle A$

**Solution** (i) In $\Delta ABC$, we have

$$AB = AC \quad \text{(Given)}$$

$\Rightarrow$ $$\angle B = \angle C$$

(∵ Angles opposite to equal sides are equal)




$\Rightarrow$ $$\frac{1}{2}\angle B = \frac{1}{2}\angle C \quad \text{(Divide by 2 on both sides)}$$

$\Rightarrow$ $$\angle OBC = \angle OCB \quad \ldots\text{(i)}$$

and $$\angle OBA = \angle OCA$$

(∵ $OB$ and $OC$ are bisectors of $\angle B$ and $\angle C$ respectively).

$$\left(\because \angle OBC = \frac{1}{2}\angle B \text{ and } \angle OCB = \frac{1}{2}\angle C\right)$$

$\Rightarrow$ $$OB = OC \quad \ldots\text{(ii)}$$

(∵ Sides opposite to equal angles are equal)

In $\Delta ABO$ and $\Delta ACD$, we have

$$AB = AC \quad \text{(Given)}$$

$$\angle OBA = \angle OCA \quad \text{[From Eq. (i)]}$$

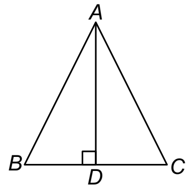
$\Rightarrow$ $$OB = OC \quad \text{[From Eq. (ii)]}$$

$\therefore$ $$\Delta ABO \cong \Delta ACO \quad \text{(By SAS congruence axiom)}$$

$\Rightarrow$ $$\angle BAO = \angle CAO \quad \text{(By CPCT)}$$

$\Rightarrow AO$ is the bisector of $\angle BAC$.

**Question 2.** In $\Delta ABC$, $AD$ is the perpendicular bisector of $BC$ (see figure). Show that $\Delta$ $ABC$ is an isosceles triangle in which $AB = AC$.




**Solution** In $\Delta ABD$ and $\Delta ACD$, we have

$DB = DC$ ($\because D$ bisect $BC$)

$\angle ADB = \angle ADC$ ($\because AD \perp BC$)

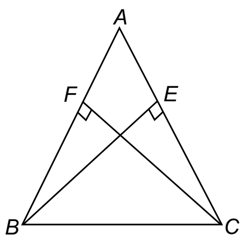
and $AD = AD$ (Common)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (By SAS congruence axiom)

$\Rightarrow$ $AB = AC$ (By CPCT)

Hence, $\Delta$ $ABC$ is an isosceles triangle.

**Question 3.** $ABC$ is an isosceles triangle in which altitudes $BE$ and $CF$ are drawn to equal sides $AC$ and $AB$ respectively (see figure). Show that these altitudes are equal.




**Solution** In $\Delta$ $ABE$ and $\Delta$ $ACF$, we have

$\angle AEB = \angle AFC$ ($BE \perp AC$, $CF \perp AB$, each 90°)

$\angle A = \angle A$ (Common)

and $AB = AC$ (Given)

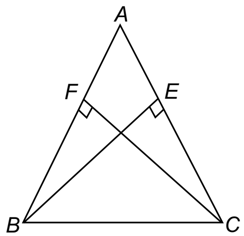
$\therefore$ $\Delta ABE \cong \Delta ACF$ (By AAS congruence axiom)

$\Rightarrow$ $BE = CF$ (By CPCT)

**Question 4.** $ABC$ is a triangle in which altitudes $BE$ and $CF$ to sides $AC$ and $AB$ are equal (see figure). Show that

(i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$ *i.e.,* $ABC$ is an isosceles triangle.




**Solution** In $\Delta ABE$ and $\Delta ACF$, we have

$$\angle AEB = \angle AFC \quad \text{(Each } 90^\circ\text{)}$$

$$\angle BAE = \angle CAF \quad \text{(Common)}$$

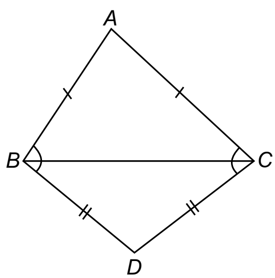
and $$BE = CF \quad \text{(Given)}$$

$\therefore$ $$\Delta ABE \cong \Delta ACF \quad \text{(By AAS congruence axiom)}$$

$\therefore$ $$AB = AC$$

So, $\Delta ABC$ is isosceles.

**Question 5.** *ABC* and *DBC* are isosceles triangles on the same base *BC* (see figure). Show that $\angle ABD = \angle ACD$.




**Solution** In $\Delta ABC$, we have

$$AB = AC \quad (\because \Delta ABC \text{ is an isosceles triangle})$$

$\therefore$ $$\angle ABC = \angle ACB \quad \ldots\text{(i)}$$

($\because$ Angles opposite to equal sides are equal)

In $\Delta DBC$, we have

$$BD = CD \quad (\because \Delta DBC \text{ is an isosceles triangle})$$

$\therefore$ $$\angle DBC = \angle DCB \quad \ldots\text{(ii)}$$

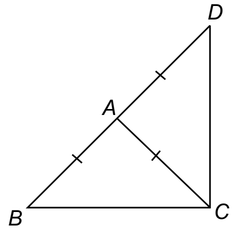
($\because$ Angles opposite to equal sides are equal)

On adding Eqs. (i) and (ii), we have

$$\angle ABC + \angle DBC = \angle ACB + \angle DCB$$

$\Rightarrow$ $$\angle ABD = \angle ACD$$

**Question 6.** $\Delta ABC$ is an isosceles triangle in which $AB = AC$. Side $BA$ is produced to $D$ such that $AD = AB$ (see figure). Show that $\angle BCD$ is a right angle.




**Solution** In $\Delta$ $ABC$, we have

$AB = AC$ (Given)

$\Rightarrow$ $\angle ACB = \angle ABC$ ...(i)

($\because$ Angles opposite to equal sides are equal)

Now, $AB = AD$ (Given)

$\therefore$ $AC = AD$ ($\because AB = AC$)

Now, In $\Delta$ $ADC$, we have

$AD = AC$ (From above)

$\Rightarrow$ $\angle ACD = \angle ADC$ ...(ii)

($\because$ Angles opposite to equal sides are equal)

On adding Eqs.(i) and (ii), we have

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$

$\Rightarrow$ $\angle BCD = \angle ABC + \angle BDC$ ($\because \angle ADC = \angle BDC$)

Adding $\angle BCD$ on both sides, we have

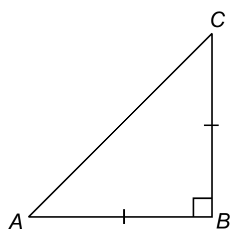
$\angle BCD + \angle BCD = \angle ABC + \angle BDC + \angle BCD$

$\Rightarrow$ $2 \angle BCD = 180°$ (By $\Delta$ property)

$\angle BCD = 90°$

**Question 7.** $ABC$ is a right angled triangle in which $\angle A = 90°$ and $AB = AC$, find $\angle B$ and $\angle C$.

**Solution** We have, $\angle A = 90°$ (Given)



$$AB = AC \quad \text{(Given)}$$

$\Rightarrow$ $$\angle B = \angle C$$

(∵ Angles opposite to equal sides are equal)

Now, we have

$$\angle A + \angle B + \angle C = 180° \quad \text{(By } \Delta \text{ property)}$$

$$90° + \angle B + \angle B = 180°$$

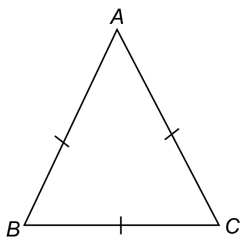
$\Rightarrow$ $$2\angle B = 90°$$

$\Rightarrow$ $$\angle B = 45°$$

$\therefore$ $$\angle B = \angle C = 45°$$

**Question 8.** Show that the angles of an equilateral triangle are 60° each.

**Solution** Let $\Delta ABC$ be an equilateral triangle, such that




$$AB = BC = CA \quad \text{(By property)}$$

Now, $$AB = AC$$

$\Rightarrow$ $$\angle B = \angle C \quad \text{...(i)}$$

(∵ Angles opposite to equal sides are equal)

Similarly, $$CB = CA$$

$\Rightarrow$ $$\angle A = \angle B \quad \text{...(ii)}$$

(∵ Angles opposite to equal sides are equal)

From Eqs. (i) and (ii), we have

$$\angle A = \angle B = \angle C \quad \text{...(iii)}$$

Now, $$\angle A + \angle B + \angle C = 180° \quad \text{(By } \Delta \text{ property)}$$

$$\angle A + \angle A + \angle A = 180° \quad \text{[From Eq. (iii)]}$$

$$3\angle A = 180°$$

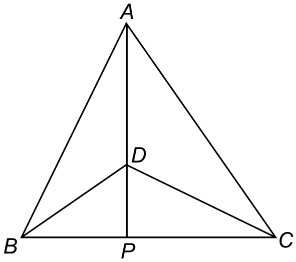
$$\angle A = 60°$$

Hence, $$\angle A = \angle B = \angle C = 60°$$

# 5 Triangles

## Exercise 5.3

**Question 1.** $\Delta ABC$ and $\Delta DBC$ are two isosceles triangles on the same base $BC$ and vertices $A$ and $D$ are on the same side of $BC$ (see figure). If $AD$ is extended to intersect $BC$ at $P$, show that




(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) $AP$ bisects $\angle A$ as well as $\angle D$

(iv) $AP$ is the perpendicular bisector of $BC$.

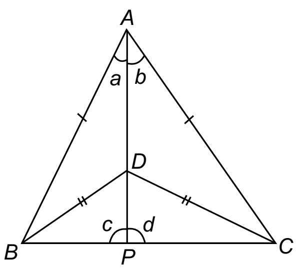
**Solution** **Given** $\Delta ABC$ and $\Delta DBC$ are two isosceles triangles having common base $BC$, such that $AB = AC$ and $DB = DC$.

**To prove** (i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) $AP$ bisects $\angle A$ as well as $\angle D$.

(iv) $AP$ is the perpendicular bisector of $BC$.




**Proof** (i) In $\Delta ABD$ and $\Delta ACD$, we have

| | | |
|---|---|---|
| | $AB = AC$ | (Given) |
| | $BD = CD$ | (Given) |
| and | $AD = AD$ | (Common) |
| $\therefore$ | $\Delta ABD \cong \Delta ACD$ | (By SSS congruence axiom) |

(ii) In $\Delta\ ABP$ and $\Delta\ ACP$, we have

$AB = AC$ (Given)

$\angle a = \angle b$ ($\because \Delta\ ABD \cong ACD$)

and $AP = AP$ (Common)

$\therefore$ $\Delta\ ABP \cong \Delta\ ACP$ (By SAS congruence axiom)

(iii) $\angle a = \angle b$ ($\because \Delta\ ABD \cong \Delta\ ACD$)

$\Rightarrow AP$ bisects $\angle A$

$\angle ADB = \angle ADC$ ($\because \Delta\ ABD \cong \Delta\ ACD$)

$\Rightarrow$ $180° - \angle ADB = 180° - \angle ADC$

$\Rightarrow$ $\angle BDP = \angle CDP$ (By linear pair axiom)

$\Rightarrow AP$ bisects $\angle D$.

(iv) $BP = CP$ ($\because \Delta\ ABP \cong \Delta\ ACP$)

and $\angle c = \angle d$ ...(i)

But $\angle c + \angle d = 180°$ (Linear pair)

$\therefore$ $\angle c + \angle c = 180° \Rightarrow \angle c = 90°$, [From Eq. (i)]

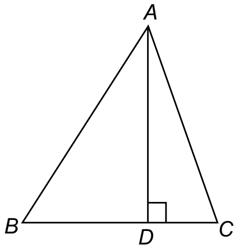
$\Rightarrow$ $\angle c = \angle d = 90°$

Hence, $AP$ is the perpendicular bisector of $BC$.

**Question 2.** $AD$ is an altitude of an isosceles triangle $ABC$ in which $AB = AC$. Show that

(i) $AD$ bisects $BC$ (ii) $AD$ bisects $\angle A$

**Solution** In $\Delta\ ABD$ and $\Delta\ ACD$, we have




$AB = AC$ (Given)

$\angle ADB = \angle ADC = 90°$ ($\because$ Given $AD \perp BC$)

and $AD = AD$ (Common)

$\therefore$ $\Delta\ ABD \cong \Delta\ ACD$ (By RHS congruence axiom)

$BD = DC$ (By CPCT)

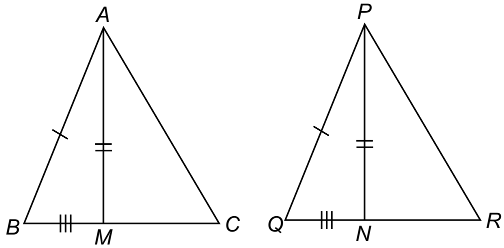
$\Rightarrow AD$ bisects $BC$.

Also, $\angle BAD = \angle CAD$ (By CPCT)

$\Rightarrow AD$ bisects $\angle A$.

**Question 3.** Two sides $AB$ and $BC$ and median $AM$ of one triangle $ABC$ are respectively equal to sides $PQ$ and $QR$ and median $PN$ of $\Delta PQR$ (see figure). Show that

(i) $\Delta ABM \cong \Delta PQN$ (ii) $\Delta ABC \cong \Delta PQR$




**Solution** We have, $AM$ is the median of $\Delta\ ABC$

$\Rightarrow \quad BM = MC = \frac{1}{2} BC \quad$ ($\because M$ bisects $BC$) ...(i)

$PN$ is the median of $\Delta PQR$.

$\Rightarrow \quad QN = NR = \frac{1}{2} QR \quad$ ($\because N$ bisects $QR$) ...(ii)

Now, we have

$BC = QR \quad$ (Given)

$\Rightarrow \quad \frac{1}{2}BC = \frac{1}{2} QR$

$\therefore \quad BM = QN \quad$ [From Eqs. (i) and (ii)] ...(iii)

(i) In $\Delta ABM$ and $\Delta PQN$, we have

$AB = PQ \quad$ (Given)

$AM = PN \quad$ (Given)

and $BM = QN \quad$ [From Eq. (iii)]

$\therefore \quad \Delta ABM \cong \Delta PQN \quad$ (By SSS congruence axiom)

(ii) In $\Delta ABC$ and $\Delta PQR$, we have

$AB = PQ \quad$ (Given)

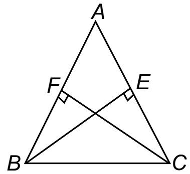
$\angle B = \angle Q \quad$ ($\because \Delta\ ABM \cong \Delta\ PQN$)

and $BC = QR \quad$ (Given)

$\therefore \quad \Delta ABC \cong \Delta PQR \quad$ (By SAS congruence axiom)

**Question 4.** $BE$ and $CF$ are two equal altitudes of a triangle $ABC$. Using RHS congruence rule, prove that the triangle $ABC$ is isosceles.

**Solution** In $\Delta BEC$ and $\Delta CFB$, we have




$\angle BEC = \angle CFB = 90°$ [$\because BE \perp AC$ and $CF \perp AB$ (Given)]

$BC = BC$ (Common)

and $BE = CF$ (Given)

$\therefore$ $\Delta BEC \cong \Delta CFB$ (By RHS congruence axiom)

$\therefore$ $EC = FB$ (By CPCT)...(i)

In $\Delta$ $AEB$ and $\Delta$ $AFC$, we have

$\angle A = \angle A$ (Common)

$\angle AEB = \angle AFC = 90°$ ($\because BE \perp AC$ and $CF \perp AB$) (Given)

and $EB = FC$ (Given)

$\therefore$ $\Delta AEB \cong \Delta AFC$ (By AAS congruence axiom)

$\therefore$ $AE = AF$ (By CPCT) ...(ii)

On adding Eqs.(i) and (ii), we have

$EC + AE = FB + AF$

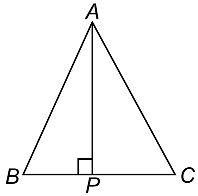
($\because AC = EC + AE$ and $AB = FB + AF$)

$\Rightarrow$ $AC = AB$

Hence, $\Delta$ $ABC$ is an isosceles triangle.

**Question 5.** $ABC$ is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$.

**Solution** In $\Delta ABP$ and $\Delta ACP$, we have




$AB = AC$ (Given)

$AP = AP$ (Common)

and $\angle APB = \angle APC = 90°$ ($\because AP \perp BC$)

$\therefore$ $\Delta ABP \cong \Delta ACP$ (By RHS congruence axiom)

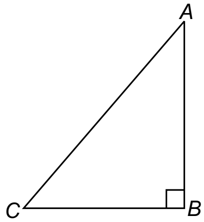
$\Rightarrow$ $\angle B = \angle C$ (By CPCT)

# 5 Triangles

## Exercise 5.4

**Question 1.** Show that in a right angled triangle, the hypotenuse is the longest side.

**Solution** Let $ABC$ be a right angled triangle, such that $\angle ABC = 90°$




We know that,

$$\angle ABC + \angle BCA + \angle CAB = 180° \quad \text{(By } \Delta \text{ property)}$$

$\Rightarrow$ $90° + \angle BCA + \angle CAB = 180°$

$\Rightarrow$ $\angle BCA + \angle CAB = 90°$

From above, we have $\angle BCA$ and $\angle CAB$ are acute angles.

$\Rightarrow$ $\angle BCA < 90°$

and $\angle CAB < 90°$

$\Rightarrow$ $\angle BCA < \angle ABC$

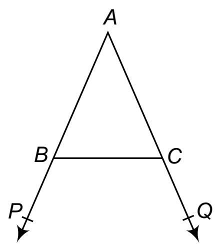
and $\angle CAB < \angle ABC$

$\Rightarrow$ $AB < AC$ and $BC < AC$ ($\because$ Side opposite to greater angle is longer)

Hence, the hypotenuse ($AC$) is the longest side.

**Question 2.** In figure, sides $AB$ and $AC$ of $\Delta ABC$ are extended to points $P$ and $Q$ respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

**Solution** We know that,




$$\angle ACB + \angle QCB = 180° \quad \text{(Linear pair)...(i)}$$

and $$\angle ABC + \angle PBC = 180° \quad \text{(Linear pair)...(ii)}$$

From Eqs. (i) and (ii), we have

$$\angle ABC + \angle PBC = \angle ACB + \angle QCB \quad ...(iii)$$

But $\angle PBC < \angle QCB$ (Given)...(iv)

From Eqs. (iii) and (iv), we have

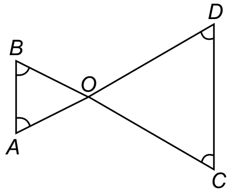
$$\angle ABC > \angle ACB$$

$\Rightarrow$ $AC > AB$

($\because$ Side opposite to greater angle is longer)

**Question 3.** In figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

**Solution** We have, $\angle B < \angle A$ (Given)




and $\angle C < \angle D$ (Given)

$\therefore$ $AO < BO$ ...(i)

and $OD < OC$

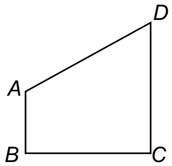
($\because$ Side opposite to greater angle is longer)...(ii)

On adding Eqs. (i) and (ii), we have

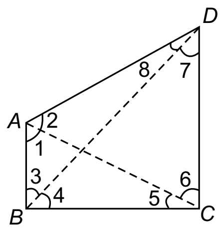
$$(AO + OD) < (BO + OC)$$

$$AD < BC$$

**Question 4.** $AB$ and $CD$ are respectively the smallest and longest sides of a quadrilateral $ABCD$ (see figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.




**Solution** **Given** $ABCD$ is a quadrilateral. $AB$ is the smallest side and $CD$ is the longest side.




**To prove** $\angle A > \angle C$ and $\angle B > \angle D$

**Construction** Join $A$ to $C$ and $B$ to $D$.

**Proof** In $\Delta ABC$, we have $AB$ is the smallest side.

$\therefore$ $AB < BC$

$\Rightarrow$ $\angle 5 < \angle 1$

($\because$ Angle opposite to longer side is greater) ...(i)

In $\Delta ADC$, we have $CD$ is the largest side.

$\therefore$ $CD > AD$

$\Rightarrow$ $\angle 2 > \angle 6$ ...(ii)

($\because$ Angle opposite to longer side is greater)

On adding Eqs. (i) and (ii), we have

$$\angle 1 + \angle 2 > \angle 5 + \angle 6$$

$\Rightarrow$ $\angle A > \angle C$

Now, in $\Delta\, ADB$, $AB$ is the smallest side.

$\therefore$ $AD > AB$

$\Rightarrow$ $\angle 3 > \angle 8$ ...(iii)

($\because$ Angle opposite to longer side is greater)

In $\Delta\, BCD$, $CD$ is the greatest side.

$\therefore$ $CD > BC$

$\Rightarrow$ $\angle 4 > \angle 7$ ...(iv)

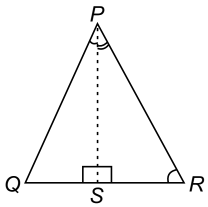
($\because$ Angle opposite to longer side is greater)

On adding Eqs. (iii) and (iv), we have

$$(\angle 3 + \angle 4) > (\angle 8 + \angle 7)$$

$$\angle B < \angle D$$

**Question 5.** In figure, $PR > PQ$ and $PS$ bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.

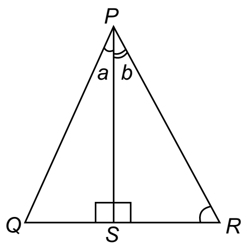



**Solution** In $\Delta\, PQR$, we have

$PR > PQ$ (Given)

$\Rightarrow$ $\angle PQR > \angle PRQ$ ...(i)

($\because$ Angle opposite to longer side is greater)




Now, $\angle a = \angle b$ ($\because PS$ bisects $\angle QPR$)...(ii)

On adding Eqs. (i) and (ii), we have

$$(\angle PQR + \angle a) > (\angle PRQ + \angle b) \quad \ldots\text{(iii)}$$

Now, $\angle PQS + \angle a + \angle PSQ = 180°$ (By $\Delta$ property)...(iv)

and $\angle PRS + \angle b + \angle PSR = 180°$ (By $\Delta$ property)...(v)

From Eqs. (iv) and (v), we have

$$\angle PQS + \angle a + \angle PSQ = \angle PRS + \angle b + \angle PSR = 180°$$

$\Rightarrow$ $\angle PQR + \angle a + \angle PSQ = \angle PRQ + \angle b + \angle PSR$ ...(vi)

$(\because \angle PRS = \angle PRQ,\ \angle PQS = \angle PQR)$

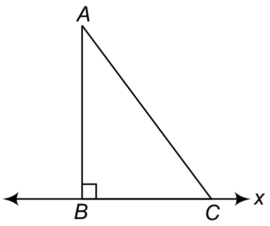
From Eqs. (iii) and (vi), we have

$$\angle PSQ < \angle PSR$$

($\because$ Side opposite to greater angle is longer)

**Question 6.** Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

**Solution** **Given** $x$ is a line and $A$ is a point not lying on $x$. $AB \perp x$, $C$ is any point on $x$ other than $B$.




**To prove** $AB < AC$

**Proof** In $\Delta ABC$, $\angle B$ is the right angle.

$\therefore \angle C$ is an acute angle.

$\therefore$ $\angle B > \angle C$

$\Rightarrow$ $AC > AB$

($\because$ Side opposite to greater angle is longer)

$\Rightarrow$ $AB < AC$

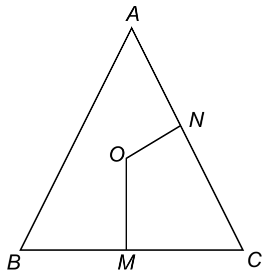
Hence, the perpendicular line segment is the shortest.

# 5 Triangles

## Exercise 5.5 (Optional)*

**Question 1.** *ABC* is a triangle. Locate a point in the interior of $\Delta ABC$ which is equidistant from all the vertices of $\Delta ABC$.

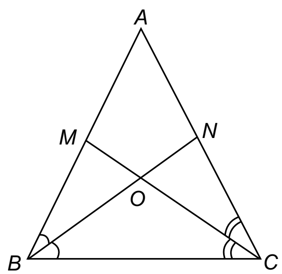
**Solution** Suppose *OM* and *ON* be the perpendicular bisectors of sides *BC* and *AC* of $\Delta$ *ABC*.




So, *O* is equidistant from two end points *B* and *C* of line segment *BC* as *O* lies on the perpendicular bisector of *BC*. Similarly, *O* is equidistant from *C* and *A*.
Hence, *O* be an orthocentre of $\Delta ABC$.

**Question 2.** In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

**Solution** Suppose *BN* and *CM* be the bisectors of $\angle ABC$ and $\angle ACB$, respectively intersect *AC* and *AB* at *N* and *M*, respectively.




Since, *O* lies on the bisector *BN* of $\angle ABC$, so *O* will be equidistant from *BA* and *BC*. Again, *O* lies on the bisector *CM* of $\angle ACB$. So, *O* will be equidistant from *CA* and *BC*. Thus, *O* will be equidistant from *AB*, *BC* and *CA*.
Hence, *O* be a circumcentre of $\Delta ABC$.

**Question 3.** In a huge park, people are concentrated at three points (see figure)

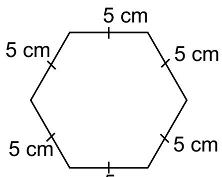
A : where these are different slides and swings for children.

B : near which a man-made lake is situated.
C : which is near to a large parking and exist.
Where should an ice-cream parlour be set up so that maximum number of persons can approach it?

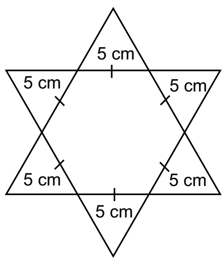

[**Hint** The parlour should be equidistant from $A, B$ and C.]

**Solution** The ice-cream parlour should be equidistant from $A, B$ and $C$ for which the point of intersection of perpendicular bisectors of $AB, BC$ and $CA$ should be situated.
So, $O$ is the required point which is equidistant from $A$, $B$ and $C$.

**Question 4.** Complete the hexagonal and star shaped Rangolies [see Fig. (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?

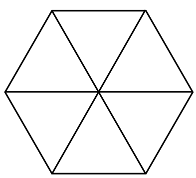



(i)

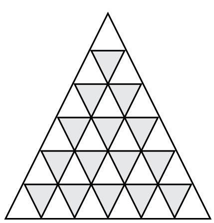



(ii)

**Solution.** We first divide the hexagon into six equilateral triangles of side 5 cm as follow.



We take one triangle from six equilateral triangle as shown above and make as many equilateral triangles of one side 1 cm as shown in the figure.



The number of equilateral triangles of side 1 cm $= 1 + 3 + 5 + 7 + 9 = 25$

So, the total number of triangles in the hexagon $= 6 \times 25 = 150$.

To find the number of triangles in the Fig. (ii), we adopt the same procedure.

So, the number of triangles in the Fig. (ii) $= 12 \times 25 = 300$.
Hence, Fig. (ii) has more triangles.