4 Triangles

Exercise 4.1

Question 1. Fill in the blanks using the correct word given in brackets.

(i) All circles are ....(congruent, similar)

(ii) All squares are ....(similar, congruent)

(iii) All .... triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are .... (b) their corresponding sides are ....(equal, proportional).

Solution (i) All circles are similar because they have similar shape but not same size.

(ii) All squares are similar because they have similar shape but not same size.

(iii) All equilateral triangles are similar because they have similar shape but not same size.

(iv) Two polygons of the same number of sides are similar, if
(a) their corresponding angles are equal.
(b) their corresponding sides are proportional.

Question 2. Give two different examples of pair of

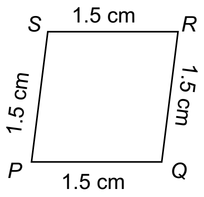
(i) similar figures.

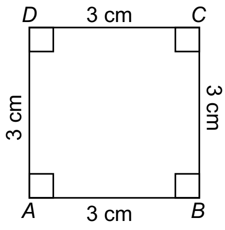
(ii) non-similar figures.

Solution (i) (a) Pair of equilateral triangle are similar figures.
(b) Pair of squares are similar figures.

(ii) (a) A triangle and a quadrilateral form a pair of non-similar figures.
(b) A square and a trapezium form a pair of non-similar figures.

Question 3. State whether the following quadrilaterals are similar or not



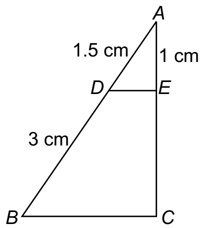


Solution The two quadrilaterals, in figure are not similar because their corresponding angles are not equal. It is clear from the figure that, $\angle A$ is 90° but $\angle P$ is not 90°.

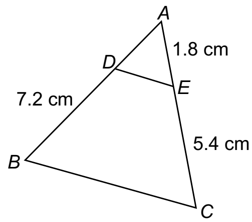
4 Triangles

Exercise 4.2

Question 1. In figures, (i) and (ii), $DE \parallel BC$. Find EC in figure (i) and AD in figure (ii).



(i)



(ii)

Solution (i) In figure (i), $DE \parallel BC$ (Given)

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{(By basic proportionality theorem)}$$

$$\Rightarrow \quad \frac{1.5}{3} = \frac{1}{EC}$$

($\because AD = 1.5$ cm, $DB = 3$ cm and $AE = 1$ cm, given)

$$\Rightarrow \quad EC = \frac{3}{1.5} = 2 \text{ cm}$$

(ii) In figure (ii), $DE \parallel BC$ (Given)

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{(By basic proportionality theorem)}$$

$$\Rightarrow \quad \frac{AD}{7.2} = \frac{1.8}{5.4}$$

($\because AE = 1.8$ cm, $EC = 5.4$ cm and $BD = 7.2$ cm, given)

$$\Rightarrow \quad AD = \frac{1.8 \times 7.2}{5.4} = 2.4 \text{ cm}$$

Question 2. E and F are points on the sides PQ and PR respectively of a ΔPQR, for each of the following cases, state whether $EF \parallel QR$

(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm.

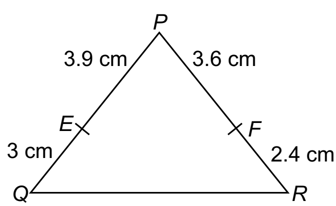
(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm.

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm.

Solution (i) In figure,

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3, \quad \frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$$

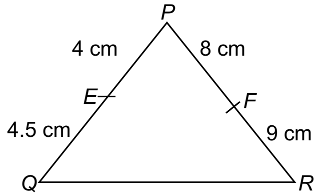
$$\Rightarrow \quad \frac{PE}{EQ} \neq \frac{PF}{FR}$$



$\Rightarrow$ EF is not parallel QR because converse of basic proportionality theorem is not satisfied.

(ii) In figure, $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$

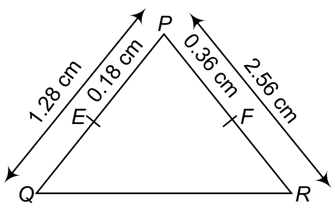
and $\frac{PF}{FR} = \frac{8}{9}$



$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$

$\Rightarrow$ $EF \parallel QR$ because converse of basic proportionality of theorem is satisfied.

(iii) In figure, $\frac{PE}{EQ} = \frac{PE}{PQ - PE} = \frac{0.18}{1.28 - 0.18} = \frac{0.18}{1.10} = \frac{9}{55}$



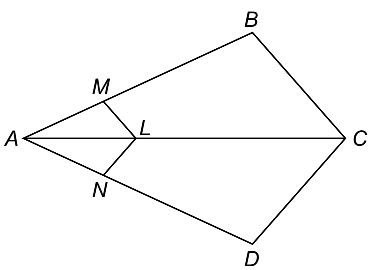
and $\frac{PF}{FR} = \frac{PF}{PR - PF}$

$$\frac{0.36}{2.56 - 0.36} = \frac{0.36}{2.20} = \frac{9}{55}$$

$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$

$\Rightarrow$ $EF \parallel QR$ because converse of basic proportionality theorem is satisfied.

Question 3. In figure, if $LM \parallel CB$ and $LN \parallel CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Solution In Δ ACB, $LM \parallel CB$ (Given)

$\Rightarrow \frac{AM}{MB} = \frac{AL}{LC}$ ...(i)

(Basic proportionality theorem)

In ΔACD, $LN \parallel CD$ (Given)

$$\Rightarrow \quad \frac{AN}{ND} = \frac{AL}{LC} \quad \ldots\text{(ii)}$$

(Basic proportionality theorem)

From Eqs. (i) and (ii), we get

$$\frac{AM}{MB} = \frac{AN}{ND} \Rightarrow \frac{MB}{AM} = \frac{ND}{AN}$$

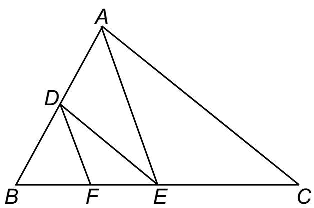
$$\frac{MB}{AM} + 1 = \frac{ND}{AN} + 1 \quad \text{(Adding both sides by 1)}$$

$$\Rightarrow \quad \frac{MB + AM}{AM} = \frac{ND + AN}{AN}$$

$$\Rightarrow \quad \frac{AM}{AM + MB} = \frac{AN}{AN + ND} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$$

Hence proved.

Question 4. In figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Solution In ΔBAC, $DE \parallel AC$ (Given)

$$\Rightarrow \quad \frac{BE}{EC} = \frac{BD}{DA} \quad \ldots\text{(i)}$$

(Basic proportionality theorem)

In ΔBAE, $DF \parallel AE$ (Given)

$$\Rightarrow \quad \frac{BF}{FE} = \frac{BD}{DA} \quad \ldots\text{(ii)}$$

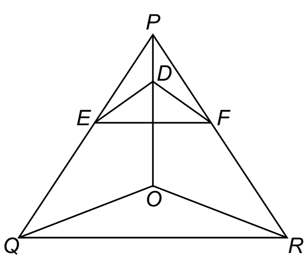
(Basic proportionality theorem)

From Eqs. (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BE}{EC}$$

Hence proved.

Question 5. In figure, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.

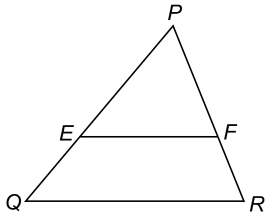


Solution In figure, $DE \parallel OQ$ and $DF \parallel OR$, then by basic proportionality theorem,

In ΔPQO, we have, $$\frac{PE}{EQ} = \frac{PD}{DO} \quad \ldots(i)$$

and in ΔPOR,

$$\frac{PF}{FR} = \frac{PD}{DO} \quad \ldots(ii)$$



From Eqs. (i) and (ii), $$\frac{PE}{EQ} = \frac{PF}{FR}$$

Now, in ΔPQR, we have proved that

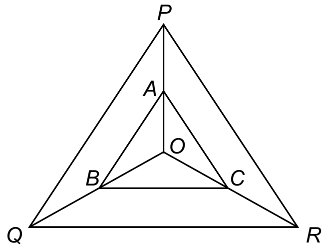
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$$EF \parallel QR$$

(By converse of basic proportionality theorem)

Hence proved.

Question 6. In figure A, B and C are points on OP, OQ and OR, respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.



Solution In figure, $AB \parallel PQ$ (Given)

$$\Rightarrow \quad \frac{OA}{AP} = \frac{OB}{BQ} \quad \ldots(i)$$

(Basic proportionality theorem)

Also, in figure, $AC \parallel PR$ (Given)

$$\Rightarrow \quad \frac{OA}{AP} = \frac{OC}{CR} \quad \ldots(ii)$$

(Basic proportionality theorem)

From Eqs. (i) and (ii), we get

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

$$\Rightarrow \quad BC \parallel QR$$

(Converse of basic proportionality theorem)

Question 7. Using theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (recall that you have proved it in Class IX).

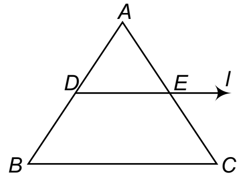
Solution In $\Delta\ ABC$, D is the mid-point of AB.

i.e., $$\frac{AD}{DB} = 1 \quad \ldots(i)$$

As straight line $l \parallel BC$.

Line l is drawn through D and it meets AC at E.

By basic proportionality theorem,



$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow$ $$\frac{AE}{EC} = 1 \quad \text{[From Eq. (i)]}$$

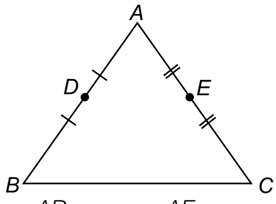
$\Rightarrow$ $$AE = EC \Rightarrow \frac{AE}{EC} = 1$$

$\Rightarrow E$ is the mid-point of AC.

Hence proved.

Question 8. Using theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (recall that you done it in Class IX).

Solution In $\Delta\ ABC$, D and E are mid-points of side AB and AC, respectively.



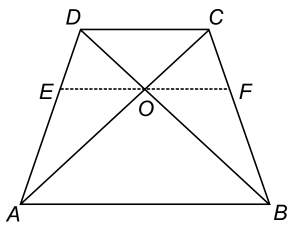
$\Rightarrow$ $$\frac{AD}{DB} = 1 \text{ and } \frac{AE}{EC} = 1 \quad \text{(See in figure)}$$

$\Rightarrow$ $$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow DE \parallel BC$$

(By converse of basic proportionality theorem)

Question 9. $ABCD$ is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Solution



We draw, $EOF \parallel AB$ (Also $\parallel CD$)

In ΔACD, $OE \parallel CD$

$\Rightarrow \dfrac{AE}{ED} = \dfrac{AO}{OC}$ (Basic proportionality theorem) ...(i)

In ΔABD, $OE \parallel BA$

$\Rightarrow \dfrac{DE}{EA} = \dfrac{DO}{OB}$ (Basic proportionality theorem)

$\Rightarrow \dfrac{AE}{ED} = \dfrac{OB}{OD}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{AO}{OC} = \frac{OB}{OD}$$

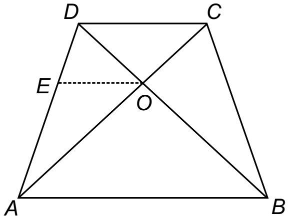
i.e., $\dfrac{AO}{BO} = \dfrac{CO}{DO}$

Hence proved.

Question 10. The diagonals of a quadrilateral *ABCD* intersect each other at the point *O* such $\dfrac{AO}{BO} = \dfrac{CO}{DO}$. Show that *ABCD* is a trapezium.

Solution In figure, $\dfrac{AO}{BO} = \dfrac{CO}{DO}$ (Given)

$\Rightarrow \dfrac{AO}{OC} = \dfrac{BO}{OD}$ (Given) ...(i)



Through *O*, we draw $OE \parallel BA$

OE meets *AD* at *E*.

In ΔDAB, $EO \parallel AB$

$\Rightarrow \dfrac{DE}{EA} = \dfrac{DO}{OB}$

$\Rightarrow \dfrac{AE}{ED} = \dfrac{BO}{OD}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{AO}{OC} = \frac{AE}{ED}$$

$\Rightarrow OE \parallel CD$

(By converse of basic proportionality theorem)

Now, we have

$$BA \parallel OE \text{ and } OE \parallel CD \Rightarrow AB \parallel CD$$

$\Rightarrow$ Quadrilateral *ABCD* is a trapezium.

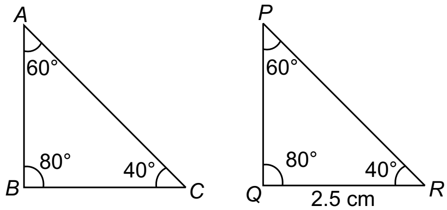
Hence proved.

4 Triangles

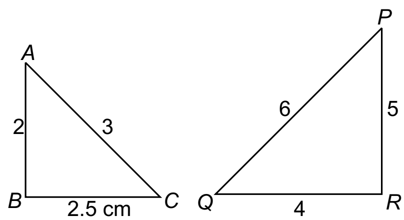
Exercise 4.3

Question 1. State which pairs of triangles in figure are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form

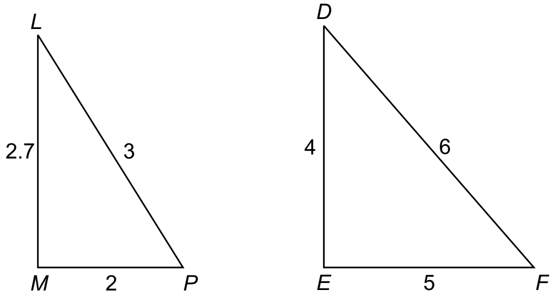
(i)



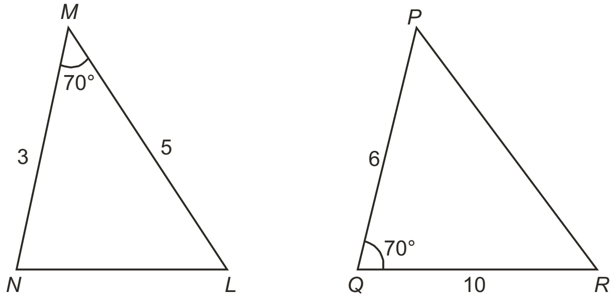
(ii)



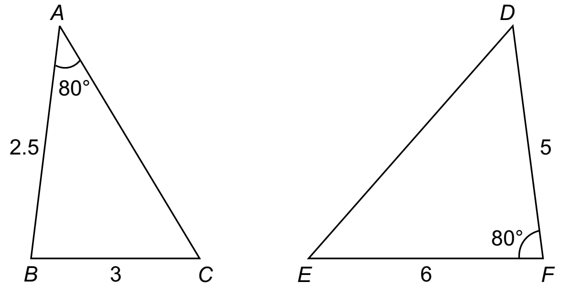
(iii)



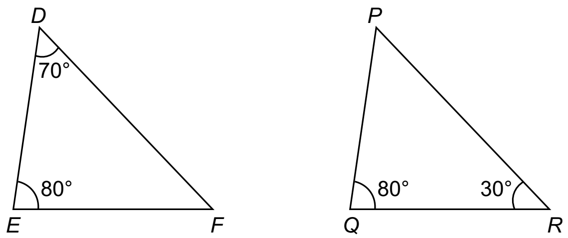
(iv)



(v)



(vi)



Solution (i) Yes. In ΔABC and ΔPQR,

$$\angle A = \angle P = 60°, \angle B = \angle Q = 80°$$

and $\qquad \angle C = \angle R = 40°$

Here, corresponding angles are equal.

Therefore, $\qquad \Delta ABC \sim \Delta PQR \qquad$ (By AAA similarity criterion)

(ii) Yes. In ΔABC and ΔPQR,

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}$$

and $\qquad \dfrac{CA}{PQ} = \dfrac{3}{6} = \dfrac{1}{2}$

Here, all corresponding sides are equal in proportional.

Therefore, $\qquad \Delta ABC \sim \Delta QRP \qquad$ (By SSS similarity criterion)

(iii) No. In ΔLMP and ΔDEF

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}, \frac{LP}{DF} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{LM}{EF} = \frac{2.7}{5} \neq \frac{1}{2}$$

i.e., $\qquad \dfrac{MP}{DE} = \dfrac{LP}{DF} \neq \dfrac{LM}{EF}$

Here, all corresponding sides are not equal in proportional.

Thus, the two triangles are not similar.

(iv) Yes. In ΔLMN and ΔPQR

$$\angle M = \angle Q = 70°, \frac{MN}{PQ} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$$

i.e., $\qquad \dfrac{MN}{PQ} = \dfrac{ML}{OR}$

Here, corresponding two adjacent sides are in proportional and one angle is equal.

Therefore, $\qquad \Delta MNL \sim \Delta QPR \qquad$ (By SAS similarity criterion)

(v) No. In $\Delta ABC, \angle A$ is given but the included side AC is not given.

(vi) Yes. $\angle D = 70°$, $\angle E = 80°$ and $\angle F = 30°$

$(\because \text{In } \Delta DEF, \angle D + \angle E + \angle F = 180°)$

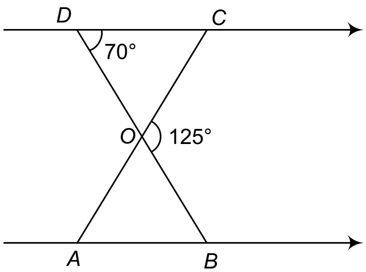
$\angle Q = 80°$, $\angle R = 30°$, then $\angle P = 70°$

$(\because \text{In } \Delta QPR, \angle Q + \angle P + \angle R = 180°)$

Here, $\angle D = \angle P, \angle E = \angle Q, \angle F = \angle R$

Therefore, $\Delta DEF \sim \Delta PQR$ (By AAA similarity criterion)

Question 2. In figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Solution $\angle DOC + 125° = 180°$ $(\because DOC$ is a straight line)

$\Rightarrow \angle DOC = 180° - 125° = 55°$

$\angle DCO + \angle CDO + \angle DOC = 180°$

(Sum of three angles of ΔODC)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO + 125° = 180°$

$\Rightarrow \angle DCO = 180° - 125° = 55°$

Now, we are given that, $\Delta ODC \sim \Delta OBA$. (Similar triangle)

$\Rightarrow \angle OCD = \angle OAB$

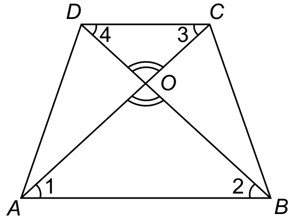
$\Rightarrow \angle OAB = \angle OCD = \angle DCO = 55°$

i.e., $\angle OAB = 55°$

Hence, we have $\angle DOC = 55°$, $\angle DCO = 55°$ and $\angle OAB = 55°$.

Question 3. Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Solution Draw $ABCD$ is a trapezium and AC and BD are diagonals intersect at O.



In figure, $AB \parallel DC$ (Given)

$\Rightarrow$ $\angle 1 = \angle 3, \angle 2 = \angle 4$ (Alternate interior angles)

Also, $\angle DOC = \angle BOA$ (Vertically opposite angles)

$\Rightarrow$ $\Delta OCD \sim \Delta OAB$ (Similar triangle)

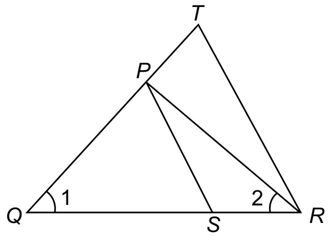
$\Rightarrow$ $$\frac{OC}{OA} = \frac{OD}{OB}$$

(Ratios of the corresponding sides of the similar triangles)

$\Rightarrow$ $$\frac{OA}{OC} = \frac{OB}{OD}$$ (Taking reciprocals)

Hence proved.

Question 4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, show that $\Delta PQS \sim \Delta TQR$.



Solution In figure, $\angle 1 = \angle 2$ (Given)

$\Rightarrow$ $PQ = PR$

(Sides opposite to equal angles of ΔPQR)

We are given that,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$\Rightarrow$ $$\frac{QR}{QS} = \frac{QT}{PQ}$$ ($\because PQ = PR$ proved)

$\Rightarrow$ $$\frac{QS}{QR} = \frac{PQ}{QT}$$ (Taking reciprocals) ...(i)

Now, in ΔPQS and ΔTQR, we have

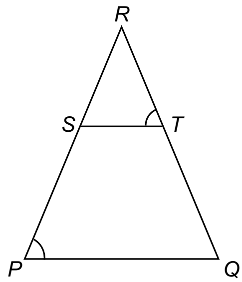
$\angle PQS = \angle TQR$ (Each $= \angle 1$)

and $$\frac{QS}{QR} = \frac{PQ}{QT}$$ [By Eq. (i)]

Therefore, by SAS similarity criterion, we have $\Delta PQS \sim \Delta TQR$.

Question 5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Solution Draw a ΔRPQ such that S and T are points on PR and QR and joining them.



In figure, we have ΔRPQ and ΔRTS in which

$\angle RPQ = \angle RTS$ (Given)

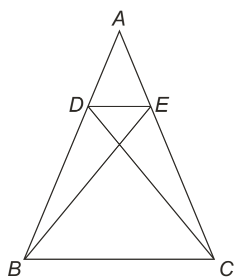
$\angle PRQ = \angle SRT$ (Each $= \angle R$)

Then, by AAA similarity criterion, we have

$$\Delta RPQ \sim \Delta RTS$$

Note *If any two corresponding angles of the triangles are equal, then their third corresponding angles are also equal by AAA.*

Question 6. In figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Solution In figure, $\Delta ABE \cong \Delta ACD$ (Given)

$\Rightarrow$ $AB = AC$ and $AE = AD$ (CPCT)

$\Rightarrow$ $\frac{AB}{AC} = 1$ and $\frac{AD}{AE} = 1$

$\Rightarrow$ $\frac{AB}{AC} = \frac{AD}{AE}$ (Each = 1)

Now, in ΔADE and ΔABC, we have

$\frac{AD}{AE} = \frac{AB}{AC}$ (Proved)

i.e., $\frac{AD}{AB} = \frac{AE}{AC}$

and also, $\angle DAE = \angle BAC$ (Each $= \angle A$)

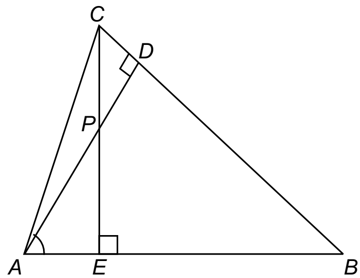
$\Rightarrow$ $\Delta ADE \sim \Delta ABC$ (By SAS similarity criterion)

Hence proved.

Question 7. In figure, altitudes AD and CE of $\Delta\, ABC$ intersect each other at the point P. Show that

(i) $\Delta\ AEP \sim \Delta\ CDP$ (ii) $\Delta\ ABD \sim \Delta\ CBE$

(iii) $\Delta\ AEP \sim \Delta\ ADB$ (iv) $\Delta\ PDC \sim \Delta\ BEC$



Solution (i) In figure, $\angle AEP = \angle CDP$ (Each = 90°)

and $\angle APE = \angle CPD$ (Vertically opposite angles)

$\Rightarrow$ $\angle\ AEP \sim \Delta\ CDP$ (By AAA similarity criterion)

(ii) In figure, $\angle\ ADB = \angle\ CEB$ (Each = 90°)

and $\angle\ ABD = \angle\ CBE$ (Each = $\angle\ B$)

$\Rightarrow$ $\Delta\ ABD \sim \Delta\ CBE$ (By AAA similarity criterion)

(iii) In figure, $\angle\ AEP = \angle\ ADB$ (Each = 90°)

and $\angle\ PAE = \angle\ DAB$ (Common angle)

$\Rightarrow$ $\Delta\ AEP \sim \Delta\ ADB$ (By AAA similarity criterion)

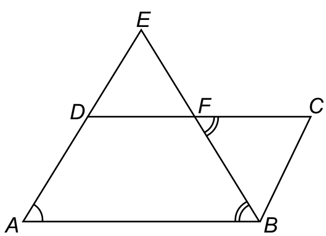
(iv) In figure, $\angle\ PDC = \angle\ BEC$ (Each = 90°)

and $\angle\ PCD = \angle\ BCE$ (Common angle)

$\Rightarrow$ $\Delta\ PDC \sim \Delta\ BEC$ (By AAA similarity criterion)

Question 8. E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Show that $\Delta\ ABE \sim \Delta\ CFB$.

Solution Draw a parallelogram *ABCD* and produce a line *AD* to *AE* and joining *BE*.



In parallelogram *ABCD*,

$$\angle\ A = \angle\ C \quad \ldots\text{(i)}$$

Now, for ΔABE and ΔCFB,
we have

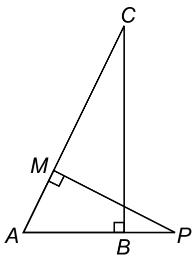
$\angle\ EAB = \angle\ BCF$ [From Eq. (i)]

$\angle\ ABE = \angle\ BFC$ (Alternate angles as $AB \parallel FC$)

$\Rightarrow$ $\Delta\ ABE \sim \Delta\ CFB$ (AAA similarity)

Question 9. In figure, ABC and AMP are two right triangles, right angled at B and M, respectively. Prove that

(i) $\Delta\, ABC \sim \Delta\, AMP$ (ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$



Solution (i) In figure, we have $\angle ABC = \angle AMP$ (Each = 90°)

Because the $\Delta\, ABC$ and $\Delta\, AMP$ are right angled at B and M, respectively.

Also, $\angle BAC = \angle PAM$ (Common angle $\angle A$)

$\Rightarrow$ $\Delta\, ABC \sim \Delta\, AMP$ (By AAA similarity criterion)

(ii) As $\Delta\, ABC \sim \Delta\, AMP$,

$$\frac{AC}{AP} = \frac{BC}{MP}$$

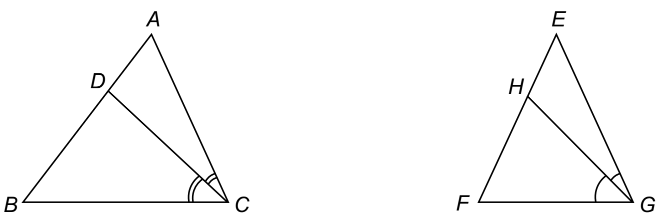
(Ratio of the corresponding sides of similar triangles)

$\Rightarrow$ $\dfrac{CA}{PA} = \dfrac{BC}{MP}$ Hence proved.

Question 10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\Delta\, ABC$ and $\Delta\, EFG$, respectively. If $\Delta\, ABC \sim \Delta\, FEG$. Show that

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ (ii) $\Delta\, DCB \sim \Delta\, HGE$ (iii) $\Delta\, DCA \sim \Delta\, HGF$

Solution Draw two ΔABC and ΔEFG along that draw two bisectors CD and GH of $\angle ACB$ and $\angle EGF$.



Since, $\Delta\, ABC \sim \Delta\, FEG$

(i) In ΔACD and ΔFGH

$\angle CAD = \angle GFH$ ...(i)

$$\left\{\begin{array}{l} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle CAB = \angle GFE \\ \Rightarrow\ \angle CAD = \angle GFH \end{array}\right.$$

$$\angle ACD = \angle FGH \quad \ldots(ii)$$

$$\left\{\begin{array}{l} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle ACB = \angle FGE \\ \Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE \end{array}\right.$$

$$\left\{\begin{array}{l} \text{(Halves of equals are equal} \\ \Rightarrow \angle ACD = \angle FGH \end{array}\right.$$

From Eqs. (i) and (ii), we get

$$\Delta ACD \sim \Delta FGH \quad (\because \text{AA similarity criterion})$$

$$\therefore \quad \frac{CD}{GH} = \frac{AC}{FG}$$

($\because$ Corresponding sides of two similar triangles are proportional)

(ii) In ΔDCB and ΔHGE,

$$\angle DBC = \angle HEG \quad \ldots(iii)$$

$$\left\{\begin{array}{l} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle ABC = \angle FEG \\ \Rightarrow \angle DBC = \angle HEG \end{array}\right.$$

$$\angle DCB = \angle HGE \quad \ldots(iv)$$

$$\left\{\begin{array}{l} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle ACB = \angle FGE \\ \Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE \end{array}\right.$$

(Halves of equals are equal)

$$\Rightarrow \quad \angle DCB = \angle HGE$$

From Eqs. (iii) and (iv), we get

$$\Delta DCB \sim \Delta HGE \quad (\because \text{AA similarity criterion})$$

(iii) In ΔDCA and ΔHGF,

$$\angle DAC = \angle HFG \quad \ldots(v)$$

$$\left\{\begin{array}{l} \because \Delta ABC \sim \Delta FEG \\ \therefore \angle CAB = \angle GFE \\ \Rightarrow \angle CAD = \angle GFH \\ \Rightarrow \angle DAC = \angle HFG \end{array}\right.$$

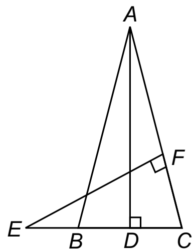
$$\angle DCA = \angle HGF \quad \ldots(vi)$$

$$\left\{\begin{array}{l} \because \Delta ABC \sim \Delta FEG, \quad \therefore \angle ACB = \angle FGE \\ \Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE \quad \text{(Halves equals are equal)} \\ \Rightarrow \angle DCA = \angle HGF \end{array}\right.$$

From Eqs. (v) and (vi), we get

$$\Delta DCA \sim \Delta HGF \quad (\because \text{AA similarity criterion})$$

Question 11. In figure, E is a point on side CB produced of an isosceles ΔABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.



Solution In figure, we are given that ΔABC is isosceles

and $$AB = AC \Rightarrow \angle B = \angle C \quad \text{...(i)}$$

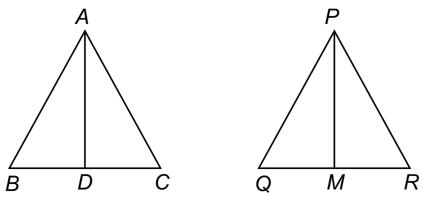
For ΔABD and ΔECF,

$$\angle ABD = \angle ECF \quad \text{[From Eq. (i)]}$$

and $$\angle ADB = \angle EFC \quad \text{[Each} = 90°\text{]}$$

$\Rightarrow$ $$\Delta ABD \sim \Delta ECF \quad \text{(AAA similarity criterion)}$$

Question 12. Sides AB and BC and median AD of a ΔABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$.



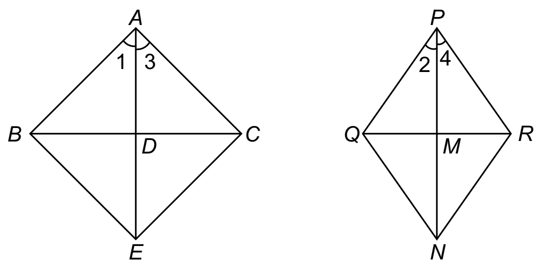
Solution Given in ΔABC and ΔPQR,

AD and PM are their medians, respectively.

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \text{...(i)}$$

To prove $\Delta ABC \sim \Delta PQR$

Construction Produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. Join BE, CE, QN and RN.



Proof Quadrilaterals *ABEC* and *PQNR* are parallelograms because their diagonals bisect each other at *D* and *M*, respectively.

$$\Rightarrow \quad BE = AC$$

and $$QN = PR$$

$$\Rightarrow \quad \frac{BE}{QN} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{BE}{QN} = \frac{AB}{PQ} \quad \text{[By Eq. (i)]}$$

i.e., $$\frac{AB}{PQ} = \frac{BE}{QN} \quad \ldots\text{(ii)}$$

From Eq. (i), $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$$

($\because$ Diagonals are bisect each other)

i.e., $$\frac{AB}{PQ} = \frac{AE}{PN} \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$$\Rightarrow \quad \Delta ABE \sim \Delta PQN$$

$$\Rightarrow \quad \angle 1 = \angle 2 \quad \ldots\text{(iv)}$$

Similarly, we can prove

$$\Delta ACE \sim \Delta PRN \Rightarrow \angle 3 = \angle 4 \quad \ldots\text{(v)}$$

On adding Eqs. (iv) and (v), we have

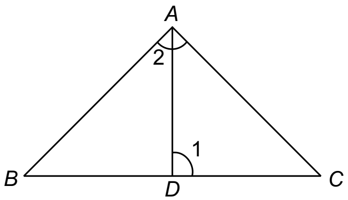
$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

$$\Rightarrow \quad \angle A = \angle P$$

$$\Rightarrow \quad \Delta ABC \sim \Delta PQR \quad \text{(SAS similarity criterion)}$$

Question 13. D is point on the side BC of a ΔABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Solution Draw a ΔABC such that D is a point on BC and join AD.



For ΔABC and ΔDAC, we have

$$\angle BAC = \angle ADC \quad \text{(Given)}$$

and $$\angle ACB = \angle DCA \quad \text{(Common } \angle C\text{)}$$

$$\Rightarrow \quad \Delta ABC \sim \Delta DAC \quad \text{(AAA similarity criterion)}$$

$$\Rightarrow \quad \frac{AC}{CB} = \frac{CD}{CA}$$

$$\Rightarrow \quad \frac{CA}{CD} = \frac{CB}{CA}$$

$$\Rightarrow \quad CA \times CA = CB \times CD$$

$$\Rightarrow \quad CA^2 = CB \times CD$$

Question 14. Sides AB and AC and median AD of a ΔABC are respectively proportional to sides PQ and PR and median PM of another ΔPQR. Show that $\Delta ABC \sim \Delta PQR$.

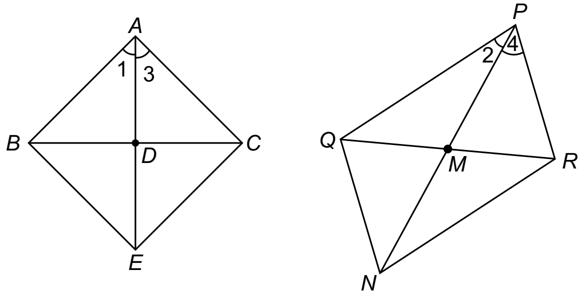
Solution Given, in ΔABC and ΔPQR,

AD and PM are their medians, respectively.

Also, $$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \text{...(i)}$$

To prove $\Delta ABC \sim \Delta PQR$

Construction Produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. Join BE, CE, QN and RN.



Proof Quadrilaterals $ABEC$ and $PQNR$ are parallelograms because their diagonals bisect each other at D and M, respectively.

$$\Rightarrow \quad BE = AC$$

and $$QN = PR$$

$$\Rightarrow \quad \frac{BE}{QN} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{BE}{QN} = \frac{AB}{PQ} \quad \text{[By Eq. (i)]}$$

i.e., $$\frac{AB}{PQ} = \frac{BE}{QN} \quad \text{...(ii)}$$

From Eq. (i), $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$$

(∵ Diagonals are bisect each other)

i.e., $$\frac{AB}{PQ} = \frac{AE}{PN} \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we have

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$\Rightarrow$ $\Delta\ ABE \sim \Delta\ PQN$

$\Rightarrow$ $\angle 1 = \angle 2$ ...(iv)

Similarly, we can prove that

$\Delta\ ACE \sim \Delta\ PRN$

$\angle 3 = \angle 4$ ...(v)

On adding Eqs. (iv) and (v), we have

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow$ $\angle A = \angle P$

$\Rightarrow$ $\Delta\ ABC \sim \Delta\ PQR$ (SAS similarity criterion)

Question 15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Solution In figure (i), AB is a pole and behind it a Sun is risen which casts a shadow of length BC = 4 cm and makes a angle θ to the horizontal and in figure ii, PM is a height of the tower and behind a Sun risen which casts a shadow of length, NM = 28 cm.

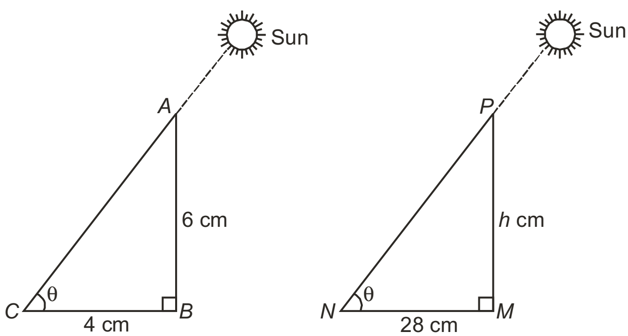
In $\Delta\ ACB$ and ΔPNM, $\angle C = \angle N = \theta$

and $\angle\ ABC = \angle\ PMN = 90°$

$\therefore$ $\Delta\ ABC \sim \Delta\ PMN$ (AAA similarity criterion)

$\Rightarrow$ $\dfrac{AB}{PM} = \dfrac{BC}{MN} \Rightarrow \dfrac{AB}{BC} = \dfrac{PM}{MN}$

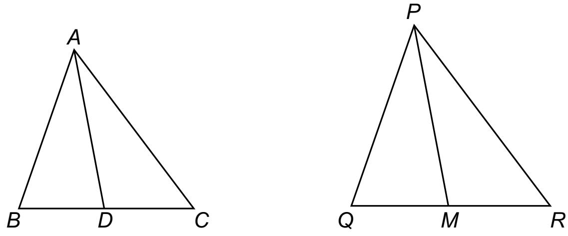
$\Rightarrow$ $\dfrac{6}{4} = \dfrac{h}{28} \Rightarrow h = \dfrac{6 \times 28}{4} = 42$ m



(i) (ii)

Question 16. If AD and PM are medians of ΔABC and ΔPQR, respectively, where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Solution Draw two ΔABC and ΔPQR taking D and M points on BC and QR such that AD and PM are the medians of the ΔABC and ΔPQR.



$$\Delta ABC \sim \Delta PQR \quad \text{(Given)}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}; \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \quad \text{...(i)}$$

Now, $BD = CD = \frac{1}{2} BC$

and $QM = RM = \frac{1}{2} QR$ ...(ii)

($\because$ D is mid-point of BC and M is mid-point of QR)

From Eq. (i), $\frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow$ $\frac{AB}{PQ} = \frac{2BD}{2QM}$ [By Eq. (ii)]

$\Rightarrow$ $\frac{AB}{PQ} = \frac{BD}{QM}$

Thus, we have $\frac{AB}{PQ} = \frac{BD}{QM}$

and $\angle ABD = \angle PQM$ ($\because \angle B = \angle Q$)

$\Rightarrow$ $\Delta ABD \sim \Delta PQM$ (by SAS similarity criterion)

$\Rightarrow$ $\frac{AB}{PQ} = \frac{AD}{PM}$

Hence proved.

4 Triangles

Exercise 4.4

Question 1. Let $\Delta ABC \sim \Delta DEF$ and their areas be, 64 cm^2 and 121 cm^2, respectively. If $EF = 15.4$ cm, find BC.

Solution $\Delta ABC \sim \Delta DEF$ (Given)

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{BC^2}{EF^2}$$

(Using property of area of similar triangles)

$$\Rightarrow \frac{64}{121} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \left(\frac{BC}{EF}\right)^2 = \left(\frac{8}{11}\right)^2 \Rightarrow \frac{BC}{EF} = \frac{8}{11}$$

$$\Rightarrow BC = \frac{8}{11} \times EF$$

$$\Rightarrow BC = \frac{8}{11} \times 15.4 = 11.2 \text{ cm}$$

Question 2. Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. If $AB = 2\,CD$. Find the ratio of the area of $\Delta\, AOB$ and $\Delta\, COD$.

Solution

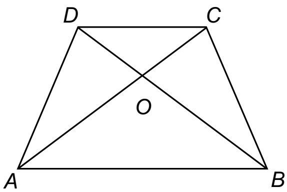
$$\frac{\text{ar}(\Delta\ AOB)}{\text{ar}(\Delta\ COD)} = \frac{AB^2}{CD^2}$$

(Using property of area of similar triangles)

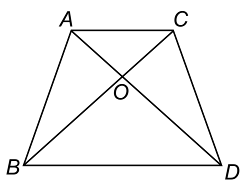
$$= \frac{(2CD)^2}{CD^2}$$

$(\because AB = 2CD)$

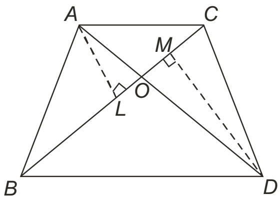
$$= \frac{4 \times CD^2}{CD^2} = \frac{4}{1}$$



Question 3. In figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\dfrac{\text{ar}(\Delta\ ABC)}{\text{ar}(\Delta\ DBC)} = \dfrac{AO}{DO}$.



Solution



Draw $AL \perp BC$ and $DM \perp BC$ (See figure)

In $\Delta\ OLA$ and $\Delta\ OMD$

$$\angle ALO = \angle DMO = 90^\circ$$

and $\angle AOL = \angle DOM$ (Vertically opposite angle)

$\therefore$ $\Delta\ OLA \sim \Delta\ OMD$ (AAA similarity criterion)

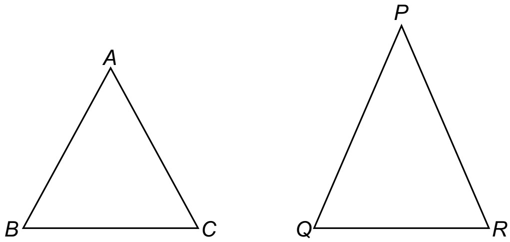
$\Rightarrow$ $$\frac{AL}{DM} = \frac{AO}{DO} \quad \ldots\text{(i)}$$

Now, $$\frac{\text{ar}(\Delta\ ABC)}{\text{ar}(\Delta\ DBC)} = \frac{\frac{1}{2} \times (BC) \times (AL)}{\frac{1}{2} \times (BC) \times (DM)} = \frac{AL}{DM} = \frac{AO}{DO}$$ [By Eq. (i)]

Hence, $$\frac{\text{ar}(\Delta\ ABC)}{\text{ar}(\Delta\ DBC)} = \frac{AO}{DO}$$

Question 4. If the areas of two similar triangles are equal, prove that they are congruent.

Solution Let $\Delta ABC \sim \Delta PQR$ and $\text{ar}(\Delta\ ABC) = \text{ar}(\Delta\ PQR)$ (Given)



i.e., $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

$\Rightarrow$ $$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{PR^2} = 1$$

(Using property of area of similar triangles)

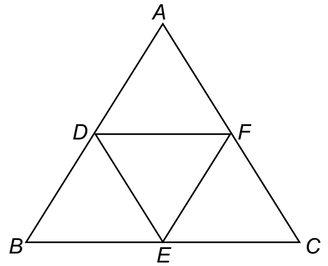
$\Rightarrow$ $AB = PQ,\ BC = QR$ and $CA = PR$

(SSS proportionality criterion)

$\Rightarrow$ $\Delta ABC \cong \Delta PQR.$

Question 5. *D, E* and *F* are respectively the mid-point of sides *AB*, *BC* and *CA* of $\Delta\, ABC$. Find the ratio of the areas of ΔDEF and ΔABC.

Solution Draw a $\Delta\, ABC$ taking mid-points *D*, *E* and *F* on *AB*, *BC* and *AC* and join them.



Here, $$DF = \frac{1}{2}BC,\ DE = \frac{1}{2}CA$$

and $$EF = \frac{1}{2}AB \quad \ldots(i)$$

($\because$ *D*, *E* and *F* are mid-points of sides *AB*, *BC* and *CA*, respectively)

$\Rightarrow$ $$\frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2}$$ (SSS proportionality criterion)

$\Rightarrow$ $$\Delta DEF \sim \Delta CAB$$

$\Rightarrow$ $$\frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta CAB)} = \frac{DE^2}{CA^2}$$

$$= \frac{\left(\frac{1}{2}CA\right)^2}{CA^2} = \frac{1}{4}$$ [From Eq. (i)]

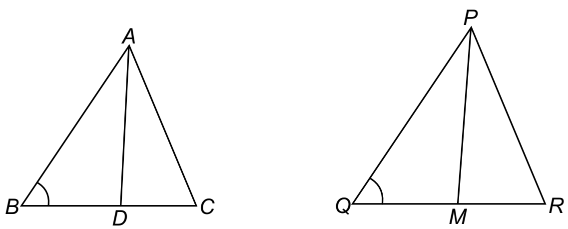
(Using property of area of similar triangle)

$\Rightarrow$ $$\frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta ABC)} = \frac{1}{4}$$ [$\because$ ar(ΔCAB) = ar(ΔABC)]

Hence, the required ratio is 1 : 4.

Question 6. Prove that the ratio of the area of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution



In figure, *AD* is a median of ΔABC and *PM* is a median of ΔPQR. Here, *D* is mid-point of *BC* and *M* is mid-point of *QR*.

Now, we have,

In figure, AD is a median of ΔABC and PM is a median of ΔPQR. Here, D is mid-point of BC and M is mid-point of QR.

Now, we have,

$$\Delta ABC \sim \Delta PQR$$

$$\Rightarrow \quad \angle B = \angle Q \quad \text{...(i)}$$

(Corresponding angles are equal)

Also, $$\frac{AB}{PQ} = \frac{BC}{QR}$$

(Ratio of corresponding sides are equal)

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{2BD}{2QM}$$

($\because D$ is mid-point of BC and M is mid-point of QR)

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} \quad \text{...(ii)}$$

In ΔABD and ΔPQM,

$$\angle ABD = \angle PQM \quad \text{[By Eq. (i)]}$$

and $$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{[By Eq. (ii)]}$$

$$\Rightarrow \quad \Delta ABD \sim \Delta PQM \quad \text{(SAS similarity criterion)}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} \quad \text{...(iii)}$$

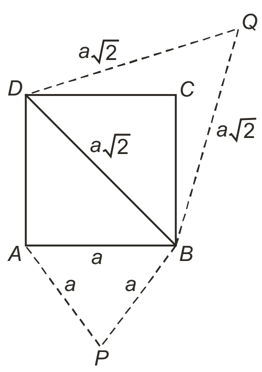
Now, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2}$$

(Using property of area of similar triangles)

$$\Rightarrow \quad \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AD^2}{PM^2} \quad \text{[From Eq. (iii)]}$$

Question 7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution



Draw $ABCD$ is a square having sides of length $= a$

Then, the diagonal, $BD = a\sqrt{2}$

We construct equilateral $\Delta\ PAB$ and ΔQBD.

$\Rightarrow \quad \Delta PAB \sim \Delta QBD$ (Equilateral triangles are similar)

$\Rightarrow \quad \dfrac{\text{ar}(\Delta PAB)}{\text{ar}(\Delta QBD)} = \dfrac{AB^2}{BD^2}$ (Using property of area of similar triangles)

$$= \frac{a^2}{(a\sqrt{2})^2} = \frac{1}{2} \Rightarrow \text{ar}(\Delta PAB) = \frac{1}{2}\ \text{ar}\ (\Delta QBD)$$

Hence proved.

Question 8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of ΔABC and ΔBDE is

(a) 2 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4

Solution (c)

Here, $AB = BC = CA = a$ (Say)

($\because$ ΔABC is an equilateral)

$BD = \dfrac{1}{2}a$ ($\because$ D is mid-point of BC)

Now, $\Delta ABC \sim \Delta BDE$ ($\because$ Both the triangles are equilateral)

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta BDE)} = \dfrac{AB^2}{BD^2}$

(Using property of area of similar to triangles)

$$= \frac{a^2}{\left(\frac{1}{2}a\right)} = \frac{4}{1}$$

i.e., The ratio is 4 : 1.

Question 9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(a) 2 : 3 (b) 4 : 9 (c) 81 : 16 (d) 16 : 81

Solution (d)

Areas of two similar triangles are in the ratio of the square of their corresponding sides

$$= \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

4 Triangles

Exercise 4.5

Question 1. Sides of some triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm , 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 2 cm , 5 cm

Solution We know that, in right triangle, sum of squares of two smaller sides is equal to the square of the third (large) side.

(i) Here, $(7)^2 + (24)^2 = 49 + 576 = 625 = (25)^2$

Therefore, given sides 7 cm, 24 cm and 25 cm make a right triangle and length of its hypotenuse is 25 cm.

(ii) Here, $(3)^2 + (6)^2 = 9 + 36 = 45$ and $(8)^2 = 64$. Both values are not equal.

Therefore, given sides 3 cm, 8 cm and 6 cm does not make a right triangle.

(iii) Here, $(50)^2 + (80)^2 = 2500 + 6400 = 8900$ and $(100)^2 = 10000$. Both values are not equal.

Therefore, given sides 50 cm, 80 cm and 100 cm does not make a right triangle.

(iv) Here, $(12)^2 + (5)^2 = 144 + 25 = 169 = (13)^2$

Therefore, given sides 13 cm, 12 cm and 5 cm make a right triangle and length of its hypotenuse is 13 cm.

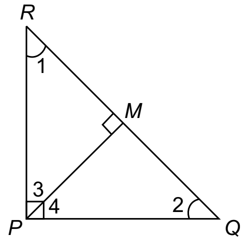
Question 2. PQR is a triangle right rangled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM \times MR$.

Solution In ΔPQR and ΔMPQ,

$$\angle 1 + \angle 2 = \angle 2 + \angle 4 \quad \text{(Each} = 90^\circ\text{)}$$

$\Rightarrow$ $\angle 1 = \angle 4$

Similarly, $\angle 2 = \angle 3$



and $\angle PMR = \angle PMQ$ (Each 90°)

$\Delta QPM \sim \Delta PRM$ (AAA criterion)

$$\Rightarrow \quad \frac{\text{ar}(\Delta QPM)}{\text{ar}(\Delta PRM)} = \frac{PM^2}{RM^2}$$

(Using property of area of similar triangles)

$$\Rightarrow \quad \frac{\frac{1}{2}(QM) \times (PM)}{\frac{1}{2}(RM) \times (PM)} = \frac{PM^2}{RM^2}$$

(Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Height)

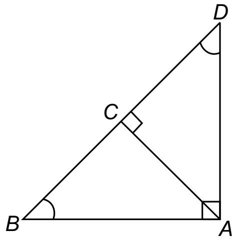
$$\Rightarrow \quad \frac{QM}{RM} = \frac{PM^2}{RM^2}$$

$$\Rightarrow \quad PM^2 = QM \times RM$$

or $$PM^2 = QM \times MR$$

Hence proved.

Question 3. In figure, ABD is a triangle right angled at A and $AC \perp BD$. Show that



(i) $AB^2 = BC \cdot BD$ (ii) $AC^2 = BC \cdot DC$ (iii) $AD^2 = BD \cdot CD$

Solution As proved in above question,

$$\Delta ABC \sim \Delta DAC \sim \Delta DBA$$

(i) $\Delta ABC \sim \Delta DBA$

Then, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBA} = \frac{AB^2}{DB^2}$$

(Using property of area of similar triangles)

$$\Rightarrow \quad \frac{\frac{1}{2}(BC) \times (AC)}{\frac{1}{2}(BD) \times (AC)} = \frac{AB^2}{DB^2}$$

(Area of triangle $= \frac{1}{2} \times$ Base $\times$ Height)

$$\Rightarrow \quad AB^2 = BC \cdot BD$$

(ii) $\Delta ABC \sim \Delta DAC$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DAC)} = \frac{AC^2}{DC^2}$$

(Using property of area of similar triangles)

$$\Rightarrow \quad \frac{\frac{1}{2}(BC) \times (AC)}{\frac{1}{2}(DC) \times (AC)} = \frac{AC^2}{DC^2}$$

(Area of triangle $= \frac{1}{2} \times$ Base $\times$ Height)

$$\Rightarrow \quad AC^2 = BC \cdot DC$$

(iii) $\Delta DAC \sim \Delta DBA$

$$\Rightarrow \quad \frac{ar(\Delta DAC)}{ar(\Delta DAC)} = \frac{DA^2}{DB^2}$$

(Using property of area of similar triangles)

$$\Rightarrow \quad \frac{\frac{1}{2}(CD) \times (AC)}{\frac{1}{2}(BD) \times (AC)} = \frac{AD^2}{BD^2}$$

(Area of triangle $= \frac{1}{2} \times$ Base $\times$ Height)

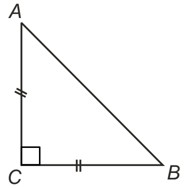
$$\Rightarrow \quad AD^2 = BD \cdot CD$$

Hence proved.

Question 4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Solution Draw ABC is an isosceles triangle right angled at C.

and $\quad AC = BC \quad$ ...(i)



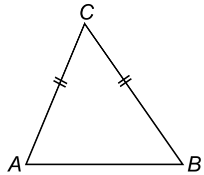
By Pythagoras theorem, we have

$$AB^2 = AC^2 + BC^2 = AC^2 + AC^2 = 2AC^2 \quad [\because BC = AC \text{ by Eq. (i)}]$$

Hence proved.

Question 5. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Solution Draw an isosceles ΔABC with $AC = BC$.



In ΔABC, we are given that

$$AC = BC \quad \ldots(i)$$

and $$AB^2 = 2AC^2 \quad \ldots(ii)$$

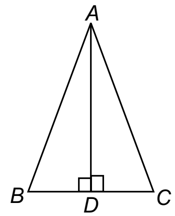
Now, $$AC^2 + BC^2 = AC^2 + AC^2 \quad \text{[By Eq. (i)]}$$

$$= 2AC^2 = AB^2 \quad \text{[By Eq. (ii)]}$$

i.e., $$AC^2 + BC^2 = AB^2$$

Hence, by the converse of the Pythagoras theorem, we have ΔABC is right angled at C.

Question 6. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.



Solution Draw equilateral ΔABC, each side is $2a$.

Also, draw $AD \perp BC$.

Where AD is an altitude.

In ΔADB and ΔADC

$$AD = AD \quad \text{(Common)}$$

and $$\angle ADB = \angle ADC = 90^\circ$$

$$\Delta ADB \cong \Delta ADC \quad \text{(RHS congruency)}$$

$\Rightarrow$ $$BD = CD = \frac{1}{2}BC = a$$

($\because$ in an equilateral triangle altitude AD is the perpendicular bisector of BC).

Now, from ΔABD by Pythagoras theorem, we get

$$AB^2 = AD^2 + BD^2$$

$\Rightarrow$ $$(2a)^2 = AD^2 + a^2$$

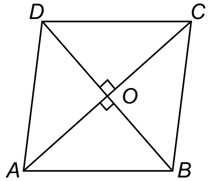
$\Rightarrow$ $$AD^2 = 3a^2$$

$\Rightarrow$ $$AD = \sqrt{3}a$$

Question 7. Prove that the sum of the square of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Solution Draw $ABCD$ is a rhombus in which $AB = BC = CD = DA = a$ (Say)

Its diagonal AC and BD are right angled bisector of each other at O.



In ΔOAB, $\angle AOB = 90^\circ$,

$$OA = \frac{1}{2}AC \text{ and } OB = \frac{1}{2}BD$$

In ΔAOB, use Pythagoras theorem, we have

$$OA^2 + OB^2 = AB^2$$

$$\Rightarrow \quad \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = AB^2$$

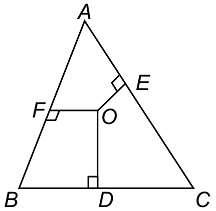
$$\Rightarrow \quad AC^2 + BD^2 = 4AB^2$$

or $$4AB^2 = AC^2 + BD^2$$

$$\Rightarrow \quad AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 \quad (\because AB = BC = CD = DA)$$

Hence proved.

Question 8. In figure, O is a point in the interior of a ΔABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

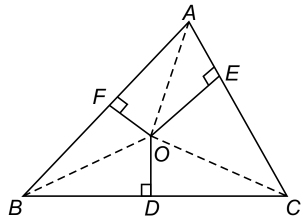


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Solution In ΔABC, from point O join lines OB, OC and OA.

(i) In right angled Δ OFA,



$$OA^2 = OF^2 + AF^2 \quad \text{(By Pythagoras theorem)}$$

$$\Rightarrow \quad OA^2 - OF^2 = AF^2 \quad \ldots\text{(i)}$$

Similarly, in ΔOBD, $OB^2 - OD^2 = BD^2$ ...(ii)

and in ΔOCE, $OC^2 - OE^2 = CE^2$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$$

(ii) From part Eq. (i), we get

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2 \quad \ldots\text{(iv)}$$

Similarly,

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = BF^2 + CD^2 + AE^2 \quad \ldots\text{(v)}$$

From Eqs. (iv) and (v), we have

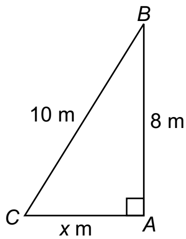
$$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$$

Question 9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Solution Let B be the position of the window and CB be the length of the ladder.

Then, $AB = 8$ m (Height of window)

$BC = 10$ m (Length of ladder)



Let $AC = x$ m be the distance of the foot of the ladder from the base of the wall.

Using Pythagoras theorem in ΔABC, we get

$$AC^2 + AB^2 = BC^2$$

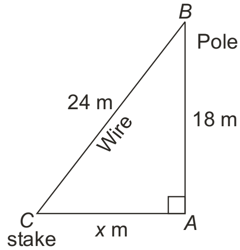
$\therefore$ $x^2 + (8)^2 = 1 + (10)^2$

$\Rightarrow$ $x^2 = 100 - 64 = 36$

$\Rightarrow$ $x = 6$, *i.e.*, $AC = 6$ m

Question 10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Solution Let AB be the vertical pole of height 18 m. A guy wire is of length $BC = 24$ m.



Let $AC = x$ m be the distance of the stake from the base of the pole.

Using Pythagoras theorem in ΔABC, we get

i.e., $AC^2 + AB^2 = BC^2$

$\therefore$ $x^2 + (18)^2 = (24)^2$

$\Rightarrow$ $x^2 = (24)^2 - (18)^2$

$= 576 - 324$

$= 252$

$\Rightarrow$ $x = \sqrt{252}$ m

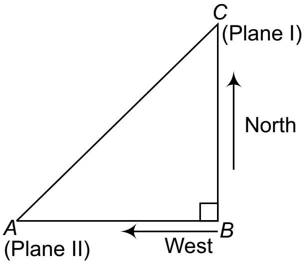
($\because$ We take positive sign because cannot be negative)

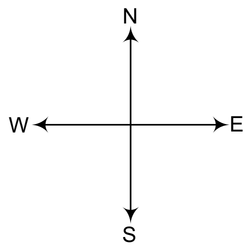
$\Rightarrow$ $x = 6\sqrt{7}$ m

Question 11. An aeroplane leaves an airport and flies due North at a speed of 1000 kmh^{-1}. At the same time, another aeroplane leaves the same airport and flies due West at a speed of 1200 kmh^{-1}. How far apart will be two planes after $1\frac{1}{2}$ h?

Solution The first plane travels distance BC in the direction of North in $1\frac{1}{2}$ h at speed of 1000 km/h.

$\therefore$ $$BC = 1000 \times \frac{3}{2} \text{ km}$$
$$= 1500 \text{ km}$$





The second plane travels distance BA in the direction of West in $1\frac{1}{2}$ h at a speed of 1200 km/h.

$\therefore$ $$BA = 1200 \times \frac{3}{2} = 1800 \text{ km}$$

In right angled ΔABC,

$$AC^2 = AB^2 + BC^2 \qquad \text{(By Pythagoras theorem)}$$
$$= (1800)^2 + (1500)^2$$
$$= 3240000 + 2250000$$
$$= 5490000$$

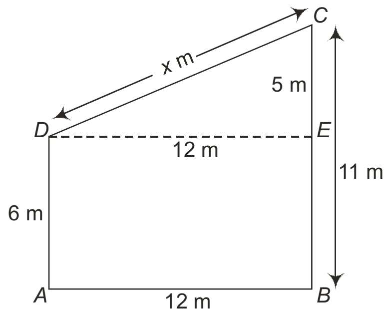
$\Rightarrow$ $$AC = \sqrt{5490000} \text{ m}$$

$\Rightarrow$ $$AC = 300\sqrt{61} \text{ m}$$

Question 12. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the foot of the poles is 12 m, find the distance between their tops.

Solution Let BC and AD be the two poles of heights 11 m and 6 m .

Then, $$CE = BC - AD$$
$$= 11 - 6$$
$$= 5 \text{ cm}$$



Let distance between tops of two poles $DC = x$ m

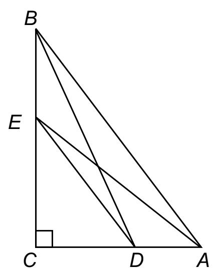
Using Pythagoras theorem in ΔDEC, we get

i.e., $DC^2 = DE^2 + CE^2 \Rightarrow x^2 = (12)^2 + (5)^2 = 169 \Rightarrow x = 13$

Hence, distance between their tops = 13 m

Question 13. D and E are points on the sides CA and CB, respectively of a ΔABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Solution Draw a right ΔABC at C. Take D and E points on the sides CA and BC and join ED, BD and EA.



In right angled ΔACE,

$$AE^2 = CA^2 + CE^2 \quad \ldots(i)$$

(By Pythagoras theorem)

and in right angled ΔBCD,

$$BD^2 = BC^2 + CD^2 \quad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$AE^2 + BD^2 = (CA^2 + CE^2) + (BC^2 + CD^2) = (BC^2 + CA^2) + (CD^2 + CE^2)$$

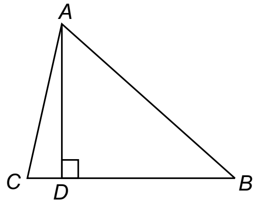
($\because$ In ΔABC, $BA^2 = BC^2 + CA^2$ and In ΔECD, $DE^2 = CD^2 + CE^2$)

$$= BA^2 + DE^2$$ (By Pythagoras theorem)

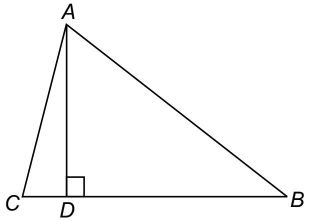
$\therefore$ $AE^2 + BD^2 = AB^2 + DE^2$

Hence proved.

Question 14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that $DB = 3\,CD$ (see in figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Solution



Given, $DB = 3CD$

$\Rightarrow$ $CD = \frac{1}{4}BC$ ...(i)

and $DB = \frac{3}{4}BC$

In ΔABD, $AB^2 = DB^2 + AD^2$ ...(ii)

In ΔACD, $AC^2 = CD^2 + AD^2$ (By Pythagoras theorem) ...(iii)

On subtracting Eq. (iii) from Eq. (ii), we get

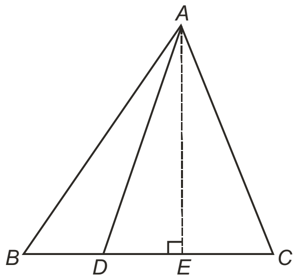
$$AB^2 - AC^2 = DB^2 - CD^2$$
$$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$
$$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{1}{2}BC^2$$

$\Rightarrow$ $2AB^2 - 2AC^2 = BC^2 \quad \Rightarrow 2AB^2 = 2AC^2 + BC^2$

Hence proved.

Question 15. In an equilateral $\Delta ABC, D$ is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Solution Draw ABC is an equilateral triangle, D is a point on side BC such that $BD = \frac{1}{3}BC$. Draw a line AE is perpendicular to BC.



$$AB = BC = CA = a \quad \text{(Say)}$$

(By property of equilateral triangle)

$$BD = \frac{1}{3}BC = \frac{1}{3}a$$

$$\Rightarrow \quad CD = \frac{2}{3}BC = \frac{2}{3}a$$

$$\because \quad AE \perp BC$$

$$\Rightarrow \quad BE = EC = \frac{1}{2}a$$

($\because$ In an equilateral triangle altitude AE is perpendicular bisector of BC.)

$$DE = BE - BD = \frac{1}{2}a - \frac{1}{3}a = \frac{1}{6}a$$

Using Pythagoras theorem in ΔADE,

$$AD^2 = AE^2 + DE^2$$

$$= AB^2 - BE^2 + DE^2$$

($\because$ Right ΔABE, $AE^2 = AB^2 - BE^2$)

$$= a^2 - \left(\frac{1}{2}a\right)^2 + \left(\frac{1}{6}a\right)^2$$

$$= a^2 - \frac{1}{4}a^2 + \frac{1}{36}a^2$$

$$= \frac{(36 - 9 + 1)a^2}{36}$$

$$= \frac{28}{36}a^2$$

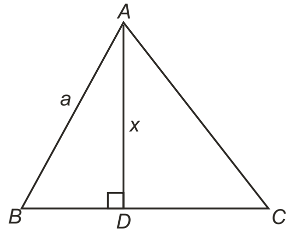
$$= \frac{7}{9}AB^2$$

$$\Rightarrow \quad 9AD^2 = 7AB^2$$

Hence proved.

Question 16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Solution Draw ΔABC is an equilateral triangle of side a. (Say)



and $AD \perp BC$

Let $AD = x$

Now, $BD = CD = \frac{1}{2} BC = \frac{1}{2} a$

(In an equilateral triangle altitude AD is a perpendicular bisector of BC)

In right angled ΔABD,

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad a^2 = x^2 + \left(\frac{1}{2}a\right)^2 \Rightarrow a^2 = x^2 + \frac{1}{4}a^2$$

$$\Rightarrow \quad 4a^2 = 4x^2 + a^2 \quad \Rightarrow \quad 3a^2 = 4x^2$$

Hence proved.

Question 17. Tick the correct answer and justify : In ΔABC, $AB = 6\sqrt{3}$, $AC = 12$ cm and $BC = 6$ cm. The angle B is :

(a) 120° (b) 60° (c) 90° (d) 45°

Solution (c)

Given $BC = 6$ cm and $AB = 6\sqrt{3}$ cm and $AC = 12$ cm

Now, $AB^2 + BC^2 = (6\sqrt{3})^2 + (6)^2$

$= 108 + 36 = 144 = (12)^2 = (AC)^2$

$\Rightarrow \Delta ABC$ is right angled at B

$\Rightarrow \quad \angle B = 90°$

Also, $BC < AB$

$\Rightarrow \angle A$ is less than $\angle C$

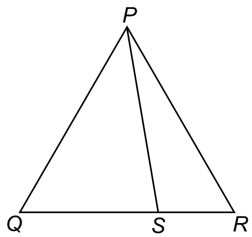
$\Rightarrow \angle A$ cannot be more than 45°

$\Rightarrow \quad \angle A = 30°$

$\Rightarrow \quad \angle B = 90° - 30° = 60°.$

Exercise 4.6 (Optional)*

Question 1. In figure, PS is the bisector of $\angle QPR$ of ΔPQR, prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.

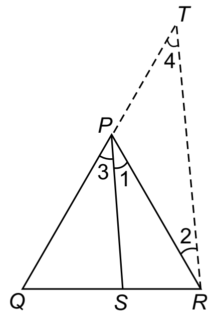


Solution Given, in figure, PS is the bisector of $\angle QPR$ of ΔPQR. Now, draw $RT \parallel SP$ to meet QP produced in T.

Proof $\because$ $RT \parallel SP$ and transversal PR intersects them

$\therefore$ $\angle 1 = \angle 2$ (Alternate interior angle)...(i)

$\because$ $RT \parallel SP$ and transversal QT intersects them



$\therefore$ $\angle 3 = \angle 4$ (Corresponding angle) ...(ii)

But $\angle 1 = \angle 3$ (Given)

$\therefore$ $\angle 2 = \angle 4$ [From Eqs. (i) and (ii)]

$\therefore$ $PT = PR$ ...(iii)

($\because$ Sides opposite to equal angles of a triangle are equal)

Now, in ΔQRT,

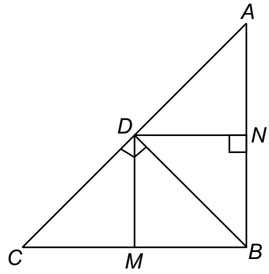
$PS \parallel RT$ (By construction)

$\therefore$ $\frac{QS}{SR} = \frac{PQ}{PT}$ (By basic proportionally theorem)

$\Rightarrow$ $\frac{QS}{SR} = \frac{PQ}{PR}$ [From Eq. (iii)]

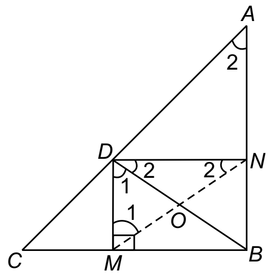
Question 2. In figure, D is a point on hypotenuse AC of ΔABC, such that $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that

(i) $DM^2 = DN \cdot MC$ (ii) $DN^2 = DM \cdot AN$



Solution Given that, D is a point on hypotenuse AC of ΔABC, $DM \perp BC$ and $DN \perp AB$.

Now, join NM. Let BD and NM intersect at O.



Proof

(i) In ΔDMC and ΔNDM,

$$\angle DMC = \angle NDM \quad \text{(Each equal to 90°)}$$

$$\angle MCD = \angle DMN$$

Let $MCD = \angle 1$

Then, $\angle MDC = 90° - \angle 1$

$(\because \angle MCD + \angle MDC + \angle DMC = 180°)$

$\therefore$ $\angle ODM = 90° - (90° - \angle 1)$

$= \angle 1$

$\Rightarrow$ $\angle DMN = \angle 1$

$\therefore$ $\Delta DMO \sim \Delta NDM$ (AA similarity criterion)

$\therefore$ $\dfrac{DM}{ND} = \dfrac{MC}{DM}$

(Corresponding sides of the similar triangles are proportional)

$\Rightarrow$ $DM^2 = MC \cdot ND$

(ii) In ΔDNM and ΔNAD,

$$\angle NDM = \angle AND \quad \text{(Each equal to } 90^\circ\text{)}$$
$$\angle DNM = \angle NAD$$

Let $\angle NAD = \angle 2$

Then, $\angle NDA = 90^\circ - \angle 2$

$$(\because \angle NDA + \angle DAN + \angle DNA = 180^\circ)$$

$\therefore$ $\angle ODN = 90^\circ - (90^\circ - \angle 2) = \angle 2$

$\therefore$ $\angle DNO = \angle 2$

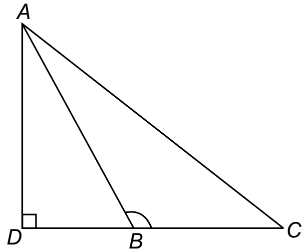
$\therefore$ $\Delta DNM \sim \Delta NAD$ (AA similarity criterion)

$\therefore$ $\dfrac{DN}{NA} = \dfrac{DM}{ND}$

$\Rightarrow$ $\dfrac{DN}{AN} = \dfrac{DM}{DN}$

$\Rightarrow$ $DN^2 = DM \times AN$

Question 3. In figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 = 2BC \cdot BD$.



Solution Given that, in figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced.

Proof In right ΔABC,

$\because$ $\angle D = 90^\circ$

$\therefore$ $AC^2 = AD^2 + DC^2$ (By Pythagoras theorem)

$= AD^2 + (BD + BC)^2$ $[\because DC = DB + BC]$

$= (AD^2 + DB^2) + BC^2 + 2BD \cdot BC$

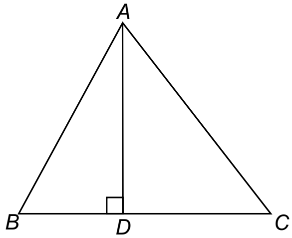
$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= AB^2 + BC^2 + 2BC \cdot BD$

($\because$ In right ΔADB with $\angle D = 90^\circ$, $AB^2 = AD^2 + DB^2$)

(By Pythagoras theorem)

Question 4. In figure, ABC is a triangle in which $\angle ABC < 90°$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 = 2BC \cdot BD$.



Solution Given that, in figure, ABC is a triangle in which $\angle ABC < 90°$ and $AD \perp BC$.

Proof In right ΔADC,

$\therefore$ $\angle D = 90°$

$\therefore$ $AC^2 = AD^2 + DC^2$ (By Pythagoras theorem)

$= AD^2 + (BC - BD)^2$ ($\because BC = BD + DC$)

$= AD^2 + BC^2 + BD^2 - 2BC \cdot BD$ [$\because (a-b)^2 = a^2 + b^2 - 2ab$]

$= (AD^2 + BD^2) + BC^2 - 2BC \cdot BD$

$= AB^2 + BC^2 - 2BC \cdot BD$

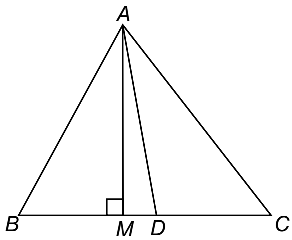
{$\therefore$ In right ΔADB with $\angle D = 90°$, $AB^2 = AD^2 + BD^2$} (By Pythagoras theorem)

Question 5. In figure, AD is a median of a ΔABC and $AM \perp BC$. Prove that

(i) $AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Solution Given that, in figure, AD is a median of a ΔABC and $AM \perp BC$.

Proof (i) In right ΔAMC,

$\because$ $\angle M = 90°$

$\therefore$ $AC^2 = AM^2 + MC^2$ (By Pythagoras theorem)

$= AM^2 + (MD + DC)^2$ ($\because MC = MD + DC$)

$= (AM^2 + MD^2) + DC^2 + 2MD \cdot DC$

[$\because (a+b)^2 = a^2 + b^2 + 2ab$]

$= AD^2 + DC^2 + 2DC \cdot MD$

[$\because$ In right ΔAMD with $\angle M = 90$, $AM^2 + MD^2 = AD^2$ (By Pythagoras theorem)]

$$= AD^2 + \left(\frac{BC}{2}\right)^2 + 2\left(\frac{BC}{2}\right)\cdot DM$$

[$\because 2DC = BC$ (AD is a median of ΔABC)]

$$\therefore \quad AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC\cdot DM \quad \ldots(i)$$

(ii) In right ΔAMB,

$\because \quad \angle M = 90°$

$\therefore \quad AB^2 = AM^2 + MB^2$ (By Pythagoras theorem)

$= AM^2 + (BD - MD)^2$ [$\because BD = BM + MD$]

$= AM^2 + BD^2 + MD^2 - 2BD\cdot MD$

[$\because (a-b)^2 = a^2 + b^2 - 2ab$]

$= (AM^2 + MD^2) + BD^2 - 2BD\cdot MD$

$= AD^2 + BD^2 - 2BD\cdot MD$

[$\because$ In right ΔAMD with $\angle M = 90°$,

$AM^2 + MD^2 = AD^2$ (By Pythagoras theorem]

$$= AD^2 - 2\left(\frac{BC}{2}\right)\cdot DM + \left(\frac{BC}{2}\right)^2$$

($\because 2BD = BC$, AD is a median of ΔABC)

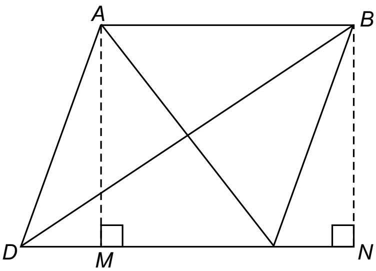
$$\therefore \quad AB^2 = AD^2 - BC\cdot DM + \left(\frac{BC}{2}\right)^2 \quad \ldots(ii)$$

(iii) On adding Eqs. (i) and (ii), we get

$$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}(BC)^2.$$

Question 6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Solution Given that, $ABCD$ is a parallelogram whose diagonals are AC and BD.



Now, draw $AM \perp DC$ and $BN \perp D$ (produced).

Proof In right ΔAMD and ΔBNC,

$$AD = BC \quad \text{(Opposite sides of a parallelogram)}$$
$$AM = BN$$
(Both are altitudes of the same parallelogram to the same base)

$\therefore \quad \Delta AMD \cong \Delta BNC$ (RHS congruence criterion)

$\therefore \quad MD = NC$ (CPCT)...(i)

In right ΔBND,

$\because \quad \angle N = 90^\circ$

$\therefore \quad BD^2 = BN^2 + DN^2$ (By Pythagoras theorem)

$= BN^2 + (DC + CN)^2$ $(\because DN = DC + CN)$

$= BN^2 + DC^2 + CN^2 + 2DC \cdot CN$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= (BN^2 + CN^2) + DC^2 + 2DC \cdot CN$

$= BC^2 + DC^2 + 2DC \cdot CN$ ...(ii)

$(\because$ In right ΔBNC with $\angle N = 90^\circ)$

$BN^2 + CN^2 = BC^2$ (By Pythagoras theorem)

In right ΔAMC, $\angle M = 90^\circ$

$\therefore \quad AC^2 = AM^2 + MC^2$ $(\because MC = DC - DM)$

$= AM^2 + (DC - DM)^2$ $[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$= AM^2 + DC^2 + DM^2 - 2DC \cdot DM$

$= (AM^2 + DM^2) + DC^2 - 2DC \cdot DM$

$= AD^2 + DC^2 - 2DC \cdot DM$

[$\because$ In right triangle AMD with $\angle M = 90^\circ$, $AD^2 = AM^2 + DM^2$ (By Pythagoras theorem)]

$= AD^2 + AB^2 = 2DC \cdot CN$ ...(iii)

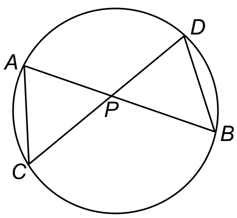
[$\because DC = AB$, opposite sides of parallelogram and $BM = CN$ from Eq. (i)]

Now, on adding Eqs. (iii) and (ii), we get

$$AC^2 + BD^2 = (AD^2 + AB^2) + (BC^2 + DC^2)$$
$$= AB^2 + BC^2 + CD^2 + DA^2$$

Question 7. In figure, two chords AB and CD intersect each other at the point P. Prove that

(i) $\Delta APC \sim \Delta DPB$ (ii) $AP \cdot PB = CP \cdot DP$



Solution Given that, in figure, two chords AB and CD intersects each other at the point P.

Proof (i) ΔAPC and ΔDPB

$$\angle APC = \angle DPB \quad \text{(Vertically opposite angles)}$$

$$\angle CAP = \angle BDP \quad \text{(Angles in the same segment)}$$

$$\therefore \quad \Delta APC \sim \angle DPB \quad \text{(AA similarity criterion)}$$

(ii) $$\Delta APC \sim \Delta DPB \quad \text{[Proved in (i)]}$$

$$\therefore \quad \frac{AP}{DP} = \frac{CP}{BP}$$

(∵ Corresponding sides of two similar triangles are proportional)

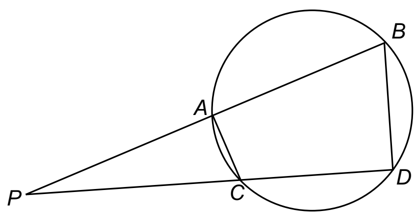
$$\Rightarrow \quad AP \cdot BP = CP \cdot DP$$

$$\Rightarrow \quad AP \cdot PB = CP \cdot DP$$

Question 8. In figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\Delta PAC \sim \Delta PDB$

(ii) $PA \cdot PB = PC \cdot PD$



Solution Given that, in figure, two chords AB and CD of a circle intersect each other at the point P (when produced) out the circle.

Proof (i) We know that, in a cyclic quadrilaterals, the exterior angle is equal to the interior opposite angle.

Therefore, $$\angle PAC = \angle PDB \quad \dots\text{(i)}$$

and $$\angle PCA = \angle PBD \quad \dots\text{(ii)}$$

In view of Eqs. (i) and (ii), we get

$$\Delta PAC \sim \Delta PDB \quad \text{(Similar)}$$

(∵ AA similarity criterion)

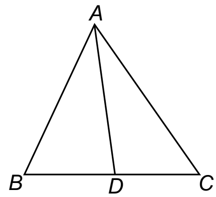
(ii) $\Delta PAC \sim \Delta PDB$ [Proved in (i)]

$$\therefore \quad \frac{PA}{PD} = \frac{PC}{PB}$$

(∵Corresponding sides of the similar triangles are proportional)

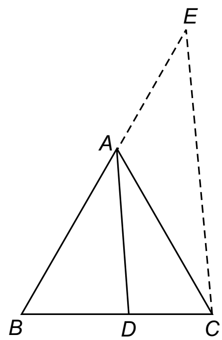
$$\Rightarrow \quad PA \cdot PB = PC \cdot PD$$

Question 9. In figure, D is a point on side BC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of $\angle BAC$.



Solution Given that, D is a point on side BC of ΔABC such that $\frac{BD}{CD} = \frac{AB}{AC}$.

Now, from BA produce cut off $AE = A$. Join CE.



Proof $$\frac{BD}{CD} = \frac{AB}{AC} \quad \text{(Given)}$$

$\Rightarrow$ $$\frac{BD}{CD} = \frac{AB}{AE} \quad [\because AC = AE \text{ (by construction)}]$$

$\therefore$ In ΔBCE,

$$AD \parallel CE$$

(By converse of basic proportionality theorem)

$\therefore$ $$\angle BAD = \angle AEC \quad \text{(Corresponding angle)...(i)}$$

and $$\angle CAD = \angle ACE \quad \text{(Alternate interior angle)...(ii)}$$

$\because$ $$AC = AE \quad \text{(By construction)}$$

$\therefore$ $$\angle AEC = \angle ACE \quad \text{...(iii)}$$

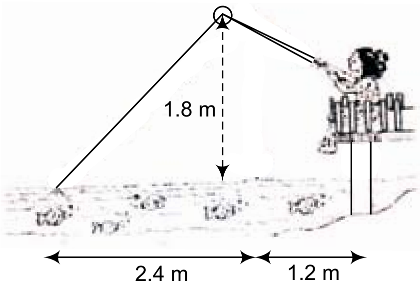
(Angles opposite equal sides of a triangle are equal)

Using Eqs. (i), (ii) and (iii), we get

$$\angle BAD = \angle CAD$$

i.e., AD is the bisector of $\angle BAC$.

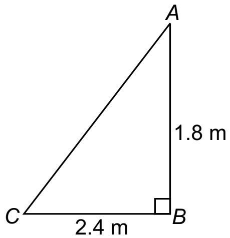
Question 10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out? If she pulls in the string at the rate of 5 cms^{-1}, what will be the horizontal distance of the fly from her after 12 s?



Solution Length of the string that she has out

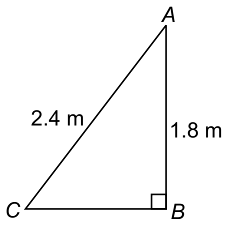
$$= \sqrt{(1.8)^2 + (2.4)^2} \qquad \text{(Using Pythagoras theorem)}$$

$$= \sqrt{3.24 + 5.76} = 3\text{ m}$$



Hence, she has 3 m string out.

Length of the string pulled in 12 s = 5 × 12 = 60 cm = 0.6 m



∴ Length of remaining string left out = 3.0 – 0.6 = 2.4 m

$$BD^2 = AD^2 - AB^2 \qquad \text{(By Pythagoras theorem)}$$

$$= (2.4)^2 - (1.8)^2 = 5.76 - 3.24 = 2.52$$

$$\Rightarrow \quad BD = \sqrt{2.52} = 1.59\text{ m} \qquad \text{(Approx.)}$$

Hence, the horizontal distance of the fly from Nazima after 12 s

$$= 1.2 + 1.59 = 2.79\text{ m} \qquad \text{(Approx.)}$$